Abstract

The aim of this dissertation is a systematic analysis of any positive map that appears in quantum information theory. In particular, we will show how any map can be characterized in terms of k-positivity using the auxiliary parameter μ_k which determines the k-positivity of the considered maps. We discuss here the problems of the structure of k-positive maps on low-dimensional matrix algebras.

The first chapter is a short introduction and presents the motivation for studying positive maps in quantum information theory.

The second chapter of the dissertation is a reminder of the basic definitions and facts related to the concept of convexity, Hilbert spaces, objects living in these spaces, and the concept of quantum entanglement, as well as the tools needed to describe the measurements and detection of entanglement in the formalism of quantum mechanics.

The third chapter is a reminder of the basic properties of positive maps, some of their classes and representations, as well as known constructions. We discuss here issues related to the characterization of k-positive maps and their additional properties such as decomposability. We show commonly used methods for studying positive maps and discuss in detail some known constructions of positive maps as well as their limitations. These limitations in particular motivate us to present a new method for constructing k-positive maps that are not $(k+1)$-positive.

Chapter four presents the original concept of characterizing any positive map using the Choi-Kraus-Stinespring representation, in particular, it presents the characterization of k-positivity using the auxiliary parameter μ_k. We show that in the special case when the dimensions between which the map differs by 1, we can analytically calculate the parameter μ_k for which the map is k-positive. We present the application of the given characterization using a broadly described example of the proposed maps. In particular, we consider modifications and generalizations of the proposed maps to obtain certain properties such as indecomposability.

In chapter five, we apply our characterization to the Miller-Olkiewicz map, which is exposed in the cone of positive maps, and we conclude that by making a certain modification, by adding the $\lambda \operatorname{Tr}$ map in such a way, that 2-positivity implies a complete positivity, which translates into the following geometrical interpretation: by moving the exposed Miller-Olkiewicz map towards the complete positive maps through the addition of $\lambda \operatorname{Tr}$, for $\lambda \in \mathbb{R}$ we show that the cone of 2-positive maps and the cone of completely positive maps must have a common face structure. Additionally, we present a procedure that allows examining the properties of the cone of k-positive maps, using the Choi map as an example.

The dissertation ends with a short summary of the presented material, containing selected suggestions for further research directions.