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Review of the doctoral dissertation of Mr. Sumit Rout entitled “On Advantages of Non-Classical Resources in Information Theoretic Tasks: Communication and Correlations”

The doctoral dissertation of Mr. Sumit Rout is devoted to advantages of non-classical resources in information-theoretic tasks, with particular emphasis on communication problems, correlation-assisted communication, and operational detection of non-classical measurements. The work is situated at the interface of quantum information theory and the foundations of quantum mechanics. The central theme is that the usefulness of a quantum or post-quantum resource is not an intrinsic property of the object alone, but is revealed by an operational scenarios based on communication tasks or characterization of correlations observed in multipartite quantum experiments. This is a timely and well-justified research topic. Understanding when and how quantum systems, entanglement, non-local correlations, or non-projective measurements outperform classical resources is important both for the conceptual foundations of quantum theory and for the development of future quantum technologies.

The dissertation is largly based on three research works in which Mr. Rout is the main author (two already published, one available as a preprint):

1. Rout, S., Sakharwade, N., Bhattacharya, S. S., Ramanathan, R., and Horodecki, P., “Unbounded quantum advantage in communication with minimal input scaling”, Physical Review Research 7, 023104 (2025).
2. Rout, S., Bhattacharya, S. S., and Horodecki, P., “Randomness-free detection of non-projective measurements: qubits & beyond”, New Journal of Physics 27, 033024 (2025).
3. Rout, S., Chaturvedi, A., Bhattacharya, S. S., and Horodecki, P., “Facets of Non-locality and Advantage in Entanglement-Assisted Classical Communication Tasks”, arXiv:2507.10830 (2025).

The list of the candidate’s publications also contains additional papers from his doctoral studies, showing that he has been active beyond the immediate scope of the dissertation.

Structure of the thesis

The dissertation has a conventional and readable structure. Chapter 1 motivates the problem of non-classical advantages in information processing and explains why communication, correlations, and measurements provide natural arenas for studying such advantages. The introductory part of the thesis is presented in chapter 2. It gives a useful overview of the basic notions needed later: quantum states and measurements, general probabilistic theories, no-signalling correlations, Bell inequalities, prepare- and measure scenarios, graph-theoretic notions such as cliques and orthogonal representations. The introduction is generally rigorous and written in an accessible way for a reader familiar with quantum information theory.

The core of the thesis is contained in Chapters 3, 4, and 5. Chapter 3 studies one-way prepare-and-measure tasks defined from orthogonality graphs and studies separations between quantum and classical resources in relation reconstruction. Chapter 4 studies randomness-free detection of measurements that are not simulable by the author’s class of projective-simulable measurements, first for qubits and then for qutrits, and also discusses an analogue for general probabilistic theories. Chapter 5 studies classical communication assisted by non-local correlations in a minimal prepare-and-measure scenario in which Bob has no input. It introduces the methods of wire-cutting and wire-reading and uses them to construct communication tasks witnessing non-locality-assisted advantage. Chapter 6 summarises the results and lists open problems. Appendices provide additional material on relaxed relation reconstruction and bounding quantum non-local correlations.

Main technical results

The most important technical contribution of the dissertation is, in my opinion, contained in Chapter 3. The author introduces communication tasks based on the distributed clique labelling problem for orthogonality graphs. For a maximum clique C, a binary coloring assigns value 1 to exactly one vertex of C and value 0 to all other vertices; the corresponding clique label is the index of the selected vertex. Alice receives a maximum clique and such a clique label, Bob receives another maximum clique, and Bob must output a clique label for his clique. The labels are consistent when they define compatible binary assignments on the two cliques: shared vertices receive the same value and adjacent vertices are not simultaneously assigned 1.

For the basic distributed computation task, Theorem 3.1 shows that there is no quantum advantage for the considered graphs: both classical and quantum one-way zero-error communication complexity without shared resources are $\log_2 \omega(G)$, where $\omega(G)$ is the clique number, i.e. the size of the largest complete subgraph. The main separation appears only for the stronger task of relation reconstruction, where Bob must not only avoid forbidden outputs but must also produce every output allowed by the relation with non-zero probability.

The key technical result is that, for suitable graphs G, relation reconstruction can be achieved with $\log_2 \omega(G)$ qubits whenever G has a faithful orthogonal representation in dimension $\omega(G)$. On the other hand, the classical communication cost without shared and private randomness for Alice grows as $\log_2 |V(G)|$ (number of vertices in G) and the classical cost without shared randomness is at least $\log_2 K(G)$, where $K(G)$ is the disjointness number of the G. Since $K(G)$ is lower-bounded by $\max\{\omega(G), \log_2 n + (1/2)\log_2\log_2 n + O(1)\}$ for graphs of order n, the classical cost grows while the quantum cost can remain constant. Theorem 3.4 shws that for graph families G_γ^(α,β), with fixed clique number β and growing number α of maximum cliques, the order $|V(G)| = \alpha(\beta - \gamma) + \gamma$ grows unboundedly while the clique number and faithful orthogonal range remain equal to β. Consequently the quantum cost remains $\log_2 \beta$ qubits, whereas the classical cost grows without bound. This is a clean example of unbounded quantum advantage in a communication-complexity-like task. It is also economical in input scaling: Alice's and Bob's inputs are specified by clique indices and a label, so for fixed β the input length scales only as $2 \log_2 \alpha + O(1)$, in contrast to earlier one-qubit versus $\log n$-bit separations where the input specification was much larger relative to the communication gap.

Chapter 3 also studies shared resources. Theorem 3.5 and the subsequent analysis show that, once communication is bounded, shared randomness can restore the ability to reconstruct the relation but the required amount may grow. For the disconnected-clique family G_disc^(α,β=2), the necessary shared randomness with a one-bit classical channel scales as $\log_2(\lceil \log_2 \alpha \rceil + 1)$. Theorem 3.7 gives an especially attractive comparison: for the same family, one e-bit of shared entanglement together with one bit of communication is sufficient, whereas the required shared randomness grows with α. This gives an unbounded separation between two kinds of shared resources in the same operational task.

Chapter 4 develops randomness-free schemes for detecting measurements that are not simulable by a single projective measurement followed by classical post-processing. The setup involves separated parties sharing systems of bounded local operational dimension and performing fixed measurements, so that no random choice of inputs is needed. The author first proves that correlations generated by the measurements he calls projective-simulable on d-dimensional quantum systems coincide with correlations achievable by shared classical systems of local dimension d. Therefore, any advantage in the corresponding tasks must come from measurements outside this class. The thesis then gives detection schemes for three- and four-outcome qubit measurements, including schemes robust against arbitrary depolarising noise, and proposes and numerically supports extensions to five-outcome qutrit measurements in bipartite and tripartite settings.

Chapter 5 studies correlation-assisted bounded classical communication. The author first introduces wire-cutting: from a communication task with a linear payoff he constructs an associated Bell functional. A violation of the corresponding Bell inequality implies an advantage in the communication task, and conversely, an advantage in the communication task gives a violation of the associated Bell inequality after cutting the classical wire. The chapter then introduces wire-reading, which uses the fact that a classical message can be read without disturbing it. With wire-reading, the author constructs families of tasks in which shared randomness is suboptimal while non-local correlations, in particular correlations lying on certain non-local faces of the no-signalling polytope, achieve the optimal payoff.

Critical comments

The thesis is rigorous and contains solid results. Nevertheless, I have a number of critical comments.

My main criticism concerns the terminology “projective-simulable measurement”. This terminology is already widely used in the literature, beginning with the work of Oszmaniec et al. from 2017, and it has a different meaning from the one used in the dissertation. The author acknowledges this difference in the preliminaries, but the choice of terminology remains unfortunate. The class of POVMs called projective-simulable in the thesis consists, essentially, of POVMs with commuting effects, i.e. POVMs obtained by post-processing the outcomes of a single standard projective measurement on the original system. Such post-processing generally requires local randomness. In my view, it is not sufficiently justified why this class deserves to be called projective-simulable, rather than, for example, “single-PVM post-processable” or “commuting-effect POVMs”. Because the term projective-simulable already has an established meaning, the terminology risks misleading readers about the scope of the results in Chapter 4.

A related point concerns the physical motivation of the definition used in Chapter 4. If the point is to treat projective measurements as classical resources in a randomness-free scenario, then it should be explained more carefully why randomising over different projective measurements, which is allowed in the usual definition of projective simulability, is excluded. Since the post-processing in the author’s definition already uses randomness, the operational distinction between post-processing randomness and randomness used to choose a projective measurement needs a clearer justification.

Another conceptual issue appears in the extension to general probabilistic theories. I do not think that the term “sharp measurement/effect” is used in a fully appropriate manner there. Sharpness should refer, at least operationally, to the possibility of obtaining a particular outcome deterministically on some state Defining sharpness simply through extremality of effects in the effect space is, in my opinion, too weak and may not capture the intended analogue of projective measurements.

At several places the author spends a significant amount of space proving statements that are either straightforward or close to tautological. One example is the proof that correlations generated by commuting measurements of the considered form coincide with correlations generated by classical systems. Another example is Theorem 5.1, where the wire-cutting equivalence is important conceptually, but the formal proof reads almost as a restatement of the definitions. Such results should perhaps be presented more economically, with greater emphasis on the intuition and on the non-trivial consequences.

The presentation is sometimes too technical in comparison with the amount of motivation provided. This is particularly visible in Chapters 4 and 5. Several tasks are defined by quite involved payoff functions or by conditions tailored to a specific family of correlations. Although this is mathematically legitimate, it makes the operational meaning of the tasks less transparent. The thesis would benefit from more explanation of why these tasks are natural, how robust they are to modifications, and whether they can be related to better-known communication or certification problems.

There thesis also suffers from a number of editorial and notational issues. Examples include typographical errors in the list of symbols, such as “vetex”, “Heritian”, and “Measurment”, small grammatical problems such as “I would also like to Aditya”, “I(X:Y) denotes the mutual information between X adn Y”, and “paticular partial Boolean function”, and awkward sentences in several proofs, for instance “using the constraints established in the, the constraints in the proof of Theorem 3.1” or “we the define following the Bell functional”.

Questions

Besides the above criticism I have also a number of questions inspired by thesis content.

- In Chapter 4, is it possible to obtain analogous randomness-free detection results for projective-simulable POVMs in the usual sense of the term, i.e. POVMs obtained by randomizing over projective measurements followed by post-processing? If not, where exactly does the proof fail, and is the obstruction conceptual or merely technical?
- For the communication-complexity problems studied in Chapter 3, does the unbounded advantage survive in a bounded-error formulation? In particular, what happens if Bob is allowed to output elements outside the relation with some small probability, as would be inevitable in an experimental implementation?
- Faithful orthogonal representations play a crucial role in the quantum protocols in Chapter 3. Does every graph that admits an orthogonal representation admit a faithful orthogonal representation in the same dimension, or at least in a controlled larger dimension? If not, what is the precise class of graphs for which the thesis construction applies, and how restrictive is this assumption?
- Chapter 5 shows that some non-local correlations give an advantage in communication tasks and that violation of the tailored Bell inequality is necessary for advantage in the associated task. Are there non-local correlations outside the classical set that never yield an advantage in any bounded classical communication task of the type considered in the thesis? Or do you expect every non-local correlation to be useful for some suitably chosen communication task?
- The tasks in Chapter 5 are often highly tailored to non-local facets or extremal correlations. Can the same non-local resources be shown to be advantageous in more natural or less tailored communication problems, for example variants of channel simulation, random access coding, or communication complexity tasks?

Summary and recommendation

Despite the reservations listed above, I evaluate the dissertation positively. The strongest part of the thesis, Chapter 3, contains a clear and original technical result: an unbounded separation between quantum and classical communication resources in relation reconstruction tasks defined from orthogonality graphs, together with a further separation between entanglement assistance and shared-randomness assistance. This result alone constitutes a substantial contribution to quantum information theory. Chapters 4 and 5 add further interesting results on randomness-free measurement detection and on the relation between non-locality and communication advantage.

My critical remarks concern mainly terminology, motivation and presentation. They do not undermine the main scientific achievements of the dissertation. In conclusion, I state that the doctoral dissertation of Mr. Sumit Rout” contains original and valuable scientific results and satisfies the requirements expected of a PhD thesis. I therefore recommend that the dissertation be accepted and that Mr. Sumit Rout be admitted to the next stages of the doctoral procedure.

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