Report concerning the procedure for conferring the habilitation degree to Dr Barbara WOLNIK

Among the various models of computation, cellular automata (CA) are discrete dynamical systems where parallelism and locality are primordial. They are formed of elementary components, the cells, which are arranged on the discrete points of a grid of an arbitrary dimension. The cells interact according to a local rule, which means that each cell only sees the state of the cells situated within a given distance. Although they have the same computing power as Turing machines, it is not yet well understood how to program cellular automata in order to obtain a given behaviour, especially when we aim at using a small number of states and simple interaction rules.

The document submitted for the habilitation procedure of Dr Barbara Wolnik consists of a synthetic text which presents a sequence of articles ([H1] to [H8]) with a common theme and a short description of other research productions related to cellular automata. The document also describes the different activities of teaching and administration.

Articles on number-conserving CA

The main problem that has been tackled by Dr Barbara Wolnik is the analysis of the number-conservation property, that is, the preservation of the sum of states from on time step through the other. Indeed, although cellular automata are simply defined mathematically, it is a difficult problem to determine which are the quantities that a given cellular automaton conserves through its evolution, or even more difficult, to enumerate all the local rules that allow a system to be number-conserving.

In Ref. [H1], the foundations of the analytical work are laid. With her collaborators, Dr. Wolnik made a decisive step by proposing a formalism adapted to the the study of number-conserving CA (NCCA) in any dimension. More precisely, the local rules are decomposed in the way they act on simple neighbourhoods, called monomers and dimers. The main benefit of this approach is to provide a first means to enumerate all the NCCA with a small number of steps and a von Neumann neighbourhood thanks to a compact set of equations.

The second paper discussed [H2] exposes the core of the contributions of Dr. Wolnik, namely the split-and-perturb decomposition method. A significant progress is made with the idea to widen the set of states beyond the original set in order to allow one to decompose any local function as the sum of two functions, called the split function and the perturbation. The author shows that this decomposition is unique and leads to a compact description. There is a continuity with the previous work [H1] in the sense that split functions are defined on monomers and perturbations on dimers. This elegant result furthers simplifies the task of enumerating all the NCCAs of a given rule space and it is interesting to note that even with three states, the author easily succeeds to give a precise account of the number of such rules while the previous methods would require an enumeration that is out of reach with an exhaustive search.

The third, fourth and fifth articles [H3, H4, H5] present a clear answer to the question to know what is the “form” of the simplest NCCAs of an arbitrary dimension, namely those with binary states [H3], or those of the ternary rules which are reversible [H4]. The author raises a negative result by stating that these rules are somehow trivial, in the sense that they all amount to one-dimensional well-known rules (shifts and traffic rules). This results help us to understand the frontier between trivial behaviour and more elaborate rules. In the fifth article [H5], other sets of ternary rules explored and the same absence of non-trivial behaviour is noted. The scope of the study is widened to the reversibility property, which is known to be undecidable in all generality.

The case of rules with four states is dealt with in the sixth article [H6]. Surprisingly enough, Dr Wolnik shows that an appropriate way to describe the dynamics of such rules is to decompose them as a two-layer system. Thanks to this tool, the author succeeds in building non-trivial four-state rules which are reversible and number-conserving, but at the moment no technique has been found to exhibit all the NCCA rules of an arbitrary dimension. The author emits an interesting conjecture about their number, which shows that in any case, it is extremely small when compared to the total number of four-state rules. This shows us, if needed, that to find such rules, only mathematical tools designed with care can lead to a complete covering of these huge spaces of rules.

To coarsen the difficulty, the author and her co-authors examines the case of two-dimensional rules with a symmetry of rotation [H7]. Interestingly, one can note that in the first construction of a cellular automaton by von Neumann and Ulam in the 1950's to study self-reproduction, this constraint of being rotation-symmetric was imposed. The techniques developed by Dr Wolnik show their efficiency as it allows one to bridge the gap between a solution built by hand with fourteen steps and a general result on five-states rules. The case of states is fully answered for the natural set Q={0,…,6} and although a non-contiguous set of states seems less natural, results are given on more general set of states in terms of their equivalence, up to morphisms, to the "natural" set. In the last article presented [H8], an analysis of the three-dimensional case with the rotation-symmetric constraint is carried out and new insights are given on the frontier between the trivial behaviour and the dynamics specific to the three-dimensional case.

Other scientific productions

The scope of the research of Dr. Wolnik also covers other topics in the field of cellular automata.

One domain that was well explored by the author and her co-authors regards the case where we extend the set of states to a continuous interval, say [0,1]. She defined affine continuous cellular automata (ACCA) and made the relevant choice to apply the mathematical techniques she developed before to find original solutions to a relaxed version of the well-studied density classification problem. This is indeed one of the central inverse problems in the field of cellular automata, where one has to decide which state compose the majority of states in an initial condition. The difficulty of the problem stems from the fact that the answer has to be given in the form of a consensus on the states of cells, which generate a form of paradox between the local decisions and the global state that needs to be attained. The author's production consists both in issuing negative results for a "natural" definition of the problem and, maybe more surprisingly, a positive result on a relaxed definition of the problem where the final density is allowed to lie in some intervals close to 0 or 1. A direct study of the dynamics of ACCAs was also carried out, with an interesting complementarity between mathematical analysis and numerical simulations.

The author also carried out a series of investigations on triangular and hexagonal grids, which allowed her to understand more precisely the role of the topology in the solutions she obtained. Among the various interesting results obtained, it is certainly quite intriguing and unexpected that she could obtain a method for enumerating the NCCAs for all the "natural" set of states (that is, of the form Q={0,…,k}).

The work she carried out in collaboration with Witold Bołt and his co-authors regarding the identification problem also requires some attention. The question is to know how to identify the local rules which generate a particular behaviour represented by a given set of observations (space-time diagrams with incomplete information). The difficulty is that there is a huge set of rules available to “reconstruct” the observations, which calls for specific techniques to select the right local rules that totally or partially match a given set of observations. The production on this theme has raised a new set of stimulating questions and we hope that they will receive more echo in the scientific community.

Other activities

The involvement of Dr Wolnik in activities of teaching, organisation of events and popularisation of science is simply outstanding. The energy she has been investing in her lessons, talks, lectures for students and pupils, etc., is tremendous and needs to be pointed out as a decisive positive point in her academic career. I think it is needless to comment all the achievements in this domain since it has been recognised by many awards. I will simply add that, as a colleague, I have always appreciated the clarity of her talks given in international conferences and I can say that listening these talks has always been instructing and stimulating moments.

As stated in the document, the work carried out with her previous PhD student, Adam Dzedzej, is of the highest quality, and has been published in high-quality international journals. It is a positive point that Dr. Wolnik is currently supervising another PhD student and we have no doubt that numerous positive productions will result from this collaboration.

Summary

Given the elements stated above, it appears that the academic profile of Dr Wolnik is solid and up to the highest international standards. As an author, Dr Wolnik has demonstrated her ability to explore a large set of challenging problems. She has produced an impressive series of well-written high-quality articles, which constitute a substantial contribution to the theory of cellular automata and discrete complex systems. As a collaborator, she has been the source of many stimulating research directions which clearly triggered a positive dynamics for the persons she has been interacting with, whether be it in her university or in her various collaborations. Beyond a well-established knowledge in her field of research, Dr Wolnik has demonstrated a wide interest in various aspects of mathematics, with an outstanding passion in transmitting this interest to a varied public, both in her teaching activities or in the popularisation activities.

For all these reasons, I recommend without any reserve to accept Dr. Barbara Wolnik as the recipient of the habilitation degree of the University of Gdansk. I sincerely hope that this step will strengthen the scientific activities she has engaged, and which deserve to receive an ample support. As a member of the same scientific community, I testify that Dr. Wolnik is an estimated and appreciated person, and that we all hope that this habilitation degree will give her an impulse to encourage her and let her continue to advance on her scientific path.

Nazim FATÈS,

researcher at the Inria Institute for Research in Digital Science and Technology,
chair of the IFIP working group on cellular automata and discrete complex systems