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Review on the doctoral dissertation of Gerardo Suárez

I had the pleasure to read the thesis *Open Quantum Systems Beyond the Markovian Regime* for the Degree Doctor of philosophy, in Physics submitted by Gerardo Suárez in June 2025.

The thesis gives a very detailed description of concepts to describe of open quantum system. This is a quickly developing and notoriously intricate field. It contains a vast amount of different approaches whose relation to each other is far from clear. Gerado Suárez accomplishes this challenge very well with only few shortcomings which are unavoidable in such a Herculean task.

For me the most interesting issue is the relevance of bath parametrization. Here, Gerado Suárez has made important technical contributions within the Qutip package. This package is well known in the community and widely used (including us) all over the world. Thus such work needs to be highly regards. Bath parametrization are a recurrent topic in the thesis, which clearly explains , where these methods turn out to very useful. In particular Gerado Suárez nicely discusses, how the use of complex bath frequencies enables the practical implementation of the HEOM (hierarchical equations of motion) methods as the higher order expression can be evaluated in matrices with respect to the bath parameters. A further application addressed is the reduced cost of Lamb-shift calculation.

Further original achievements reported in the thesis are the observation that the lamb shift is relevant for energy transport in the two-qubit model and several benchmark calculation assessing the accurateness and numerical effort for different approaches. All these are useful contributions to the advance of the field.

Some issue could have been formulated a bit more accurately. Examples are

- Eq. (6.25) seems to me not of GKLS type yet. This requires non-negative eigenvalues of the gamma matrix, which is far from obvious.

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- Chapter 15: The name non-equilibrium Spin-Bose model is misleading for the choice of a special Hamiltonian. (Non-equilibrium should relate to excitations)
- First line of section 16.2: Ref [43] did only show superiority compared to a few models studied and did not make a fully general claim, as indicated by the formulation here.
- At several placed data on computation time is presented. This is of high benefit for the reader. However, I am missing a brief summary of the underlying machinery and programming (degree of parallelization etc., software used).
- An ordinary amount of typos. I will provide a list with those I found directly to Gerado Suárez.

In summary, I strongly recommend the acceptance of the thesis, which contains significant progress in the field. A few optional improvements would be of benefit for the future reader.

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