Review of scientific achievements of Maciej Mroczkowski

This document is a review of the scientific achievements of Dr. Maciej Mroczkowski (University of Gdańsk) in the procedure of conferment of the degree of habilitated doctor.

Summary

Dr. Maciej Mroczkowski's field of research is low-dimensional topology. His publication record counts the ten papers in the cycle "Arrow diagrams with applications to skein modules and classical knots" and four more publications, two publications before PhD (as single author) and two after PhD (in collaboration). His results have been published in top level international journals and they have been presented in international conferences on several occasions. Dr. Maciej Mroczkowski has also been contractor for research grants in five occasions (two at international level). This is evidence that his research output is recognised internationally.

Cycle of papers "Arrow diagrams with applications to skein modules and classical knots"

This cycle of papers consists of seven single-authored papers and three in collaboration. The major part of the papers in the cycle study problems in a branch of low-dimensional theory called skein theory. Skein modules were introduced independently by Przytycki and Turaev in the early 90s to study 3-manifolds using tools from knot theory. Skein modules are usually very difficult to compute, which makes hard to study their properties, like for example having or not torsion. At the current state of affairs they have been computed for some families of 3-manifolds. Computation techniques are therefore desirable. One of the main ingredients of the results obtained in this cycle of ten papers is the development a technology called *arrow diagrams*: it consists of a theory of diagrams together with a Reidmeister theorem for links in 3-manifolds that are more general than the 3-sphere. As in knot theory in ordinary 3-space, *arrow diagrams* have the advantage of allowing the use of combinatorial methods to do computations. The arrow diagrams that are introduced in this cycle of publications allow computing skein modules for several manifolds. The results obtained are nontrivial and were inaccessible before the development of *arrow diagrams*. Dr. Maciej Mroczkowski concentrates initially on the *Kauffman Bracket Skein Module* (KBSM) and later in the HOMFLYPT skein module and the Dubrovnik and Kauffman polynomial skein modules, covering the skein modules corresponding to the four classical families of quantum link invariants (associated to the Lie algebras of types A_n, B_n, C_n and D_n). Among other important results (mentioned below) in the first six papers, the first paper **answers an open question** by Przytycki (2002) about torsion of a certain skein module. The set of the last four papers is mainly dedicated to the study of link diagrams on the 2-sphere that are defined from links in the 3-sphere using the Hopf map. It worth to mention that *arrow diagrams* are omnipresent and play a fundamental role. **Several open questions about algebraic knots are answered** (these questions were posed by Fiedler in 1991). The last paper concentrates on the Jones polynomial and approaches the question of finding whether a given Laurent polynomial is the Jones polynomial of some link. Among other results, he constructs infinite families of knots whose Jones polynomial have a certain form, with Mahler measure equal to one, **answering an open question** by Champanerkar and Kofman (2005) about the existence of such a family.

Here is a summary of the results in this cycle.

H1: *Topology and its Applications*, 2009 (with M. K. Dabkowski)

Computes the KBSM for the 3-manifolds $D_k \times S^1$, with D_k being a disk with k holes, for $k = 0, 1, 2$. They also show that in the case of D_2 the module is free. The latter result **answers a question** of Przytycki from the list in *Problems on invariants of knots and 3-manifolds* [Oht02, Problem 4.4] published in 2022 (and supports Conjecture 4.3 in [Oht02]). A fundamental role is played by *arrow diagrams* (link diagrams together with a Reidemeister theorem) for links in $F \times S^1$ with F an oriented manifolds, introduced in this paper.

H2: ***Journal of Knot Theory and its Ramifications*****, 2011 (single authored)**

Computes the KBSM of the connected sum of projective spaces $\mathbb{R}P^3\#\mathbb{R}P^3$, which is the first computation of this module for a closed non-prime manifold. It is found that the aforementioned skein module has torsion, providing the second ever fully computed KBSM with torsion. The techniques developed for the computation mentioned above are also used to provide alternative computations of the KBSM of $S^1 \times S^2$ and the lens space $L(p, 1)$.

H3: ***Journal of Knot Theory and its Ramifications*****, 2011 (single authored)**

This paper computes the KBSM of prism manifolds with first homology of order 4. Prism manifolds form a family of Seifert manifolds (3-manifolds admitting a certain fibration). This is made with the aid of the technology of arrow diagrams links in (oriented) Seifert manifolds, introduced in this paper. The arrow diagrams in this paper generalize existing constructions for links in $F \times S^1$ with F a surface (oriented or not. The case of F oriented was introduced by the author in H1).

H4: ***Topology and its Applications*****, 2014 (with B. Gabrovšek)**

This paper uses the technology of arrow diagrams (introduced in the first paper in this cycle) to compute the HOMFLYPT skein module of $L(p, 1)$. Bases of these spaces are also computed.

H5: book chapter, 2017 (with B. Gabrovšek)

This is a survey paper on H1 to H4. It also contains new sets of generators for the KBSM and the HOMFLYPT skein module of the solid torus and lens spaces.

H6: ***Journal of Knot Theory and its Ramifications*****, 2018 (single authored)**

This paper studies skein modules of lens spaces $L_{p,1}$ based on the Dubrovnik and Kauffman two-variable skein relations. While the KBSM and the HOMFLYPT skein module can be interpreted in the realms of the representation theory of (a quantized version of) Lie algebras of type A, the Dubrovnik and Kauffman polynomials have their interpretation on the B, C and D families. The work uses *arrow diagrams*, as developed in previous publications. It is proved that the Dubrovnik skein module of $L_{p,1}$ is free while the Kauffman 2-variable skein module is free if and only if for p is odd. Bases are exhibited in the cases where the corresponding skein module is free.

H7: ***Osaka Journal of Mathematics*****, 2020 (single authored)**

This publication studies diagrams in S^2 obtained from links in S^3 using the Hopf map. The minimal crossing number of such diagrams, called the *Hopf crossing number* and denoted $h(k)$, is considered. A classification of knots admitting a diagram with $h(k) \leq 1$ is provided. The Hopf fibration gives rise to a crossing number C_{alg} for algebraic knots that are realised around complex singularities. For an algebraic link L one has $h(L) \leq C_{alg}(L)$. The author **answers a question** posed by Fiedler [Fie91] for algebraic links with $h(L) \leq 1$, namely it is shown that the inequality is in fact an equality for knots K with $h(K) \leq 1$, answering Fiedler's question for this class of knots.

H8: ***Mediterranean Journal of Mathematics*****, 2021 (single authored)**

In this paper Dr. Mroczkowski introduces k-moves ($k \in \mathbb{N}$) on links in S^3. These moves are defined with the help of arrow diagrams from H1 and H7. Equivalence classes of links in S^3 under these moves are studied and it is shown that a k-move multiplies by $(-1)^{k+1}$ the Jones polynomial evaluated at a kth primitive root of unity. This result is then used to deduce that for $k > 5$ the set of equivalence classes under k-moves is infinite. They are also applied to study Hopf crossing number of knots: some families with unbounded h are exhibited. Finally, it is given an interpretation of k-moves in terms of some identification of links in different lens spaces $L_{p,1}$.

H9: *Topology and its Applications*, 2022 (single authored)

By finding an upper bound on the Hopf crossing number on some families of knots the author exhibits families of algebraic knots for which the difference $C_{alg}(K) - h(K)$ can be arbitrarily large, therefore **answering a question** by Fiedler [Fie91] concerning algebraic knots and the Hopf fibration.

H10: *Geometriae Dedicata*, 2022 (single authored)

Studying the zeros of Jones polynomials is useful to address the question whether a given polynomial is the Jones polynomial of a knot. The results in this article prove that the locus of the zeros of Jones polynomials intersected with the unit circle is dense in the unit circle. This paper studies knots which have Jones polynomials of Mahler measure equal to 1 and construct several infinite families of knots whose Jones polynomial has a prescribed form with Mahler measure 1. This **answers a question** of finding families of knots having Jones polynomials of Mahler measure 1 was posed bt by Champanerkar and Kofman in [ChCO05].

Conclusion

In my opinion, from the described above, Dr. Maciej Mroczkowski's scientific output in the field of low-dimensional topology gives a **significant contribution** to mathematics.

References

[ChCO05] Champanerkar, A. and Kofman, I. On the Mahler measure of Jones polynomials under twisting. *Algebr. Geom. Topol.* 5, 1–22 (2005).

[Fie91] Fiedler, T. Algebraic links and the Hopf fibration. *Topology* 30, No. 2, 259–265 (1991).

[Oht02] Ohtsuki, T. (ed.), Problems on invariants of knots and 3-manifolds. *Geom. Topol. Monogr.* 4, 377–572 (2002).