Autoreferat

Dr John H. Selby

Summary of Professional Accomplishments

October 10, 2024

Contents

1 NAME AND SURNAME

John Selby

2 DIPLOMAS, DEGREES

- **PhD in Physics** January, 2018
  Thesis: *"A process theoretic triptych: two roads to the emergence of classicality, reconstructing quantum theory from diagrams, looking for post-quantum theories"*.
  Advisors: Prof. Bob Coecke and Prof. Terry Rudolph
  Centre for Doctoral Training in Controlled Quantum Dynamics
  Imperial College London, London, UK.

- **MRes in Controlled Quantum Dynamics** – Distinction (80%) November 2014
  Thesis: *"A process theory approach to possibilistic theories"*.
  Advisors: Prof. Bob Coecke and Prof. Terry Rudolph
  Centre for Doctoral Training in Controlled Quantum Dynamics
  Imperial College London, London, UK.

- **MSc in Theoretical Physics – Perimeter Scholars International** – Pass[1]September 2013.
  Thesis: *"Tomographic locality in reconstructions of quantum theory"*.
  Advisors: Prof. Lucien Hardy
  Perimeter Institute and University of Waterloo, Waterloo, Canada

- **MSci in Physics** – First Class Honours (81%) August 2012
  Thesis: *"Modelling methods for predictive microbiology"*.
  Advisors: Prof. Peter Török
  Imperial College London, London, UK.

- **ATCL Recital Diploma** – Distinction July 2008.
  Performance diploma for classical guitar
  Trinity College London

3 INFORMATION ON PREVIOUS EMPLOYMENT IN SCIENTIFIC INSTITUTIONS

- **Team leader** March 2022 - present.
  Compositional foundations of physics team
  International Centre for Theory of Quantum Technologies, University of Gdańsk, Poland.

- **Adiunkt** October 2019 - present.
  New Quantum Resources and Thermodynamics group
  International Centre for Theory of Quantum Technologies, University of Gdańsk, Poland.

[1] Note that this course is graded only pass/fail.

- **Postdoctoral Research Fellow** September 2017- September 2019
  Perimeter Institute for Theoretical Physics

4 DESCRIPTION OF THE ACHIEVEMENTS ACCORDING TO ART. 219 PARA 1 POINT 2 OF THE ACT

4.1 Title of the achievement

Single-themed series of publications, titled *Foundational insights on physics and information processing arising from the paradigm of generalised probabilistic theories.*

4.2 List of selected publications

The list of publications related thematically:

1. *Linear program for testing nonclassicality and an open-source implementation*
   John H. Selby, Elie Wolfe, David Schmid, Ana Belén Sainz, Vinicius P. Rossi
   Physical Review Letters, 132, 050202 (2024) – total pages: 7

2. *Accessible fragments of generalized probabilistic theories, cone equivalence, and applications to witnessing nonclassicality*
   John H. Selby, David Schmid, Elie Wolfe, Ana Belén Sainz, Ravi Kunjwal, Robert W. Spekkens
   Physical Review A, 107, 062203 (2023) – total pages: 21

3. *Contextuality without incompatibility*
   John H. Selby, David Schmid, Elie Wolfe, Ana Belén Sainz, Ravi Kunjwal, Robert W. Spekkens.
   Physical Review Letters, 130, 230201 (2023) – total pages: 6

4. *A structure theorem for generalized-noncontextual ontological models*
   David Schmid, John H. Selby, Matthew F. Pusey, Robert W. Spekkens
   Quantum 8, 1283 (2024) – total pages: 41

5. *Decomposing all multipartite non-signalling channels via quasiprobabilistic mixtures of local channels in generalised probabilistic theories*
   Paulo J. Cavalcanti, John H. Selby, Jamie Sikora, Ana Belén Sainz.
   Journal of Physics A: Mathematical and Theoretical 55 (40), 404001 (2022) – total pages: 28

6. *Uniqueness of noncontextual models for stabilizer subtheories*
   David Schmid, Haoxing Du, John H. Selby, Matthew F. Pusey.
   Physical Review Letters 129 (12), 120403 (2022) – total pages: 6

7. *A no-go theorem on the nature of the gravitational field beyond quantum theory*
   Thomas D. Galley, Flaminia Giacomini, John H. Selby.
   Quantum 6, 779 (2022) – total pages: 21

8. *Post-quantum steering is a stronger-than-quantum resource for information processing*
   Paulo J. Cavalcanti, John H. Selby, Jamie Sikora, Thomas D. Galley, Ana Belén Sainz.

npj Quantum Information 8 (1), 76 (2022) – total pages: 10

9. *Characterization of Noncontextuality in the Framework of Generalized Probabilistic Theories*
   David Schmid, John H. Selby, Elie Wolfe, Ravi Kunjwal, Robert W. Spekkens.
   PRX Quantum 2, 010331 (2021) – total pages: 13

10. *Impossibility of coin flipping in generalized probabilistic theories via discretizations of semi-infinite programs*
    Jamie Sikora, John H. Selby.
    Physical Review Research 2 (4), 043128 (2020) – total pages: 7

11. *Compositional resource theories of coherence*
    John H. Selby, Ciarán M. Lee.
    Quantum 4, 319 (2020) – total pages: 43

12. *How to make unforgeable money in generalised probabilistic theories*
    John H. Selby, Jamie Sikora
    Quantum 2, 103 (2018) – total pages: 27

13. *Simple proof of the impossibility of bit commitment in generalized probabilistic theories using cone programming*
    Jamie Sikora, John H. Selby.
    Physical review A 97 (4), 042302 (2018) – total pages: 5

5 PRESENTATION OF SIGNIFICANT SCIENTIFIC ACTIVITY

The scientific achievement is a part of collective publications. My contribution is described in point I2 of the annex "List of scientific or artistic achievements which present a major contribution to the development of a specific discipline". The contribution statements provided by the coauthors are presented in the attached documents.

Hereon, the publications that belong to the series will be cited with Roman letters, e.g. [A], publications by the applicant that do not correspond to the series will be cited with numbers, e.g. [1], and other references will be cited following author-year convention, e.g. [Bel64,KS67].

5.1 Introduction

Despite the fact that we are now over a century on from the crystallisation of the mathematical formalism of quantum theory [VN13], and that we are now becoming quite adept at using quantum phenomena for technological purposes, we are still a long way from having a complete understanding of what quantum theory is actually telling us about our world.

There is a long history of gaining insight into quantum theory by considering counterfactual alternatives, i.e., by considering quantum theory as just one potential theory amongst a vast landscape of hypothetical theories of nature. By doing so we can find out what makes quantum theory special, what singles it out from this landscape, what distinguishes it from classical physics, and what are its limitations? While this kind of research programme has a long history [Mac13, Lud12, Mie68, CMW00], it has always been a struggle to adequately deal with compositionality of physical theories [CMW00]. That is, to go beyond the study of a single system in isolation. Of course, on some level one can argue that the universe itself can be viewed as a single system in isolation and so this should suffice, but, with the advent of quantum information theory and the vital role of entanglement therein [HHHH09], together with the famous theorem of Bell [Bel64], we now know that understanding quantum compositionality is crucial to understanding the theory as a whole.

Much more recently, there has been the development of a new research programme known as Categorical Quantum Mechanics [AC09]. This provides a new mathematical formalism for quantum theory which puts composition to the fore. Technically, it is built on the mathematics of symmetric monoidal categories [ML13], but one of the great dividends of this is that one can use their representation as string diagrams [JS91, Sel11] to provide an intuitive language for proving results and for performing calculations [vdW20, CHKW22]. Although this formalism was originally motivated by foundational ideas [Coe11], it has nonetheless led to great progress in practical topics in quantum information theory, for example, it is now used by many companies in the quantum industry and underpinning state of the art quantum compilation tools such as tket [vdW20, CHKW22].

In my research I bring together these two aspects, viewing quantum theory as a particular theory within a landscape of compositional generalised probabilistic theories of physics. I am by no means the first to have done so, indeed I am very much following in the footsteps of Lucien Hardy [Har11] and the so-called Pavia group [CDP10], however, the research I present here, represents a collection of insights that I have obtained from this paradigm.

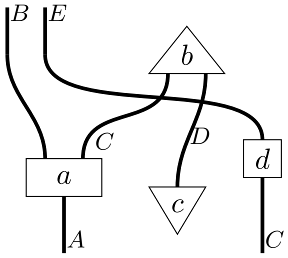
5.1.1 A brief introduction to generalised probabilistic theories

I will now briefly introduce the formalism of generalised probabilistic theories (GPTs) as it is one of the key tools – both conceptually and mathematically – underpinning this work. In this introduction (which I based on that of [B]) I will describe the mathematical structure of a GPT, illustrating this with two key examples, namely quantum theory and classical theory. This mathematical structure can also be derived from well motivated axioms (see, for example, my PhD thesis [1]) however I will not get into this here. There are two key aspects to formalism, a compositional aspect and a probabilistic aspect, which are demanded to interact with each other in a coherent way.

To being I will focus on the *compositional* aspect. This can be captured by the notion of a *process theory*[2] [Coe11], which came out of the study of categorical quantum mechanics [CK17]. There are two parts to a process theory, the *systems*, and the *processes* which act on these systems. Very conveniently, process theories have a diagrammatic representation which we will make use of here.

In this diagrammatic representation systems are depicted as labelled wires and processes as labelled boxes which have some input wires and some output wires. Here we follow the convention of drawing the inputs at the bottom and the outputs at the top. These processes can be wired together to form diagrams such as:

[2]For our purposes this can be thought of as being the same thing as a symmetric monoidal category, see Ref. [2], however, for a discussion of some of the subtleties of this.



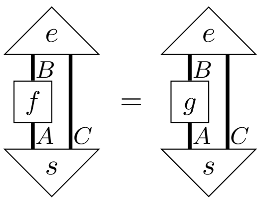
. (1)

Where such a diagram is itself a valid process in the theory; in this case with a *composite* system AC as an input and BE as output. In short, a process theory is a collection of processes which is closed under forming diagrams.

Importantly, however, that there are certain limitations on what constitutes a valid diagram, we must only wire an output of a particular type of system to an input of the same type, and we must not create any cycles in the diagram. Moreover, there is a degree of redundancy in the graphical depiction of a given diagram, that is, the precise position that we draw each process on the page is irrelevant, all that matters is the *connectivity* of the diagram, i.e., how the processes are wired to each other and also to the 'boundary' of the diagram.

Generalised probabilistic theories are a particular kind of process theory in which we think of the systems as representing physical systems (for example, quantum systems or classical systems) and the processes as being physical processes which act on these (for example, unitary transformations). In this case, we think of processes which have no input as representing the *state* of the system, and those with no outputs as *effects* (i.e., a particular outcome of a demolition measurement). When we, for instance, wire together a state and an effect for a given system, then we obtain a process with neither inputs nor outputs, this is taken to be a number in the unit interval, and conceptually we think of this as being the probability that the outcome occurs given the state of the system. At this point, however, we have not actually imposed any suitable normalisation constraints, and so at this point general scalars (i.e., processes with neither inputs nor outputs) are given by arbitrary non-negative real numbers.

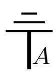
Very importantly, especially for Sec. 5.5, is the fact that the processes in a GPT do not contain all of the information about the physical implementation of the process, they only contain information which is accessible by doing tomography on the inputs and outputs of the process. In terms of the GPT this means that we can characterise processes by probabilities, that is, $f = g$ if and only if



$\forall C, s, e.$ (2)

We return to this point in Sec. 5.5 where we discuss the relevance of this for the study of generalised noncontextuality [Spe05].

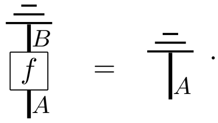
Generalised probabilistic theories also come with a way to discard (or simply to ignore) a given system. We denote this by a special effect for each system:



. (3)

For consistency we demand that discarding a pair of systems together is the same as discarding each of them individually, and that discarding nothing is represented by the number 1. In quantum theory discarding is provided by the (partial) trace operation.

These discarding effects allow us to characterise processes as being *discard-preserving* (i.e., the analogue of trace-preserving in quantum theory), which means they are the kind of process which we can choose to implement, via the constraint

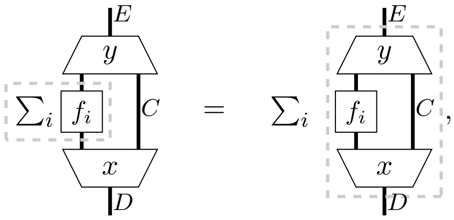


(4)

One can show that the only discard-preserving effects are the discard effects themselves, and that the only discard-preserving scalar is the number 1 [CK18]. Moreover, it can be shown that discard-preserving processes are closed under forming diagrams [CK18].

It is also often useful to consider processes which are *discard-nonincreasing* (i.e., the analogue of trace-nonincreasing in quantum theory), which are those that we cannot directly choose to implement but which occur as one of some number of possibilities.

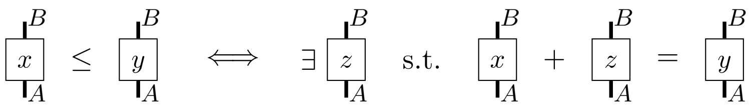
In order to formally capture this idea we need to define the notion of a diagrammatic sum of processes. What distinguishes a diagrammatic sum from any other kind, is that we require it to interact well with forming diagrams via the following distributivity condition:



(5)

for all x, y, C, D, and E.

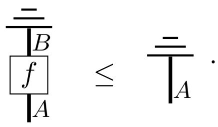
This allows us to define a partial order on our processes, which in turn will allow us to define the discard-nonincreasing condition. Let us define:



(6)

where z is another process in the theory.

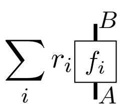
Using this partial order we can then define a process to be discard-nonincreasing if and only if:



(7)

It can be shown that these discard-nonincreasing processes are closed under forming diagrams, and that the discard-nonincreasing numbers are the unit interval. It is therefore these discard-nonincreasing processes which we will ultimately use in order to compute probabilities of certain events occurring.

These sums, together with the scalars in the theory, lead to an interesting geometric structure on the sets of processes with a given input and output. For example, we can form arbitrary non-negative linear combinations via:

 , for $r_i \in \mathbb{R}^+$, (8)

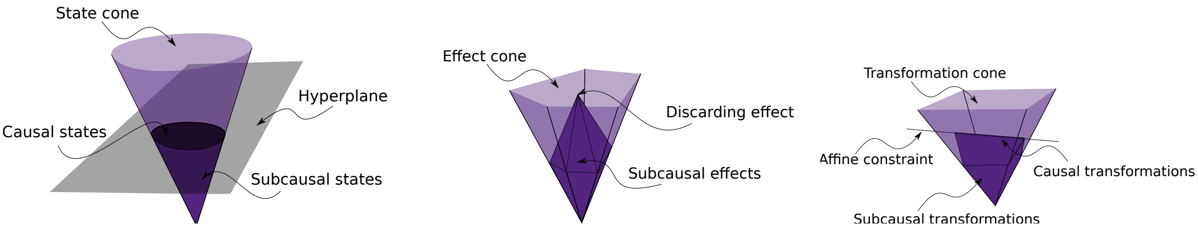
and this must itself be a valid process in the theory. In other words, the sets of processes with a given input and output define a convex cone K_A^B, which can be canonically extended to the real vector space which it spans, denoted V_A^B. This also applies equally well to states and effects, where we will denote these cones as K^A and K_A respectively.

The discard-preserving transformations from A to B form a convex set living inside K_A^B, as it is simple to demonstrate that if f and g are discard-preserving then

$$p\ \boxed{f}_{A}^{B} + (1-p)\boxed{g}_{A}^{B} \tag{9}$$

is also discard preserving. The partial order $\leq$ can then be seen as the partial order defined by viewing the cone as a positive cone, and the discard-nonincreasing processes are those that live within the cone but under the discard-preserving convex set.

Pictorially we can think about three different cases, states, effects, and transformations as is depicted in Fig. 1 [B].



(a) States (b) Effects (c) Transformations

Figure 1: (a) In general the state cone can be an arbitrary convex cone, the causal states are given by the intersection of this cone with a single hyperplane and the subcausal states lie between this hyperplane and the zero vector (i.e. the point), as depicted below. (b) We have only a single causal effect, the discarding effect itself. The subcausal effects lie between the discarding effect and the zero vector. (c) The set of causal processes is a more complicated set, but importantly, it is still an *affine* constraint, namely Eq. (4) , on the transformation cone. The subcausal transformations lie between this affine constraint and the zero vector.

To finish this section, see Table 1 [K] for an account of what all of these elements correspond to in the more familiar quantum and classical worlds.

Elements of a GPT	Quantum theory	Classical theory
Systems	Hilbert spaces	finite sets
States	density operators	probability distributions
Effects	POVM elements	$[0,1]-$valued functions
Discarding effect	(partial) trace	marginalisation
Discard preserving transformations	CPTP linear maps	stochastic maps
Discard nonincreasing transformations	CPTNI linear maps	substochastic maps
Composition rule	tensor product	Cartesian product

Table 1: Elements that define a generalised probabilistic theory, and how they are defined for the particular case of quantum and classical theories viewed as GPTs.

5.2 Motivation and Scientific goals

My main motivation in research is the pursuit of a deeper foundational understanding of the world around us. As quantum theory is, at the very least, an extremely good approximation of fundamental physics in a wide range of circumstances, a great deal of my research has therefore focused on developing an understanding of quantum theory. The area in which quantum theory seems most likely to break down, however, is when it comes to how it interacts with gravity, studying the interaction of these two theories is therefore also one of my main interests. Of course, while my motivation is mainly foundational, the dividends of developing a deeper understanding of the world can have profound impact on science, technology, and ultimately on the society that we live in.

Since the end of my PhD my research focus has been on asking what we can learn about the foundations of physics and of information processing, by considering quantum theory as just one theory within the landscape of generalised probabilistic theories. In particular, the research presented in this Habilitation thesis aims at making substantial progress within the following topics:

- exploring the possibilities for information processing in generalised compositional theories;
- developing a geometric and compositional understanding of the nonclassicality of nature;
- developing compositional approaches to quantifying crucial quantum resources;
- understanding what modern experimental proposals for probing the nature of the gravitational field can teach us.

5.3 Summary

There are four key topics which I have contributed to since my PhD via the paradigm of generalised probabilistic theories. I will discuss each of these in turn.

Firstly, I, together with Dr Jamie Sikora, studied primitives of information processing within the landscape of generalised probabilistic theories. In particular, we studied integer-commitment [Gol04] (a natural generalisation of the more widely studied bit-commitment), Wiesner's unforgeable money [Wie83], and coin-flipping [Blu83] respectively in Refs. [A, B, C]. In each case we showed how certain quantum results could be recovered in a broader class of generalised probabilistic theories. This, on the one hand, shows that quantum theory is not special in any of these protocols, and, on the other hand, shows exactly which aspects of the quantum formalism are necessary for each of these results.

Secondly, I, together with a number of collaborators, studied a notion of nonclassicality known as generalised contextuality [Spe05]. We recast this notion into the language of generalised probabilistic theories [G] which has led to a plethora of new results [H], new techniques [I, M], and a much deeper understanding of the topic [E]. For instance, we were able to use this new formalisation in order to prove that generalised contextuality is a necessary resource for computational advantage in the magic state-injection model for quantum computation [L]. This is analogous to the celebrated result of Ref. [HWVE14] which was proven for the case of Kochen-Specker contextuality. Our result was enabled by the fact that the generalised probabilistic theory formalism allows us to define generalised contextuality in a fully compositional way, so that it can be applied to arbitrarily complicated scenarios such as those that are found in a complicated circuit involved in a quantum computation.

Thirdly, I have studied quantum resources from the perspective of generalised probabilistic theories. To begin, I, together with Dr Ciarán Lee, considered resource theories of coherence. We showed that these could be defined for arbitrary generalised probabilistic theories, for arbitrary processes therein, and in a way that does not require one to make an arbitrary choice of basis [D]. I then, together with a number of colleagues, studied steering as a resource. In particular, we constructed a generalised probabilistic theory which permits post-quantum steering, and demonstrated that this is a resource for the task of remote state preparation [J]. With some of the same collaborators we then turned to the study of multipartite non-signalling channels which are often considered as resources in the study of entanglement and nonlocality [SRB20], we showed that these can always be simulated as affine combinations of product channels in a broad class of generalised probabilistic theories [K].

Finally, I, together with Dr Thomas D. Galley and Dr Flaminia Giacomini, asked what we can learn about recent experimental proposals aimed at probing the nature of the gravitational field [BMM+17, MV17], from the perspective of generalised probabilistic theories. We analysed these experimental proposals in the generalised probabilistic theory framework and proved a new no-go theorem [F]. Roughly speaking, what we show is that if entanglement is witnessed in these experiments, then this does indeed mean that the gravitational field is not classical. However, this does not mean it is necessarily quantum. That is, we show that there are other potential theories (in particular Refs. [HG20, BGW20]) which can mediate entanglement between quantum systems.

5.4 Insights into information processing

By now it is well known that quantum theory offers advantages in information processing. This realisation can be traced back to Wiesner's quantum money scheme [Wie83] which provides a protocol for creating banknotes which provably cannot – according to the laws of quantum mechanics – be forged. However, it is also well known that there are cryptographic tasks, for example bit-commitment [Gol04] and coin-flipping [Blu83], which cannot be achieved even with arbitrary quantum resources. It is therefore important to developing an understanding of the possibilities and limitations of quantum theory for information processing. In this section I will discuss three papers (Refs. [A, B, C]) which we tackle this issue within the framework of generalised probabilistic theories.

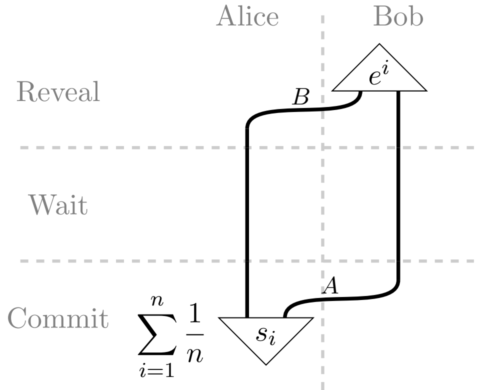
The key insight underpinning these three papers, is that the transition from classical to quantum to GPT, is precisely mirrored by a hierarchy of optimization problems, from linear to semi-definite to conic programs [A]. That is, if we find that an quantum optimisation problem is described by a semi-definite program, then often it will be the case that the analogous classical optimisation problem is a linear program, and that for a given GPT it will be a conic program.

Conic programs are well studied by the optimisation community [BV04], and there are a collection of important analytic results and techniques which can be applied which often reduce to well known results and techniques in semi-definite programming as a special case. Therefore, if one can prove a result using only these particular results and techniques for semi-definite programs in the case of quantum theory, then there is a good chance that such a result can be lifted to the study of generalised probabilistic theories. I will now discuss three particular applications of this idea.

5.4.1 Integer commitment

Integer-commitment [Gol04] is a generalisation of the better known cryptographic protocol of bit-commitment. It is a two party (e.g., Alice and Bob) cryptographic task consisting of two phases known as the commit and reveal phases. In the commit phase, Alice must generate a random integer from $\{1, ..., n\}$ and pass Bob some physical token to commit this integer. The idea at this stage in the protocol is that Alice should be unable to change which integer she committed to, but that Bob should be unable to learn which integer it is. Intuitively there is a tension between these two desiderata, the obvious way to stop Alice being able to change the committed integer is for the integer to be perfectly encoded in the token, but, by doing so, Bob would be able to learn its value too soon. In the reveal phase Alice should then pass a second token to Bob, at which point Bob should be able to perform some measurement on the pair of tokens to reveal the value of the integer which Alice committed.

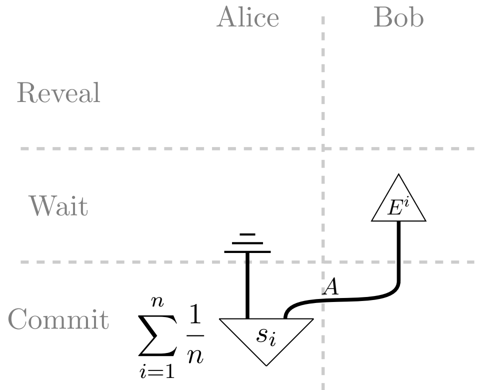
Diagrammatically we can depict this protocol in an arbitrary generalised probabilistic theory as follows [A]:



$$= \quad 1 \tag{10}$$

here, A represents the first token and B the second, s_i determines the state of this pair of tokens for a given integer i which Alice has committed to, and e_i represents Bob obtaining the measurement outcome corresponding to the integer i when he performs his measurement in the reveal phase. If the protocol were working properly then Bob should always obtain the correct outcome and so this diagram should be equal to 1.

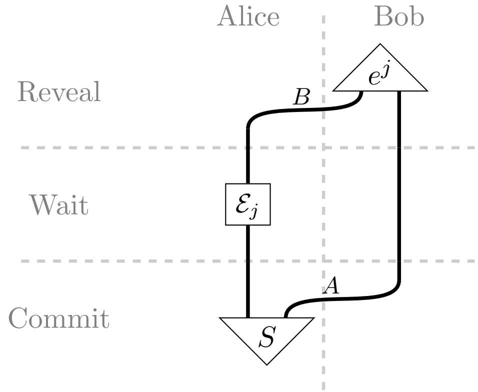
Now let us consider how each of Alice and Bob could try to cheat such a protocol. Let us start with Bob as this is the simpler case. All that he could do to cheat would be to perform a measurement prior to the reveal phase, that is, before Alice sends the second token. Diagrammatically we can depict this as:



$$\leq \quad \frac{1}{n} \tag{11}$$

where E^i represents the measurement outcome that a cheating Bob believes corresponds to Alice committing integer i. If, however, the protocol is working perfectly then Bob should not be able to learn anything about i, and so this probability should be upper bound by a random guess, i.e., $1/n$.

Alice has more possible cheating strategies, but the one that is relevant for us here is depicted diagrammatically as follows:



$$\leq \frac{1}{n}. \tag{12}$$

That is, rather than choosing a state or the tokens based on the integer i, she instead prepares a fixed state S, and later on, applies some transformation on the second token in order to choose some other integer j. Again if the protocol is working properly, then the ability for Alice to cheat should be upper bound by a random guess by Bob, again, $1/n$.

By using this particular cheating strategy for Alice, and using properties of cone programs we prove the following result [A].

Theorem 1. *In a generalised probabilistic theory satisfying the no-restriction hypothesis and the purification postulate, we have that either Alice or Bob can necessarily cheat with probability* $\sqrt{\frac{1}{n}} \geq \frac{1}{n}$.

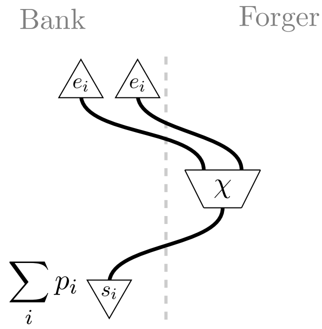
This means that in such theories, for example quantum theory, that there is no perfect integer-commitment protocol. The no-restriction hypothesis and the purification postulate therefore pick out a class of generalised probabilistic theories which are 'quantum-like' with respect to integer-commitment. Informally, the no-restriction hypothesis demands that the theory admits all logically consistent measurements, i.e., those which would not lead to negative probabilities of certain measurement outcomes occurring. Whereas the purification postulate is a generalisation of the notion of a purification of a quantum state, it states that every mixed state in the theory can be viewed as the marginalisation of a pure bipartite state, and that any two such purifications are related by reversible transformations.

This shows us that there is a broad class of theories for which we can prove an upper bound on how well one can perform the task of integer-commitment. What is particularly interesting about this is that it is not a simple impossibility result, and that instead we find a numerical upper bound which must be satisfied by an entire class of theories. This was, to our knowledge, the first instance of such a result, which highlights the role that cone programs can play in studying information processing in generalised probabilistic theories.

5.4.2 Unforgeable money

Wiesner's quantum money scheme [**?**]elies on the quantum no-cloning theorem in order to create quantum banknotes which provably cannot be forged. The original scheme actually only showed that any attempted forgery could be detected by the bank with some non-zero probability, later developments of the scheme, however, showed how the detection probability of any attempted forgery can be pushed up to 1.

We can think of this as a two party adversarial cryptographic task, one party, the bank, creates some physical bank note which the other party, the forger, attempts to copy. Diagrammatically we can represent a particular strategy of the bank and forger as follows [B]:



$$< \quad 1. \tag{13}$$

The bank prepares some physical system in some state s_i and encodes it in a bank note with a label which the bank can look up in their private database to find out the value of i for the given note. When the bank note is returned they can then measure it with some measurement, which depends on i, in order to check that the state is indeed s_i as they expect. The state that they use should be chosen randomly according to some probability distribution. All the forger can do, is to try to copy the bank note according to some physical transformation χ. For the bank to have a chance of detecting the forgery, it must be the case that the probability of both notes passing the banks test is strictly less than 1.

We, again by using techniques from cone programs, prove a dichotomy theorem for generalised probabilistic theories [B]:

Theorem 2. *In any generalised probabilistic theory, either unforgeable money is impossible (such as is the case for classical theory) or it can be achieved perfectly, that is, with arbitrarily small probability of the forger escaping detection (such as is the case for quantum theory).*

What is surprising about this result is that it holds for any generalised probabilistic theory at all, which is to be contrasted with our theorem for integer-commitment which held only for the class of generalised probabilistic theories satisfying a pair of extra assumptions. What is also surprising is finding such a strict dichotomy, as within the framework of generalised probabilistic theories typically theories can always be carefully tuned to achieve any given success probabilities in any given task of interest.

5.4.3 Coin flipping

Coin flipping is a two party cryptographic task where the two parties (e.g., Alice and Bob) aim to generate a shared uniformly random bit over some communication channel. Unlike in the previous two protocols, there is no fixed structure as to how this communication channel can be used, and so it is not straightforward to set up an explicit diagrammatic representation of the space of strategies for either party. Nonetheless, we still have sufficient structure around to be able to define an optimisation problem which we can then tackle analytically. To do so, however, we must go beyond even cone programs, and work instead with a kind of optimisation problem known as a *semi-infinite program* (not to be confused with the semi-definite programs commonly used in quantum information theory).

More formally, a coin-flipping protocol consists of a triple of strategies for Alice $(A_0, A_1, A_{\text{abort}})$ which correspond to the three possible outcomes of some deterministic strategy A_{det}, and a triple of strategies for Bob $(B_0, B_1, B_{\text{abort}})$ which correspond to the three possible outcomes of some deterministic strategy B_{det}, such that

$$\mathsf{Prob}(A_b, B_b) = \frac{1}{2} \tag{14}$$

for $b \in \{0, 1\}$. This ensures that the bit b is uniform and perfectly correlated between Alice and Bob. If perfect coin flipping were possible, then it should be the case that if neither Alice nor Bob can cheat by deviating from these strategies in order to bias the distribution of b.

We show how to set up this problem in the framework of generalised probabilistic theories and prove the following theorem [C]:

Theorem 3. *Any coin-flipping protocol in a generalised probabilistic theory satisfying the generalised no-restriction hypothesis (for either Alice or Bob) allows for either Alice or Bob to force a given outcome with probability of at least* $\frac{1}{\sqrt{2}}$.

Here, the generalised no-restriction hypothesis is like the no-restriction hypothesis for measurements, but lifted to the level of strategies for Alice or Bob. That is, any logically possible strategy (in the sense of not leading to negative probabilities for certain outcomes) is a physically permitted strategy.

Like in the case of integer-commitment, what is particularly interesting about this result is that we have managed to find a numerical lower bound which holds for any generalised probabilistic theory, as opposed to simple possibility or impossibility results which are more common in the literature. Typically, in order to obtain numerical results one must focus attention to a particular generalised probabilistic theory of interest rather than dealing with an entire class of them. What is also interesting about this work is the utilisation of semi-infinite programs to obtain the result, this is a new technique that we introduced to the study of generalised probabilistic theories which, like with cone programs, we expect to be much more widely used in the future.

5.5 Insights into generalised contextuality

The eponymous theorem of Kochen and Specker [KS90] demonstrated that there is no classical hidden variable model for quantum theory satisfying a constraint known as non-contextuality. Hence, this contextuality became well known as an important signature of the non-classicality of quantum theory. This assumption states that if two outcomes of two different quantum measurements are represented by the same projector, then these outcomes should have the same representation in the classical hidden variable model. However, since the discovery of this theorem there have always been questions as to how natural this assumption actually is. Indeed, Bell discussed how this "apparently innocent assumption" might not be so innocent after all [Bel66], and scant justification is given in the original works of Kochen and Specker beyond an appeal to a certain mathematical naturalness of the assumption.

Much later on, a justification for this assumption was provided by Spekkens [Spe05] which appeals to Leibniz's principle of the identity of indiscernibles [Spe19]. The idea being that if two things cannot in principle be distinguished then the best explanation for this fact is that they two things are in fact the same. Taking this principle to it's logical conclusion, however, gives us much more than the assumption of Kochen and Specker, it gives a constraint not just on the representation of outcomes of measurements in a classical hidden variable model, but also on the representation of states, transformations, and general processes in the theory.

Since it's formulation by Spekkens, this notion of *generalised contextuality* has generated much interest in the quantum foundations research community. It has been shown to be the most generally applicable notion of nonclassicality, and has been demonstrated to be a crucial resource underpinning many information processing advantages afforded by quantum theory [SBK+09, SHP19, SS18, FLC+22, LS20, Los20]. However, the mathematical formalisation of the idea remained somewhat imprecise, and, perhaps as a consequence, there were a lack of systematic tools for exploring the phenomena. For example, each proof of generalised contextuality had to be tailored to the situation at hand, rather than having a generic test which could be straightforwardly applied in any situation.

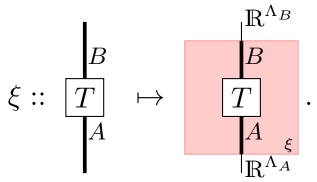
In this section I will discuss how we applied the paradigm of generalised probabilistic theories to overcome these problems, and the results that we discovered having done so.

5.5.1 Formalising generalised contextuality

The key insight underpinning this new approach is that the processes in a generalised probabilistic theory really represent equivalence classes of physical processes [CDP10, Har11]. Two physical processes are said to be operationally equivalent if and only if they cannot be distinguished by doing tomography on the input and output systems. A process in a generalised probabilistic theory therefore can be thought of as an equivalence class of operationally equivalent physical processes. This is precisely what was captured by Eq. (2) in the brief introduction to generalised probabilistic theories. Indeed, as shown in [CDP16] one can really think of a generalised probabilistic theory as an operational theory which is quotiented with respect to this operational equivalence relation.

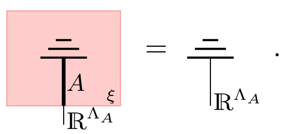
Now, a noncontextual representation of a theory, is one where operationally equivalent physical processes are represented in the same way in the underlying ontological (i.e., classical hidden variable) theory. Or, to put it another way, a noncontextual representation is one which factors through the theory quotiented by operational equivalences. Or, to put it a third way, a noncontextual representation of an operational theory exists if and only if a representation of the quotiented theory exists [G, E]. This means that we can study noncontextual representations of operational theories via representations of generalised probabilistic theories. This is convenient as there is a great deal of mathematical structure which has been well studied for generalised probabilistic theories which we can then exploit. With this in mind, we can formally define an ontological representation of a generalised probabilistic theory as follows [E]:

Definition 4 (Ontological models of GPTs)**.** *An ontological model ξ of a generalised probabilistic theory* $\mathbf{G}$ *is a diagram-preserving map*[3] $\xi : \mathbf{G} \to \mathbf{ClassicalTheory}$ *, where* $\mathbf{ClassicalTheory}$ *is the classical generalised probabilistic theory. We depict the action of ξ as*

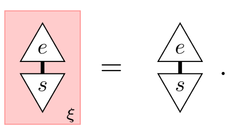


Moreover, this map must satisfy three further properties:

1. *It represents the deterministic effect appropriately:*



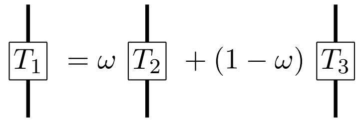
2. *It reproduces the operational predictions of the generalised probabilistic theory (i.e., is empirically adequate), so that for all closed diagrams,*



(15)

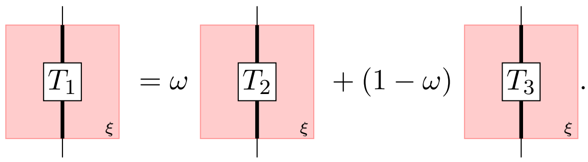
[3] a.k.a. a strict symmetric monoidal functor

3. *It preserves the convex and coarse-graining relations between operational procedures. E.g., if*



(16)

then it must hold that

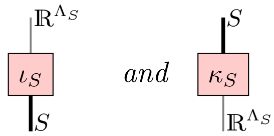


(17)

If such an ontological representation exists, then any operational theory which quotients to **G** is said to be noncontextual. If it does not exist, then any operational theory which quotients to **G** is said to be contextual.

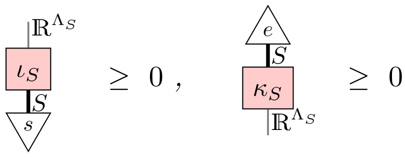
This provides a formal way to, at least in principle, test whether a given operational theory is contextual or not. However, very often we are not interested in the theory as a whole, but just in some particular scenario within the theory. In particular, the literature has very often focused on the simplest scenario which is a prepare-measure scenario for a single system. In this case, we obtain a much simpler definition [G]:

Definition 5. *An ontological representation of a prepare-measure scenario for a system S is defined by a set of ontic states Λ_S and a pair of linear maps*

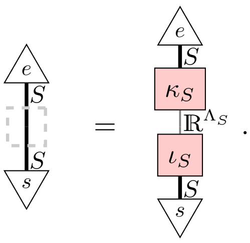


(18)

such that for all $s \in K^S$ and for all $e \in K_S$ we have

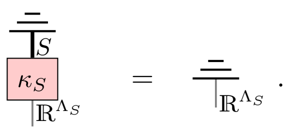


(19)



(20)

and such that



(21)

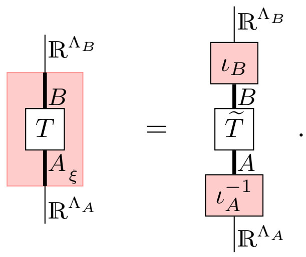
In the remainder of this section I will discuss the results that we have obtained from this new perspective on generalised contextuality.

5.5.2 Structure theorem

In the original definition of a noncontextual ontological model [Spe05], and even in the new definition given above, it is quite unclear what the scope of possibilities are. For instance, do we need to independently define the representation for the states and effects or does one fix the other? What is the freedom in specifying the representation of the transformations? Can Λ be arbitrarily large or can we bound its cardinality?

In Ref. [E] we answer all of these questions in the case of tomographically local generalised probabilistic theories [Har01]. This is an important class of generalised probabilistic theories, which includes quantum and classical theory as special cases. Such theories are defined by the property that in them we can characterise bipartite states just by the (typically correlated) statistics which can be observed by performing local measurements. That is, if two bipartite states are distinguishable, then there will be some local measurement that reveals this distinction, that is, we will not need to perform some entangled measurement in order to do so.

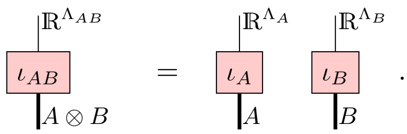
For such theories we find that the possibilities for ontological models become extremely constrained. Firstly, for every system S we find that $\mathsf{dim}[V_S] = \mathsf{dim}[\mathbb{R}^{\Lambda_S}] = |\Lambda_S|$ that is, that the cardinality of Λ_S is necessarily specified by the dimension of the state space of S. Secondly, we find that for every system that ι_S and κ_S are inverses of one another, hence, that fixing the representation of states via ι_S uniquely fixes the representation of the effects via $\kappa_S = \iota_S^{-1}$ (or vice versa), and, finally that the representation of transformations is also uniquely fixed by the representation of states (or effects) via:



(22)

Essentially, all that we are doing is picking a choice of basis for the state space such that all of the states are positive with respect to this basis, and such that all of the effects are positive with respect to the dual basis. As such, this can be thought of as a generalisation of the idea of an exact frame representation which has previously been studied in the context of quantum information theory [FE08].

An important point to note, however, is that these ι_S cannot be chosen independently for the different systems in the theory, as they must satisfy the further condition that:



(23)

In other words, the basis for a composite system must be the tensor product of the bases of the components. One must then take care to check that such a product basis is positive on all composite states within the theory, and similarly, that the basis dual to this is positive on all composite effects within the theory. As such, it remains a highly non-trivial task to actually construct an ontological model for the entirety of a generalised probabilistic theory, as, even once one has found a representation of the local systems it is not guaranteed that this will work for composite systems.

5.5.3 Computational advantage from generalised contextuality

In the previous section we discussed how, at least for tomographically local theories, ontological representations are extremely constrained. In fact, in certain cases one can prove that they are unique. Of particular relevance is the fact that this is the case when one considers the stabilizer subtheory in odd dimensions as we demonstrate in Ref. [L]. That is, we show that:

Theorem 6.

(a) For any stabilizer subtheory (single- or multi-particle) in odd *dimensions, the unique ontological representation is Gross's representation.*

(b) For any stabilizer subtheory (single- or multi-particle) in even *dimensions, there is no ontological representation at all.*

Now, it is well known [Got98] that the stabilizer subtheory is efficiently simulable, however, if one adds in appropriate non-stabilizer states then it achieves universal quantum computation, and any such state *must* have negativity with respect to Gross's representation[4]. This means that if we supplement the stabilizer subtheory with such a state then we end up with a theory which is necessarily contextual. We therefore arrive at a result akin to that of Ref. [HWVE14] (which considered Kochen Specker contextuality) but now for generalised contextuality [L]:

Theorem 7. *Consider any state ρ which promotes the stabilizer subtheory to universal quantum computation. There is no generalised noncontextual ontological model for the stabilizer subtheory together with ρ.*

In this sense we find that generalised contextuality is a necessary resource for the quantum speedup. The key open question that this leads to, is whether or not this result is only due to the particular model of computation (i.e., a state injection scheme) or whether it can be shown to hold in arbitrary models of computation.

5.5.4 New possibilities for witnessing nonclassicality

Let us now turn away from considering general compositional theories, and instead focus on witnessing generalised contextuality in a particular prepare-measure scenario, and let us ask what exactly do we need in order to witness this phenomena. In the context of Bell experiments, we know that we need quite a lot to observe the nonclassicality of quantum theory. For example, we need entangled states, incompatible measurements, the ability to freely choose the measurement settings, and highly efficient detectors [San05]. In experimental tests of generalised contextuality, however, it was known that entangled states were not necessary, and in recent work we have shown that none of the other things are necessary either [H, I].

That is, we show that there are proofs of generalised contextuality which do not have any incompatibility [H], which do not require freedom to choose settings [I], and where the detectors used can be arbitrarily inefficient [I]. These counter-intuitive results came directly from our new understanding of the phenomena which we gained from the perspective of generalised probabilistic theories. In this language what this boils down to is the following [I].

Theorem 8. *An ontological representation of a prepare measure scenario exists if and only if there exists a set of ontic states Λ_S and linear maps ι_S and κ_S such that conditions 19 and 20 of Def. 5 are satisfied.*

[4] Recall that Gross's representation is a quasiprobability representation, which means that quantum states are represented by quasiprobability distributions, i.e., probability distributions which can take negative values.

In other words, we do not need to additionally demand that condition 21 of Def. 5 is satisfied. We prove this by showing that if one can find such linear maps which do not satisfy condition 21 that there necessarily also exist linear maps which do. Hence, this does not actually serve as an additional constraint.

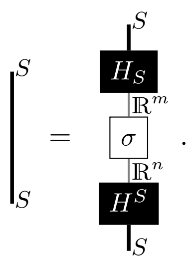
What this in turn means is that to determine whether a given prepare-measure scenario admits of an ontological representation, we need only specify the positive cone generated by the set of states, and the positive cone generated by the set of effects, and not any further details pertaining to the overall scaling of these. Hence, if two scenarios generate the same cones, then either both are generalised noncontextual or neither are. We say such scenarios are *cone-equivalent* [I].

The results mentioned above are then quite simple applications of this observation. For example, if we have inefficient detectors, then all this is doing is scaling down the effects compared to the perfectly efficient case – hence the perfectly inefficient an the arbitrarily inefficient scenarios are cone-equivalent. As we know that there are proofs of contextuality in the perfectly efficient case then we know that there are also such proofs in the arbitrarily inefficient case. Similarly, there is a technique known as *flag-convexification* whereby some setting variable is turned into an outcome variable, that is, we select the setting at random and keep a record of this as an additional outcome. This means we can turn a scenario with settings into one without settings, and so there is no possibility for incompatibility or necessity of freedom of choice in this new scenario. It can then be shown that this new scenario is cone-equivalent to the original scenario. Hence, if we take any known proof of contextuality, it can straightforwardly be turned into a new proof which has no settings, and hence, no incompatibility or necessity to make free choices.

5.5.5 New tests for generalised contextuality

While it remains a difficult problem to find an ontological representation of the entirety of a generalised probabilistic theory (as discussed in Sec. 5.5.2), it turns out that if we focus on just the prepare-measure scenario, that (even without appealing to tomographic locality) there is an efficient way to construct an ontological representation (should it exist).

Specifically, we show that finding an ontological representation of the prepare-measure scenario within a generalised probabilistic theory boils down to finding a matrix of non-negative real entries, σ satisfying the condition [M]:



(24)

Here, H^S is the facet characterisation of the cone K^S and H_S is the facet characterisation of the cone K_S. Finding such a σ is a linear program, and we introduce an open source code in order to solve it. The ontological representation, namely Λ_S, ι_S and κ_S, can then be directly computed from σ, H^S, and H_S.

An consequence of this result, is that we can then (following ideas from Ref. [GW22]) apply Carathéodory's theorem [Car11], in order to show that $|\Lambda_S| \leq \mathsf{dim}[V_S]^2$. That is, we can obtain a bound on the number of ontic states for prepare-measure scenarios without needing to appeal to tomographic locality. It remains an important open question as to whether this upper bound is tight.

We can, however, take things further than simply testing for the existence or not of a generalised noncontextual ontological model. Specifically, in the case that such a model does not exist, we can compute how much noise must be added such that one does. In other words, we can compute the *robustness* of contextuality to a given noise model N [M]. We represent this noise model as a particular process within the generalised probabilistic theory. In the case of quantum theory the natural choices to explore are either depolarising or dephasing noise, however these are not always well defined for some arbitrary choice of generalised probabilistic theory. Once we have such a noise model, however, the robustness can be computed as:

$$r^*_N := \mathsf{inf} \left\{ r \;\middle|\; \begin{array}{rcl} r\,\boxed{N}_{S}^{S} + (1-r)\, \Big|_{S}^{S} &=& \boxed{H_S}^{S}_{\mathbb{R}^m}\, \boxed{\sigma}^{\mathbb{R}^m}_{\mathbb{R}^n}\, \boxed{H^S}^{\mathbb{R}^n}_{S} \\ \boxed{\sigma}_{\mathbb{R}^n}^{\mathbb{R}^m} &\geq& 0 \\ r &\in& [0,1] \end{array} \right\}, \qquad (25)$$

where in general the robustness will depend on the choice of noise model N. This again is a linear program and we provide an open source code to solve it for a given noise model.

5.6 Insights into quantum resources

We have already mentioned that generalised contextuality can be viewed as a resource in many situations, and, in particular, how this is the case in the state-injection model of quantum computation. There are, however, many other quantum resources which one can consider. In this section I discuss three such resources, how my research has furthered our understanding of them, and how this was enabled by taking the perspective of generalised probabilistic theories. Firstly we studied coherence and developed a compositional resource theory of coherence [D] . Secondly we studied steering, and in particular post-quantum steering, showing how this is a resource for the task of remote state preparation [J]. Finally we developed tools [K] for the study of type-independent resource theories of either local operations and shared randomness [SRB20] or local operations and shared entanglement [SDM+21]. These last resource theories subsume, for example, the study of Bell nonclassicality and the study of entanglement in a common framework.

5.6.1 Coherence

Coherence is one of the critical resources for quantum technologies, and battling against decoherence in a myriad of quantum systems is one of the key experimental challenges of our time. Gaining a deep understanding of the best way to quantify and manipulate coherence is therefore an important theoretical challenge that we can undertake in order to support the development of future technologies. It is therefore not surprising that there has been a great deal of interest in formalising the quantification of coherence as a resource theory [SAP17]. Indeed, a large number of different resource theories of coherence have been proposed and studied in detail [SAP17].

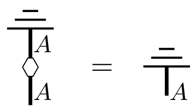
Typically, however, each of these has suffered from a number of limitations. For example, they tend to require an arbitrary choice of basis, only quantify the coherence in quantum states, and only apply to single quantum systems. In this work we endeavoured to overcome these limitations by recasting the study of coherence in the language of generalised probabilistic theories. We show in Ref. [D] that resource theories of coherence can be defined for arbitrary generalised probabilistic theories, and in fact, this sheds light also on coherence specifically within quantum theory. In particular, in the case of quantum theory we find that certain resource theories of coherence which have been proposed in the past are inconsistent with certain desiderata whilst other resource theories can in fact be seen as special cases of the resource theory that we introduce. Moreover, the resource theory that we define can be applied to the quantification of coherence not just of states, but of arbitrary collections of quantum processes. Finally, all of this can be done in a way that does not require one to make any arbitrary choice of basis.

The key insight which underpins this is the realisation that it is much easier to understand decoherence than coherence in generalised probabilistic theories, but that once we understand decoherence we can turn it around to enable the study of coherence. The study of decoherence in generalised probabilistic theories formed an important component of my PhD studies, see Sec. 7.2. Specifically, in any generalised probabilistic theory we can define decoherence processes as follows:

Definition 9. *A decoherence process for a system A, denoted:*

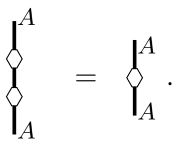
(26)

is a discard-preserving process, i.e.:



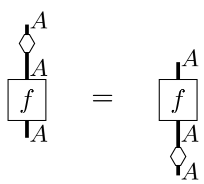
(27)

which is idempotent, i.e.:



(28)

The key principle behind the resource theories that we define [D] is that: *"Free processes must, at a minimum, preserve the set of decohered processes"*. This can be shown to imply the following condition:

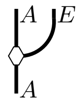


(29)

which, in quantum theory for specific choices of decoherence processes, reduces to the notion of decoherence invariant operations [CG16, MS16]. Therefore, our resource theories can be seen as an extension and generalisation of the resource theory of decoherence invariant operations.

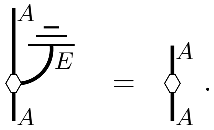
We do not simply have to consider the impact of decoherence on the system of interest, we can also consider the impact on the environment which is causing the decoherence. Doing so we define a generalisation of the notion of a decoherence process which we call a decoherence mechanism [D]:

Definition 10 (Decoherence mechanism)**.** *A decoherence mechanism is a process denoted*



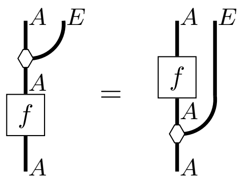
(30)

which induces a decoherence process on the system A when the environment system E is discarded, i.e.:



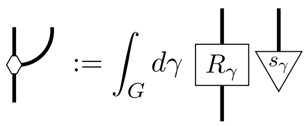
(31)

The natural generalisation of Eq. (29) gives us the constraint on free processes:



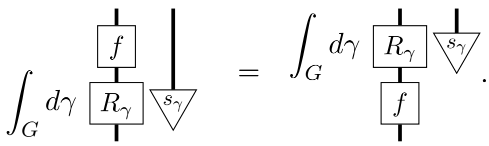
(32)

A particularly interesting case to consider is when the decoherence is due to loss of a reference frame, in which case the decoherence mechanism is given by:



(33)

which we show leads to the condition on free processes:



(34)

which, in the case of quantum theory reduces to the notion of translationally invariant operations [GS08]. Hence our formalism provides a unification of (and in fact an interpolation between) translationally invariant operations and decoherence invariant operations within a common framework.

The construction of the formal resource theories follows standard techniques from category theory and the categorical approach to resource theories of [CFS16]. The details of this go beyond the scope of this summary. But, roughly speaking, the idea is that we freely construct a new generalised probabilistic theory in which systems are labelled by a pair constituted of: i) a system from the original generalised probabilistic theory, and ii) a decoherence mechanism on that system, and in which the processes are arbitrary processes from the original theory. We can then define the free processes as those that satisfy equation (32). To construct the resource theory, we must then choose what we want to quantify the resourcefulness of, e.g., of states, of measurements, of general processes, or collections thereof, and follow the recipe laid out in [CFS16].

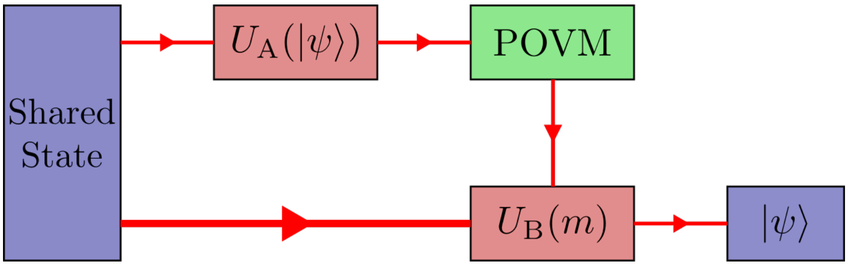


Figure 2: Pictorial representation of the remote state preparation protocol. Alice performs $U_A(|\psi\rangle)$ on her share of a physical system – a unitary operation that depends on $|\psi\rangle$ – and then a measurement (POVM). She sends a classical message m to Bob, who, in turn, performs an unitary transformation (which depends on m) on his share of the system. The outcome of the protocol is a quantum state $|\psi\rangle$ in Bob's lab.

5.6.2 Steering

The study of post-quantum steering has been an important topic in quantum foundations since it was first discovered in Ref. [SBC+15]. However, since its discovery there have been two open questions. Firstly, we know that steering is a resource in certain tasks, but does post-quantum steering provide any advantage over quantum steering in any task? Secondly, we can define post-quantum steering in a way that is compatible with no-signalling constraints, but can we find a hypothetical physical theory which could give rise to this kind of steering?

In Ref. [J] we resolve both of these open questions. We construct a generalised probabilistic theory which we call *Witworld*, in which we can show that there is post-quantum steering which, moreover, provides an advantage over quantum theory for the task of remote state preparation.

Crucial to the definition of Witworld is the notion of the max-tensor product. This is a way of combining systems from arbitrary generalised probabilistic theories to create a new theory. Specifically, given two systems A and B we can construct the composite system $A \otimes_{\max} B$ such that $V_{A\otimes_{\max}B} = V_A \otimes V_B$, where the cone of effects $K_{A\otimes_{\max}B}$ is taken to contain only conic combinations of product effects, and where the cone of states $K^{A\otimes_{\max}B}$ is taken to be the largest possible cone which does not give negative probabilities for the effects.

We define Witworld as the theory that one obtains by taking the max-tensor product of systems from three different pre-existing generalised probabilistic theories, namely, quantum, classical, and Boxworld (hence the 'world' in Witworld) [J]. In the case of taking the max-tensor product of two quantum systems we end up with the state cone coinciding with the space of entanglement witnesses (hence the 'Wit' in Witworld).

Within Witworld we show that we can reproduce all analytically discovered examples of post-quantum steering. Hence, this answers the second question, namely, we have constructed a hypothetical physical scenario which can give rise to post-quantum steering. It remains an open question, however, as to whether we can reproduce all examples which were discovered through numerical techniques, or even whether it can reproduce the full scope of possible examples of post-quantum steering.

To answer the first question we consider the task of remote state preparation. An illustration of this task is given in Fig. 2 [J]. Remote state preparation is a protocol in which Alice wants to send a state to Bob via performing some action on part of some shared entangled resource and classical communication. The difference with teleportation is that she is allowed to know which state it is that she wishes to send (although not in advance of setting up the shared entangled resource). In quantum theory this takes at least two bits of communication to remotely prepare the state of a qubit, whilst we show that within Witworld we can achieve the same result with only a single bit [J]. The crux of the protocol within Witworld, is that the universal-NOT gate for a qubit is a valid transformation within Witworld (whereas it is not within quantum theory), and so we can use this within the protocol. Whether this result can be generalised beyond the case of a single qubit is an open direction for future research. What this means, however, is that at least in this special case we have shown that post-quantum steering is a resource for remote state preparation.

5.6.3 Type-independent non-signalling channels

Non-signalling channels recently been considered as resources in Ref. [SRB20]. They work in a type-independent paradigm, which means to say that it allows for the inputs and outputs of the channels for each party to be quantum, or classical, or to be trivial (i.e., no input or output). This means that they can study Bell nonclassicality, entanglement, steering, and many more kinds of resources, in a coherent unified way.

In Ref. [K] we prove a useful theorem for this line of research, and actually do so for a broad class of generalised probabilistic theories, namely, those that are tomographically local [Har01]. Specifically, we prove that:

Theorem 11. *A multipartite channel in a tomographically local generalised probabilistic theory is non-signalling if and only if it is an quasiprobabilistic (i.e., affine) combination of product channels.*

This had previously been established in Ref. [ASS13] for the case of multipartite classical channels, and in Ref. [CDPV13] for the case of bipartite quantum channels.

On a practical level this result can provide a tool for performing calculations involving multipartite non-signalling channels, which is useful in the context of the aforementioned resource theories. On the other hand, on a conceptual level this result can be seen as stating that it is really a kind of negativity, i.e., the negativity of the quasiprobability distribution, which underpins all of these resources. It remains to be seen in future work whether this can provide a handle on quantification of these resources.

5.7 Insights into the nature of gravity

One of the deepest problems in the foundations of physics is to discover exactly how quantum theory and general relativity interact with one another. This is rarely a practical impediment, as in typical regimes where we work either quantum or gravitational effects dominate. Recently, however, experiments have been proposed which would probe the intermediate regime, and which are expected to be achievable within the next decade [BMM+17, MV17]. A careful analysis of such experimental proposals is important, such that when they are performed we will be in a position to correctly interpret whatever results we find.

Critical to such an analysis, is that we don't presuppose the result in the tools that we use to perform the analysis. For instance, we should not presume the correctness of either quantum theory or general relativity in the analysis of the experiment, as we need to entertain the possibility that either, or both, could break down in this intermediate regime. This is where the paradigm of generalised probabilistic theories comes into the picture, if we can analyse the experiment using these tools, then we are doing so in a way which makes minimalist assumptions about the underlying theory of nature.

The experimental set-up to be considered is illustrated in Fig. 3 [F], and the idea is to test whether or not the final state of the two masses is entangled or not. Typically, the argument is then made that: if we observe entanglement, then the gravitational field must be quantum, whilst if we do not observe entanglement, then the gravitational field could be intrinsically classical.

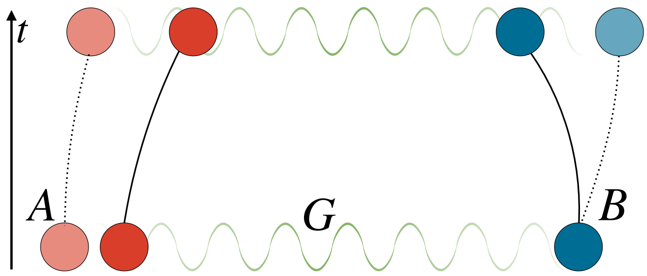


Figure 3: Illustration of the experimental situation. Two masses A and B are initially prepared in a separable state (where at least one of them, A in the figure, is in a superposition state in position basis). The masses interact via the gravitational field G. After some time, the full state becomes entangled.

Formalising this experiment within the framework of generalised probabilistic theories allows us to prove the following no-go theorem [F]:

Theorem 12. *We consider two GPT systems A and B, initially in a separable state, and the gravitational field G in a product state. We assume that the systems A and B only interact gravitationally. Then the following statements are incompatible:*

1. *The gravitational field G is able to generate entanglement;*
2. *A and B interact via the mediator G;*
3. *G is classical.*

Therefore, if we are confident in the experiment that there is no interaction between the masses aside from the gravitational field, and we observe entanglement at the end of the experiment, then we are forced to conclude that the gravitational field is non-classical, whilst if we do not observe entanglement, then it remains a possibility that the gravitational field is classical.

Importantly, however, in the preceding paragraph, we have not claimed that an observation of entanglement tells us that the field is quantum, only that it is non-classical. Indeed, we also demonstrate that there are other possible generalised probabilistic theories which can mediate entanglement between quantum systems. There are two such generalised probabilistic theories in the literature which have this property [F], namely Ref. [BGW20], and Ref. [HG20].

We can use this no-go theorem as a way to classify various models of interaction between quantum theory and gravity by which of our three assumptions in the no-go theorem are violated [F]. For example, models of gravitational decoherence or spontaneous collapse models violate condition 1 as decoherence or collapse would occur thereby destroying any entanglement before it could be observed in the experiment. In contrast, the Schrödinger-Newton approach (if it could be suitably formalised as a generalised probabilistic theory) would violate condition 2, that is, any observed entanglement could not be mediated by the gravitational field. Finally, a failure of condition 3 is what predicted by the kind of analysis given in Fig. 3 which comes from a naïve application of quantum theory to the scenario.

6 PRESENTATION OF TEACHING, ORGANISATIONAL, AND 'POPULARISATION OF SCIENCE' ACHIEVEMENTS

6.1 Teaching achievements

Academic teaching:

- *University of Gdańsk* 2021-present.
  Lecturer in the Quantum Information Technologies masters course
  - "Advanced topics in quantum foundations: resource theories". 2 lectures, 4 hours total (spring 2023-24).
  - "Categorical Quantum Mechanics a.k.a. Diagrammatic Quantum Theory". 15 lectures, 30 hours total (spring 2022-24).
  - "Signatures of nonclassicality". 15 recitation classes, 30 hours total (winter 2022).
- *University of Gdańsk* 2020.
  Teaching assistant in the Doctoral School
  - "Signatures of nonclassicality" development and marking of 10 exercise sheets
- *Perimeter Institute for Theoretical Physics* July 2018
  Lecturer for tutorial series
  - Process theories and diagrammatic mathematics
  - An introduction to symmetric monoidal category theory
  - Reformulating quantum theory as a process theory
  - The landscape of process theories

Invited lectures:

- Solstice of Foundations – *ETH Zürich, Switzerland.* June, 2022.
  "Generalised physical theories"
  https://foundations.ethz.ch/

6.2 Organisational achievements

Conferences, workshops, and seminars: organisation

- **Co-organiser** *Picturing quantum weirdness* August, 2023.
  University of Gdańsk, Poland.
  https://gqi.ug.edu.pl/schools/

- **Main organiser** *Quantum Physics and Logic* June, 2021.
  ICTQT, University of Gdańsk, Poland.
  https://qpl2021.eu/

- **Programme committee member** *Quantum Physics and Logic* 2021-present.
  Gdańsk, Poland (2021), Oxford, UK (2022), Paris, France (2023).
  https://qpl2021.eu/, https://www.qplconference.org/, https://qpl2023.github.io/

- **Organiser** *Quantum foundations seminar series* 2018-2019.
  Perimeter Institute for Theoretical Physics, Waterloo, Canada.

- **Co-organiser** *Foundations of quantum mechanics conference* 2018.
  Perimeter Institute for Theoretical Physics, Waterloo, Canada.
  https://perimeterinstitute.ca/events/foundations-quantum-mechanics

- **Programme committee member** *Q-Turn: changing paradigms in quantum scence* 2018-2020.
  Florianópolis, Brazil (2018), Online (2020)
  https://qturnworkshop.wixsite.com/2018, https://www.q-turn.org/

6.3 Popularisation of science achievements

Science communication:

- **Cover article:** "Can we solve quantum theory's biggest problem by redefining reality?" Sept, 2024.
  Cover article about my work with David Schmid and Robert W. Spekkens
  New Scientist
  https://www.newscientist.com/article/mg26335070-700-can-we-solve-quantum-theorys-bigge

- **News article:** "Diagramming quantum weirdness" January, 2022.
  Article about my work with David Schmid and Robert W. Spekkens
  APS Physics 15, 11
  physics.aps.org/articles/v15/11

- **News article:** "We have hints of a theory beyond quantum mechanics" June, 2018.
  Featured article about my work with Ciarán M. Lee
  New Scientist
  newscientist.com/article/mg23831820-300-we-have-hints-of-a-theory-beyond-quantum-physi

- **News article:** "Entanglement is an inevitable feature of reality" September, 2017.
  Featured article about my work with Jonathan G. Richens and Sabri W. Al-Safi
  Phys.org
  phys.org/news/2017-09-entanglement-inevitable-feature-reality.html

- **News article:** "Scientists finally prove strange quantum physics idea Einstein hated" September, 2017.
  Front page article about my work with Jonathan G. Richens and Sabri W. Al-Safi
  Gizmodo
  gizmodo.com/scientists-finally-prove-strange-quantum-physics-idea-e-1798433666

- **Blog post:** "If you think quantum physics is weird, try these theories" September, 2017.
  Blog about my work with Jonathan G. Richens and Sabri W. Al-Safi
  George Musser
  spookyactionbook.com/2017/08/31/if-you-think-quantum-physics-is-weird-try-these-theori

7 OTHER SCIENTIFIC ACHIEVEMENTS

7.1 Bibliometric data

Source: Google Scholar (19.09.2024)

- Number of peer-reviewed publications: 31 (21 after PhD)
- Number of online pre-prints: 8
- Total number of citations: 985
- H-index: 18

Source: Web of Science (19.09.2024)

- Number of peer-reviewed publications: 21 (11 after PhD)
- Total number of citations: 372 (281 without self citations)
- H-index: 14

7.2 Track record before PhD

The research output within my PhD thesis [1] may be presented in the three following topics.

1. Emergence of classicality.–

We studied the emergence of classicality within quantum theory, in particular, within the decoherence paradigm [Zur09]. Typically, this approach focuses just on the emergence of classical states, i.e., via the observation that when quantum states undergo decoherence they become diagonal in some basis, and so the space of decohered states is isomorphic to a space of probability distributions. In our work, on the other hand, we take a step beyond this by understanding how the entire theory of classical stochastic processes emerges from quantum theory [3]. In fact, we do so in two different ways which we show are equivalent to one another. The first is via the Karoubi envelope, which can be thought of as a formalisation of the decoherence paradigm, the second is via the biproduct completion, which can be thought of as a way to incorporate classical uncertainty about which system one has into the theory [4]. In each case we obtain the category of complex matrix algebras, which, contains classical stochastic processes as a full subcategory. Looking to more general theories, we showed that a necessary feature of a theory in order to permit for classicality to emerge via a decoherence mechanism was that the theory must have entangled states [5]. This was quite a surprising result as entanglement is normally viewed as an obstruction to classicality rather than something that is needed for the emergence of the classical world.

2. Reconstructing quantum theory.–

In Ref. [6] we show how one can reconstruct the mathematical formalism of quantum theory from diagrammatic postulates. Aside from the focus on diagrammatic postulates, the key differentiating feature of this reconstruction to any others at the time, was that it reproduced the full process-theoretic description of quantum theory, consisting of composite classical-quantum systems, superselected quantum systems, and their interactions. Having reconstructed this version of quantum theory, we then presented two ways to single out the fully quantum systems (i.e., those with no superselection rule), the first is a ‘no-leaking’ postulate (roughly, that information gain causes disturbance), and the second, a ‘purity-of-cups’ postulate (roughly the existence of maximally entangled states).

3. Investigating post-quantum theories.–

The final part of my thesis involved asking whether it is possible that there could be some post-quantum theory which has a decoherence-like mechanism which serves as an explanation for why we have not yet seen any evidence for the theory in our current experiments. Such a mechanism is known as *hyperdecoherence* and was first proposed in Ref. [Życ08]. We prove a no-go theorem for such a theory, namely that any theory which hyperdecoheres to quantum theory must violate either the principle of causality, the purification postulate, or both [7]. What is particularly interesting about this result, is that it allows us to gain insight into a more fundamental physical theory without needing to propose any specific model for it.

During my PhD I worked on a number of other projects which did not form part of the PhD thesis.

4. Computation in generalised probabilistic theories.—

We investigated various aspects of quantum computation within the framework of generalised probabilistic theories. In particular, we showed how the phase kick-back mechanism [8] and Grover's lower bound [9] could be derived from physical principles, and showed how one could understand computational oracles within the generalised probabilistic theory framework [10].

5. Higher-order interference.—

While quantum interference has long been considered a hallmark of quantum theory, there is a limit to how far quantum interference diverges from our classical intuitions. Specifically, it is limited to being second-order in a hierarchy defined by Sorkin in Ref. [Sor94]. There have been various attempts to experimentally verify that quantum theory is indeed limited to second-order interference, and so far it has withstood these tests [SCM+09, PML12]. It is interesting, however, to see if we can theoretically understand why nature should be limited to only second-order interference. In Ref. [11] we put forward an argument based on physical principles as to why this should be the case. In Ref. [12] we also investigate this phenomena, and show that there would be a computational advantage if nature shows higher than second order interference.

7.3 Additional track record after PhD

Research that I carried out after my PhD, and which is not part of this Habilitation achievement, includes the following topic.

1. Complete extension postulate.–

In Ref. [13] we investigate an alternative to the purification postulate which was introduced in [CDP10] within the context of a reconstruction of quantum theory. This postulate is extremely useful for proving many results regarding generalised probabilistic theories, however, it substantially limits the scope of applicability of the results. For instance, neither classical theory nor boxworld [Bar07] satisfy this postulate. It is therefore interesting to ask whether there is some alternative postulate which preserves many of the useful features of the purification postulate, but which applies to a broader class of theories. We show that this is indeed the case by defining such a postulate, which we call the complete extension postulate. We investigate its properties, its relation to purification, and show that we can reprove certain results (such as the result for integer-commitment presented in Sec. 5.4.1) which previously relied on the purification postulate.

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