Abstract

The dissertation presents the analysis of specific properties of many–level quantum systems. By many–level system we mean the system, whose sub–systems are described by Hilbert space of dimension greater than 2. We call such states qudits. Two–level quantum systems (called qubits) are the easiest to analyse, therefore we know a lot about them as opposed to many–level systems. That's why we investigate the latter.

The purpose of this dissertation is to provide new tools for analysing many–level quantum states. Especially we are interested in characterizing their non–classical properties, such as entanglement.

In chapter 1 we introduce all the necessary concepts and definitions, that are used in the rest of this dissertation. We also discuss basic assumptions of local and realistic hidden variable models. Next, we introduce the EPR paradox, which played a fundamental role in the development of quantum information. We end chapter 1 with a discussion of Bell type inequality introduced by Clauser, Horn, Shimony and Holt.

Chapters 2 – 5 are based on results published in papers:

1. P. Kurzynski, A. Kolodziejski, W. Laskowski, M. Markiewicz, *Three-dimensional visualization of a qutrit*, Phys. Rev. A **93**, 062126 (2016).
2. M. Markiewicz, K. Kostrzewa, A. Kolodziejski, P. Kurzynski, W. Laskowski, *Investigating nonclassicality of many qutrits by symmetric two-qubit operators*, Phys. Rev. A **94**, 032119 (2016).
3. M. Markiewicz, A. Kolodziejski, Z. Puchala, A. Rutkowski, T. Tylec, W. Laskowski, *Unified approach to geometric and positive-map-based non-linear entanglement identifiers*, Phys. Rev. A **97**, 042339 (2018).
4. P. Kurzynski, W. Laskowski, A. Kolodziejski, K. F. Pal, J. Ryu, T. Vertesi, *Disproving hidden variable models with spin magnitude conservation*, arXiv:1806.06637.

In chapter 2 we present a method of visualisation of a three–level quantum state. First, we point out problems which arise when one tries to generalize the Bloch ball picture. Next, we use the isomorphism between spaces $\mathbb{C}^3$ and $\mathrm{Sym}(\mathbb{C}^2 \otimes \mathbb{C}^2)$ to construct a graphical representation of a qutrit. Every three–level system can be represented as the ellipsoid with the Bloch vector lying

inside it. We show that our method is well suited for distinguishing pure states from mixed states. We also give instructive examples and discuss the unitary evolution in our representation.

In chapter 3 we use the formalism developed in last chapter to analyse the CGLMP inequality for two qutrits. We express Bell operators corresponding to qutrit Bell inequalities in terms of symmetric two–qubit operators and analyse the maximal quantum violation of a given Bell inequality from the qubit perspective. We show that the two-qutrit CGLMP inequality can be seen as a combination of Merminâ€™s and CHSH qubit Bell inequalities, and therefore the optimal state violating this combination differs from the one which corresponds to the maximally entangled state of two qutrits.

In chapter 4 we use the Choi–JamioĹ‚kowski isomorphism to provide an analysis of whether there exists a relation between two of the most popular types of entanglement identifiers: the first one based on nonlinear entanglement identifier and the second one on positive maps. We show that each nonlinear entanglement identifier can be translated to a positive-map-based criterion. We provide examples of entanglement identifiers and positive maps and we investigate their efficiency in detection of entanglement.

In chapter 5 we re–examine hidden variable models. We argue that a physical HV model should not only be local and noncontextual, but should also obey the conservation laws. In case of spin systems the model should conserve angular momentum, i.e.,the length of spin vectors should be fixed. As a result, our model extends the nonclassical behaviour to a broader class of quantum states. Additional constraints put on a HV model lead to a stronger tests of local realism. It turns out that standard Bell inequalities are violated by fewer states.

The dissertation ends with a summary, where we gather all obtained results.