SUMMARY OF PROFESSIONAL ACCOMPLISHMENTS

1. Name: **Maciej Mroczkowski**

2. Diplomas, degrees conferred in specific areas of science or arts, including the name of the institution which conferred the degree, year of degree conferment, title of the PhD dissertation:
• Doctorate of Philosophy in Mathematics awarded by the University of Uppsala in 2004. Title of the PhD dissertation: Projective Links and Their Invariants
• Engineering degree obtained in 2000 (promotion year 1999) at the Higher National School of Telecommunications of Brittany (ENST Bretagne)

3. Information on employment in research institutes or faculties/departments or school of arts: Assistant Professor at the Institute of Mathematics of the University of Gdańsk, employed since October 2004

4. Description of the achievements, set out in art. 219 para 1 point 2 of the Act:

Monothematical cycle of 10 publications under the title:

Arrow diagrams with applications to skein modules and classical knots

consisting of the following publications:

[H1] M. Mroczkowski, M. Dąbkowski, *KBSM of the product of a disk with two holes and S^1*, Topology and its Applications **156** (2009), 1831–1849.

[H2] M. Mroczkowski, *KBSM of the connected sum of two projective spaces*, J. Knot Theory and its Ramifications **20** (2011), no. 5. 651–675.

[H3] M. Mroczkowski, *KBSM of a family of prism manifolds*, J. Knot Theory and its Ramifications **20** (2011), no. 1. 159–170.

[H4] B. Gabrovsek, M. Mroczkowski, *The HOMFLYPT skein module of the lens spaces*, Topology and its Applications **175** (2014), nr. 9, 72-80

[H5] B. Gabrovsek, M. Mroczkowski, *Link diagrams and applications to skein modules*, Algebraic Modeling of Topological and Computational Structures and Applications, Springer Proceedings in Mathematics & Statistics book series **219** (2017).

[H6] M. Mroczkowski *The Dubrovnik and Kauffman skein modules of the lens spaces $L_{p,1}$*, J. Knot Theory Ramifications **27** (2018), no. 3, 1840004.

[H7] M. Mroczkowski, *Knots with Hopf crossing number at most one*, Osaka J. Math. **57** (2020), no. 2, 279–304.

[H8] M. Mroczkowski, *On some moves on links and the Hopf crossing number*, Mediterr. J. Math. **18**, 7 (2021).

[H9] M. Mroczkowski, *On two crossing numbers of algebraic knots under Hopf fibration*, Topology and its Applications, **312** (2022), 108084.

[H10] M. Mroczkowski, *Infinitely many roots of unity are zeros of some Jones polynomials*, Geom. Dedicata **216** (2022), no. 4, Paper No. 43.

In the following summary, the publications [H1]-[H10] refer to the cycle above, the publications [AD1], [AD2], [PD1], [PD2] refer to my other publications (page

7 of this summary), whereas the remaining citations concern publications of other authors (at the end of the summary).

4.1 Introduction

The classical knot theory very often makes use of the notion of *diagrams*, or projections of knots (or links) from $\mathbb{R}^3$ to $\mathbb{R}^2$. On the *shadows* or immersed curves obtained in this way, the extra information of tunnel-bridge is added for each crossing. Ambient isotopies of links correspond to series of *Reidemeister moves* on diagrams [Rei27]. There are three types of such moves. Link diagrams together with Reidemeister moves give a very useful way for constructing and studying *link invariants*: a map from diagrams to arbitrary objects (set of numbers, polynomials...) will be an invariant, if its value remains unchanged under Reidemeister moves. The blossoming in the study of such invariants took place in the eighties of the last century, then later on, and it starts with the Jones polynomial [Jon85].

The generalization of link invariants discovered after the Jones polynomial, from links in $\mathbb{R}^3$ (or S^3) to links in other 3-manifolds, was initiated by the introduction of *skein modules* [Prz91, Tur88]. The computations of the different types of skein modules turned out to be quite difficult in general. The most popular of them, the Kauffman bracket skein module, is computed, until now, for a very modest number of families of 3-manifolds (for instance, it is not computed for all Seifert fibered spaces). Other skein modules are not computed even for the simplest families, such as lens spaces. Thanks to the introduction of the arrow diagrams in [H1], some of these problems could be partially solved.

In the recent years there has been a great interest in the study of skein modules. It is worth mentionning the papers [BW16], [FKBL19] which concern Kauffman bracket skein algebras, or [GJS23], where the authors solve the Witten conjecture concerning the finiteness of the dimension of the Kauffman bracket skein modules for closed 3-manifolds (with coefficients in $\mathbb{Q}(A)$).

For links in S^3 there is a possibility of projecting them to S^2 by using the Hopf fibration. This gives naturally arrow diagrams of classical links with six Reidemeister moves. Such alternative projections of classical links were considered in [Fie91] and [Tur92]. Except for a few papers from the end of the last century, such projections were not studied until the papers [H7]-[H10], where new invariants, moves on links and examples of knots with interesting Jones polynomials were discovered. Also, a problem from [Fie91] was solved.

4.2 Arrow diagrams

The arrow diagrams were introduced in [H1]. In that paper, links in manifolds $F\times S^1$, where F is an orientable surface (with or without boundary), are represented with arrow diagrams. These are closely related to the so called *gleams* introduced by Turaev in [Tur92]. Arrow diagrams consist of generically immersed curves on the surface F, with extra information tunnel-bridge on the crossings and some additional arrows on the curves, outside the crossings. Except for the arrows, these diagrams look like classical diagrams for $F \times I$, where $I = [0, 1]$. Arrow diagrams are obtained by cutting $F \times S^1$ - in the $F \times I$ thus obtained, the arrows keep the information about the places in which the link in $F \times S^1$ was cut. Ambient isotopies of links correspond to Reidemeister moves: three classical and two extra ones, shown in fig. 1. In this way, the study of links in $F \times S^1$ is reduced to the study of arrow diagrams together with five Reidemeister moves.

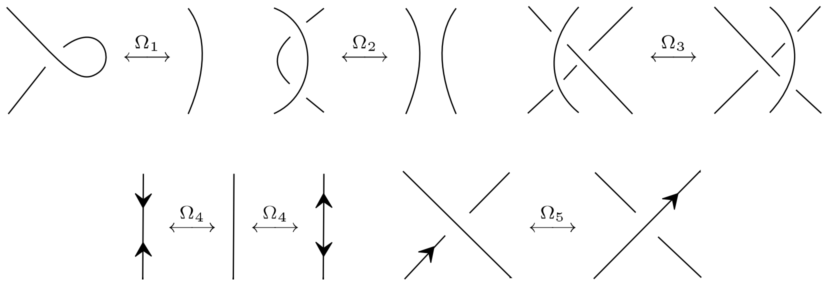

FIGURE 1. Reidemeister moves

The arrow diagrams were extended for links in twisted S^1 bundles over the unorientable surfaces and, more generally, for links in Seifert fibered spaces in [H2] and [H3] (with a summary in [H5]). In each case one adds some appropriate Reidemeister moves to the basic five moves. The arrow diagrams are a useful tool for studying links in these different manifolds. They have been mainly used in the computation of skein modules of some manifolds (section 4.3), as well as in the study of classical knots, i.e. knots in S^3 (section 4.4).

4.3 SKEIN MODULES

Skein modules where introduced by J. Przytycki [Prz91] and V. Turaev [Tur88]. Through skein modules, one can generalize in a natural way some invariants of links in S^3 to links in other 3-manifolds, orientable in the majority of cases. On the one hand, a concrete skein module of some 3-manifold M allows to study links in M; on the other hand, computing the skein modules of whole families of 3-manifolds can yield some information about these 3-manifolds. There exist several skein modules, their definition depending on the chosen skein relation.

The most studied skein modules are the Kauffman bracket skein modules (or KBSM for short), [Prz99]. Other important cases are HOMFLYPT skein modules or Kauffman skein modules. In their definition one uses the skein relation of the HOMFLYPT polynomial [FYH$^+$85, PT88] and, respectively, the definition of the Kauffman polynomial [Kau90].

The paramount importance of the KBSM's is due to their relationship with the representations of the fundamental groups of the 3-manifolds in $SL_2(\mathbb{C})$, discovered by D. Bullock [Bul97a]. The KBSM's, which depend on the choice of the coefficients such as $\mathbb{Z}[A, A^{-1}]$, $\mathbb{C}[A, A^{-1}]$ or, possibly, by allowing some elements to be invertible, have been computed in a few cases only, such as: the lens spaces [HP93], $S^1 \times S^2$ [HP95], the complements of torus knots $(2, 2p+1)$ [Bul95], surgeries on the trefoil [Bul97b], the quaternionic space [GH07], $\mathbb{R}P^3 \natural \mathbb{R}P^3$ [H2], an infinite family of the prism manifolds [H3], the complements of the torus knots [Mar10] or the complements of the 2-bridge links [LT14]. For more general families, generating sets of the KBSM's have been found, for example for a family of Seifert fibered spaces in [AF22].

J. Przytycki has studied the relationship beetween the existence of incompressible nonboundary surfaces in a 3-manifold and the torsion in the Kauffman bracket skein module of this 3-manifold. He has exhibited such a relationship under some assumptions [Prz99]. Here, an interesting case is the manifold $M = F \times S^1$, where F is a disk with two holes. M contains an immersed torus, but it does not contain an embedded one. Przytycki was interested in the presence of torsion in the KBSM of M. In the paper [H1] this skein module has been computed: it is free with an infinite basis. The main tool in the proof is the notion of arrow diagrams, introduced in this paper. In several steps are computed the KBSM's of: the solid torus, the thickened torus (these cases were computed before) and, finally, the manifold M. Thanks to the arrow diagrams it is possible to keep track of all the skein relations in the module, which allows to find the generators and prove that the skein module is free.

In the papers [H2] and [H3] the arrow diagrams were extended for links in twisted S^1 bundles over unorientable surfaces and, more generally, for links in Seifert fibered spaces. In [H2], the KBSM of the twisted S^1 bundle over the real projective plane, homeomorphic to $\mathbb{R}P^3 \sharp \mathbb{R}P^3$, was computed. It was the first complete computation of the KBSM of a connected sum of two 3-manifolds, as well as of a manifold containing a separating surface. The structure of this skein module is relatively complicated: it contains torsion elements and it does not split into a sum of cyclic modules. Also in [H2] are re-computed the KBSM's of the lens spaces $L_{p,1}$ and of $S^2 \times S^1$. In the second case, especially, the proof is much shorter and simpler than in [HP95].

By adding fibers to $\mathbb{R}P^3 \sharp \mathbb{R}P^3$, one gets prism manifolds. In some simple cases, when the fiber is of type $(p, 1)$ so that it is not special, one gets the prims manifolds with first homology group of order 4. For this family of manifolds, the KBSM's have been computed in [H3]. In this case they have a simple structure: they are free, finitely generated, and the dimension depends in a simple way on p. It is worth mentionning, that this family of prism manifolds contains the quaternionic manifold, for which the KBSM has been computed in [GH07].

The arrow diagrams method has been used by other authors. Carrega has shown in [Car17], that the Kauffman bracket skein module with coefficients in $\mathbb{Q}(A)$ (thus allowing all polynomials in A to be invertible) of the 3-torus is generated by 9 elements. This result has been generalized by Detcherry and Wolff, in [DW21], to the product of S^1 and an arbitrary orientable surface without boundary. In both papers arrow diagrams are essential in the proofs. Also, Arand and Ferguson have made use in [AF22] of arrow diagrams to find some sets of generators for the KBSM's of some family of Seifert fibered spaces.

Further computations of the KBSM's with coefficients in $\mathbb{Q}(A)$, such as carried out in [Det21], reinforced the so called Witten hypothesis, which states, that the KBSM with coefficiens in $\mathbb{Q}(A)$ of a closed 3-manifold M is finite dimensional (for a formulation see for instance [Det21]). The Witten hypothesis has been recently proved in [GJS23]. This opens new fields for some very interesting research on 3-manifolds. In particular, the computation and interpretation of the finite dimension for concrete manifolds is in its beginnings.

Also in [Det21] some generalizations of the Witten hypothesis to manifolds with boundary are proposed (Conj. 3.2 i 3.3). In this paper, the manifold M from [H1] constitutes an important example reinforcing hypothesis 3.3.

The Witten hypothesis has been also extended in a different direction, namely to the KBSM of a closed manifold with coefficients in $R = \mathbb{Z}[A, A^{-1}]$ (Marche hypothesis or hypothesis 1.1 in [DW21]). This extended hypothesis says, that the KBSM splits into R^d (d a natural number) and a sum (possibly infinite) of modules N_k, where N_k is a $A^k - A^{-k}$-torsion module. It was observed in [BP22], that there is a counterexample to this extended hypothesis, namely the manifold $M = \mathbb{R}P^3 \sharp \mathbb{R}P^3$, for which the KBSM has been computed in [H2]. It was shown in [H2] (Prop. 4.19), that the KBSM of M does not split into free and cyclic modules (such as the modules N_k). It follows that Marche hypothesis is not true - the skein module structure may be more complicated.

Apart from the Kauffman bracket skein modules, other important examples are, as was mentionned at the beginning of this section, the HOMFLYPT and the Kauffman skein modules. The computations of these skein modules are more complicated than in the case of the Kauffman bracket: there are very few cases of 3-manifolds with such computations carried out completely. None of these skein modules has been computed for all lens spaces [DLP16, DL17, DL19]. An important result in this direction is the computation in [H4] of the HOMFLYPT skein module of the lens spaces $L_{p,1}$, $p > 1$. These skein modules are free. Infinite bases are given for each of them. For the lens spaces $L_{p,q}$, with $q > 1$, the situation gets very complicated. The methods from [H4] may be used only to find a set of generators for $L_{p,2}$. When $q > 2$, these methods do not yield even such a set of generators.

By extending the methods of [H4], it was possible in [H6] to compute the Kauffman skein modules of $L_{p,1}$, for odd p. It was also shown, that there is torsion in these skein modules when p is even. There exists a variant of the Kauffman polynomial, the so called Dubrovnik polynomial. For classical links, it is equivalent to the Kauffman polynomial. By using in [H6] the Dubrovnik skein relation instead of the Kauffman one, the Dubrovnik skein modules of all lens spaces $L_{p,1}$, $p > 1$, have been computed. It turns out, that in this case all modules are free, infinitely generated, similarly to the case of HOMFLYPT skein modules from [H4].

Both in [H4] and [H6], the computation of the skein modules relies completly on arrow diagrams and the related Reidemeister moves.

4.4 Knots and the Hopf fibration

Another important use of arrow diagrams is the study of classical knots or links (i.e. in S^3). Traditionally, the links in $\mathbb{R}^3$ (which is equivalent to links in S^3) are studied with the help of diagrams - these are projections of links from $\mathbb{R}^3$ to $\mathbb{R}^2$, keeping the extra information tunnel-bridge on the crossings. Another possibility is to use the Hopf fibration $p : S^3 \to S^2$ with which one may project links along the fibers S^1 onto the sphere S^2. By cutting the fibers, one gets in a natural way arrow diagrams in a disk, with an extra Reidemeister move, fig. 2. This is explained in detail in [H7]. By studying some properties of the arrow diagrams of knots, for example the minimal number of the crossings, one gets new invariants of classical knots (and links).

The idea of studying knots with the help of the Hopf fibration has been Fiedler's in [Fie91]. Later, Turaev introduced in [Tur92] the notion of *gleams* - diagrams in S^2 with the corresponding 2-cells being colored with integer numbers. Such gleams correspond to arrow diagrams of links in S^1 bundles over S^2 without special fibers (i.e. of links in lens spaces $L_{p,1}$, in particular $L_{1,1} = S^3$). Two interesting results about gleams are related to Vassiliev invariants (or finite type invariants). In 1990, Viro formulated the following hypothesis: the Vassiliev invariants are polynomials

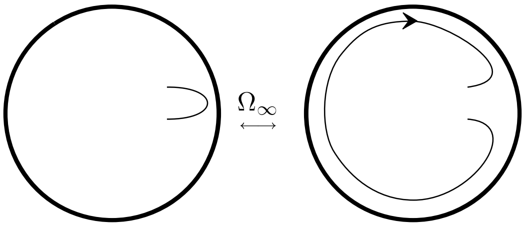

FIGURE 2. extra Reidemeister move

in the gleams (meaning, that they are polynomials in the variables assigned to the 2-cells of a fixed diagram, taking integer values). Burri in [Bur97] has shown that the hypothesis is true for some special Vassiliev invariants coming from the Jones polynomial. In particular, he obtained an interesting formula for the Vassiliev invariant of degree 2: it is a polynomial of degree 4 in the gleams. Then Goussarov, in [Gou98], has proven the Viro hypothesis, by showing that any Vassiliev invariant of degree n is a polynomial of degree $2n$ in the gleams. In the following years, there were no studies on classical knots using the Hopf fibration. Such studies were only renewed in papers [H7]-[H10].

In [Fie91], Fiedler considers two types of crossing numbers for algebraic knots (i.e. knots obtained from singularities of algebraic curves). For an algebraic knot K, the first crossing number, $C_{alg}(K)$, is the minimal number of crossings of K under the Hopf fibration, where we allow only the realizations of K coming from algebraic curves, whereas for the second crossing number, $h(K)$ (the notation is not from [Fie91]), we allow any (topological) realization of K. Obviously, $h(K) \leqslant C_{alg}(K)$. Fiedler finds some lower bounds for $C_{alg}(K)$ for some algebraic knots K and asks the question, whether this inequality is always an equality. He also observes, that almost nothing is known about the invariant h.

In [H7], using arrow diagrams, all knots K with $h(K) \leqslant 1$ are classified. It is possible thanks to a closed formula for the Jones polynomial of such knots, which distinguishes them all (except for a pair that can be easily distinguished in a different way). The knots from the Rolfsen table (up to 10 classical crossings) with $h(K) \leqslant 1$ are identified. It is also shown, that $h(K) = C_{alg}(K)$ for algebraic knots which satisfy $h(K) \leqslant 1$. In particular, this requires the classification of all algebraic knots satisfying $h(K) \leqslant 1$.

It turns out, that, in general, for algebaric knots the equality $h(K) = C_{alg}(K)$ does not always hold. In [H9] examples of knots K are constructed for which $C_{alg}(K) - h(K)$ can be arbitrarly large. This answers the question posed by Fiedler. The proof is based on the realization that for a classical diagram of a knot K with $c(K)$ crossings, containing a twist with m crossings, $h(K)$ is lower thant $c(K)$ by an amount of at least (roughly) $m/2$. This yields examples for which $C_{alg}(K)$ is estimated from below (thanks to an inequality in [Fie91]), $h(K)$ from above and the difference between the estimates can be arbitrarly large. It follows also, that there is a large gap between $c(K)$ and $h(k)$ for torus knots $K = T(2, n)$ or twist knots (roughly $h(K) \leqslant c(K)/2$). However, no lower bounds for $h(K)$ are known for these knots.

In knot theory one considers often different families of moves which divide links or knots into equivalence classes (two links being in the same class if one can pass from one to the other with a series of the chosen moves). Examples of such moves appear already in Fox [Fox58]. Some of the simplest such moves, the so called t_k moves, which consist in adding or removing a twist with k crossings between two strands in a link, are analyzed in [Prz88]. It is shown there, that the values of the Jones polynomial in some appropriate roots of unity (depending on k) are unchanged (or change in a highly controled way), when one performs t_k moves.

Using arrow diagrams, new moves were introduced in [H8], the k-moves, defined in a simple way by adding or removing k arrows on a single strand (without crossings separating such arrows). In that paper, it is shown that the values of the Jones polynomial in the k-th roots of unity do not change under a k-move, if k is odd, whereas they are multiplied by -1, if k is even. One of the fundamental questions to consider for some chosen moves it the question whether all knots lie in a single class, in other words, whether any knot can be made trivial by applying a series of the chosen moves. Simiarly to the t_k moves, it truns out that for the k-moves the most difficult cases are $k = 3$ and $k = 4$, the only cases for which the above question is open. When $k < 3$ any knot can be made trivial with k-moves. When $k > 4$ there are infinitely many equivalence classes of knots under k-moves. If studying the k-moves is interesting in itself, they appear also as a useful tool in studying the invariant h, since an infinite number of knots with bounded h gives a finite number of equivalence classes after dividing with k-moves. This tool may be very useful in the computations of h for concrete knots (for instance the knots from Rolfsen's table).

By studying knots with small h and their Jones polynomials, interesting families of knots with cyclotomic Jones polynomials were discovered in [H10]. Such knots were considered in [CK05] i [CK06]. In both papers the authors study the Mahler measure of the Jones polynomial. It is shown, that under some operations this measure behaves in a similar way to the hyperbolic volume. In [CK05], the authors pose the problem of constructing knots that have Jones polynomials with Mahler measure 1 (so that these polynomials are cyclotomic or products of cyclotomic polynomials). The paper [H10] gives a solution to this problem: four infinite families of knots having Jones polynomials with Mahler measure 1 are constructed. It is also shown, that inifinitely many roots of unity are zeros of Jones polynomials of knots. Furthermore, such zeros are dense in the unit circle. This can be compared to the theorem of [JZDT10], whch states that zeros of Jones polynomials are dense in $\mathbb{C}$. The knots in [H10] have relatively simple arrow diagrams. This is a good motivation to study such simple diagrams, for example when the number of crossings is low. In some cases, it is possible to get closed formulas for the Jones polynomial.

5. Summary of other research achievements.

Articles in journals before the PhD

[AD1] M. Mroczkowski, *Diagrammatic unknotting of knots and links in the projective space*, Journal of Knot Theory and Ramifications **12** (2003), no. 5. 637–651.

[AD2] M. Mroczkowski, *Polynomial invariants of links in the projective space*, Proceedings of Knots in Poland 2003, Fundamenta Mathematicae **184** (2004), 223–267.

Articles in journals after the PhD

[PD1] J. Malesic, M. Mroczkowski *Meridional number of a link and skein modules of the solid torus*, Topology and its Applications **159** (2012), no. 8, 2021-2031.

[PD2] B. Gabrovsek, M. Mroczkowski, *Knots in the solid torus up to 6 crossings*, Journal of Knot Theory and Ramifications **21** (2012), no. 11. 1-43.

5.1 Knots and links in the real projective space

The papers [AD1] and [AD2], forming the doctorate, are about links in the real projective space $\mathbb{R}P^3$. The research on links in this 3-manifold was initiated by J. Drobotukhina in [Dro90, Dro91, Dro94]. It is based on diagrams of links in the real projective space: these are diagrams in a disk consisting of closed curves and arcs; the endpoints of the arcs lie on the boundary of the disk, grouped into antipodal pairs. Ambient isotopies of links correspond to five Reidemeister moves on such diagrams (three classical and two extra moves, which are *not* related to the Reidemeister moves of arrow diagrams). Using a generalization of the Jones polynomial to links in $\mathbb{R}P^3$ [Dro90], Drobotukhina obtains the classification of links up to 6 crossings [Dro94].

In [AD1] the problem of *descending diagrams* for knots and links in $\mathbb{R}P^3$ is considered. Since $H_1(\mathbb{R}P^3) = \mathbb{Z}_2$, there are 2 types of knots. A natural choice of 2 trivial knots are knots which have diagrams with no crossings. In the case of links which consist of at least two non homologically trivial components, the choice of a trivial link is not obvious, though one can choose a definition of such a link in a natural way. It is shown in [AD1], that from any diagram of any knot one can obtain the trivial knot in the homology class of the original knot, by changing some subset of the crossings of the diagram. This is no longer possible with links, which is demonstrated with a counterexample consisting of 4 non homologically trivial components. In [AD2], the tools from [AD1] are used to generalize the HOMFLYPT polynomial to links in $\mathbb{R}P^3$. As an application, a conctruction of knots with any *distance from affinity* is given. This invariant is defined as the minimal number of antipodal pairs of endpoints of arcs amongst all diagrams of a knot (or link).

5.2 Knots and links in the solid torus

In [PD1] the *meridional number* of links in the solid torus T is considered. It was introduced in [Mal95]. For a given link L, this number, denoted $v(L)$, is the maximal number of meridional disks in T such that L can be isotoped into a link for which no arc is joining two different discs. For instance, taking a simple chain C_n going aroung T with n components, $v(C_n) = n$. To get bounds from below for v, one has to construct an appropriate number of disks so that no arc joins two different discs. The bounds from above for v are obtained from appropriate coefficients of links in different skein modules of the solid torus: the HOMFLYPT, Kauffman, Kauffman bracket and homotopy skein modules. Using the Kauffman bracket skein module, it is shown, that $v(B_n) = 2n$ for the Bing links B_n.

In [PD2] the prime knots up to 6 crossings in the solid torus where classified (there are 526 such knots). In this case the knots are presented with classical diagrams in the annulus (and not arrow diagrams in a disk). To distinguish between diagrams skein modules are used, mostly KBSM but, in a few cases, also Kauffman and HOMFLYPT skein modules. Amphicheiral knots are detected (24 cases). For a knot K in the solid torus, its *wrapping number* is the minimal number of intersections between K and a meridional disk, up to isotopy. The *wrapping conjecture* states, that the wrapping number equals the degree in x of K expressed in the KBSM of the solid torus (where x^k stands for k concentric circles going around the torus).

From the computations in [PD2] it follows, that the wrapping conjecture is true for knots up to 6 crossings.

6. Presentation of significant scientific or artistic activity carried out at more than one university, scientific or cultural institution, especially at foreign institutions
• Cooperation with M. Dąbkowski from UT Dallas resulting in the paper [H1]. As part of this cooperation I gave the following talks during seminars at UT Dallas:
- Skein modules of twisted I bundles over unoriented surfaces, September 2007
- KBSM of the product of a disk with two holes and S^1, September 2008
• Cooperation for several years with J. Malesic and B. Gabrovsek from the University of Ljubljana, resulting in papers [H4], [H5], [PD1], [PD2]. As a result of this cooperation, I was also a co-promotor for the doctorate of B. Gabrovsek. During this cooperation, I gave a talk in Ljubljana, February 2009, Kauffman bracket skein module of the product of a disk with two holes and S^1.
• Cooperation with S. Lambropoulou and I. Diamantis from National Technical Univeristy of Athens and B. Gabrovsek from the University of Ljubljana during the project: THALIS - NTUA - Algebraic modeling of topological and computational structures and applications, since 2015, in progress.

7. Presentation of teaching and organizational achievements as well as achievements in popularization of science or art
• Promotor (auxilliary) for the doctorate of Bostjan Gabrovsek of the University of Ljubljana, 2013
• Promotor for 9 MSc and several BSc students.
• During many years I have taught several courses (lectures and exercise sessions) in mathematics and computer sciences both before my PhD at the University of Uppsala, and after at the University of Gdansk: Fourier series, functional analysis, differential geometry, general topology, number theory, calculus, linear algebra, introduction to programming, data bases...
• Co-organizer of 3 conferences *Knots in Gdansk*, during 2017-2019, with, as participants, world specialists in knot theory.
• Jury member for the contest: mathematical fairy tale, organized by the University of Gdansk in 2015.
• Organizer of the contest: mathematical comics during The Year of Mathematics in Pomerania 2015.

Maciej Mroczkowski
(Applicant's signature)

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