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On a Comparative Analysis of Industrial Credit Portfolio Risk Models Versus a New Support Vector Machine - Based Approach.

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STRESZCZENIE

ANALIZA PORÓWNAWCZA PRZEMYSŁOWYCH MODELI RYZYKA PORTFELA KREDYTOWEGO Z NOWYM PODEJŚCIEM OPARTYM NA MASZYNIE WEKTORÓW WSPIERAJĄCYCH (SUPPORT VECTOR MACHINE).

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Celem niniejszej pracy jest porównanie różnych modeli portfolio kredytowego i wykazanie, że zastosowanie podejścia opartego o maszynę wektorów wspierających (support vector machine) jest adekwatne do pomiaru ryzyka portfolio kredytowego, w tym, że dla części z modeli portfolio wykazuje skuteczność lepszą niż powszechnie stosowane modele przemysłowe. Aby zrealizować cel pracy, definiuje się różne klasy aktywów, ich grupy oraz ryzyko związane z obrotem nimi. Zaprezentowane zostają różne rodzaje ryzyka występujące w bankowości, współczesne wymogi zarządzania ryzykiem oraz stosowane w praktyce modele oceny i zarządzania ryzykiem wraz z ich formalnymi podstawami. W tym kontekście zostają przedstawione modele wyceny aktywów oraz budowania portfela inwestycyjnego. W szczególności praca dotyczy instrumentów dłużnych i związanego z nimi ryzyka kredytowego. W pierwszej kolejności dokonuje się oceny ryzyka pojedynczego emitenta długu w sposób ustrukturalizowany i skwantyfikowany, co z reguły następuje poprzez wykorzystanie ratingów i modeli skoringowych transformowalnych do ryzyka upadłości pojedynczego przedsiębiorstwa. Omówiony zostaje proces nadawania ratingu oraz wymogi związane z wewnętrznymi i zewnętrznymi czynnikami wpływającymi na jego kształtowanie. W celu ekstrapolacji ryzyka związanego z pojedynczym przedsiębiorstwem na poziom ryzyka całego portfolio skorelowanych papierów wartościowych, omówione zostają modele portfolio kredytowego. Klasyczne modele ryzyka i modele strukturalne zostają porównane z wykorzystaniem przeglądu literatury. Omówione zostają także współczesne modele oparte o sztuczną inteligencję jako potencjalne narzędzia analizy, w szczególności sztuczne sieci neuronowe (SNN) oraz maszyny wektorów wspierających (SVM). Dla wszystkich czterech portfolio testowych odrzucona zostaje hipoteza zerowa o takiej samej skuteczności zbudowanego modelu opartego o SVM w porównaniu do modelu liniowego, co potwierdzają testy Kruskalla-Wallisa oraz miara RMSE. Co więcej, odrzucona zostaje analogiczna hipoteza dla najpowszechniej stosowanego w praktyce modelu CreditMetrics® biorąc pod uwagę różnice pomiędzy rzeczywistym a przewidywanym Value-at-Risk (VaR). Tym samym wskazane zostaje, że regresja oparta o maszyny wektorów wspierających charakteryzuje się wysoką skutecznością i może być wartościowym narzędziem oceny ryzyka kredytowego.

Słowa kluczowe: Ryzyko kredytowe, ratingi, modele portfolio kredytowego, SVM, regresja

ABSTRACT

ON A COMPARATIVE ANALYSIS OF INDUSTRIAL CREDIT PORTFOLIO RISK MODELS VERSUS A NEW SUPPORT VECTOR MACHINE - BASED APPROACH.

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The aim of the thesis is to compare credit portfolio models and to show that a novel approach based on support vector regression is suitable to measure credit portfolio risks and is even of superior performance compared to current industrial models for certain portfolios. Therefore, asset classes, further collections of assets in portfolios and funds as well as their underlying risk and return characteristics are defined and described. The various kinds of risks appearing in banking are presented, modern (credit) risk management requirements are discussed, and concrete risk measures and their mathematical foundations are explained. Afterward, as application and overarching context, current asset pricing and portfolio (risk) optimization models are considered. Thereby, the focus lies on debt or bond portfolios and the metrics utilized for credit risk. As the first component, the credit risk of single obligors has to be judged in a structured, quantifiable way, which is commonly achieved via rating or scoring functions, transformed into an individual probability of default. That rating process is thoroughly examined, and requirements for bank internal as well as external ratings are illustrated to build solid rating models. To put these in the context of a bank's model inventory, risk models for various other banking risks are briefly touched on. In order to come from a single obligor point of view to a (whole) portfolio level with correlated bonds in the next step, credit portfolio risk models are introduced and treated in-depth. Hazard rate and structural models (and further econometric ones) are compared by means of a comprehensive literature review. Modern artificial intelligence techniques are presented as additional possible model candidates, especially ANNs and SVMs (SVRs). The null hypothesis that SVR performs not better than a linear model is clearly denied for all four test portfolios, employing a Kruskal-Wallis test and RMSE measure for comparison. Moreover, the null hypothesis that CreditMetrics® as a leading model performs better than SVR in most cases is also denied in terms of comparing the distance of the predicted VaRs to the real VaR. Support vector regression shows superior performance and can be a valuable tool for banks to quantify credit portfolio risk.

Keywords: Credit Risk, Ratings, Credit Portfolio Models, SVM, Support Vector Regression

INTRODUCTION

This thesis deals with the topic of comparing credit portfolio risk measurement approaches – various “classical” methods used within the global banking industry versus an artificial-intelligence-based method. Thereby the author develops and verifies a new concept that is measuring credit portfolio risk with the help of an existing “machine learning” technique, more precisely utilizing support vector (machine) regression (SVM regression, SVR).

Due to the fact that not only do capital requirements set by the regulators and especially within the finalization of Basel III[1] and the CRR III[2] framework have to be fulfilled by banks, but also internal (credit) risks need to be economically measured for controlling and risk management purposes, and even for the task of pricing credit and loans themselves, the measurement and quantification of credit risks within a bank’s portfolio is of eminent importance (Bouteillé & Coogan-Pushner, 2021, pp. 19-20; Camba-Méndez & Mongelli, 2021, pp. 2-3; Hull, 2015, pp. 33-36; McKinsey, 2016; Witzany, 2017, pp. 4-5, 32, 115). Some of the most recent changes within the regulatory framework especially regarding operational risk, (counterparty) credit risk, and the market risk framework (“FRTB”[3]) will be only part of the CRD VI/CRR III banking package which was originally supposed to be approved by the European Parliament by 2020 but postponed due to the COVID-19 pandemic. The authorities’ rationale was to support banks in reaching out new loans and credit lines to the severely hit goods and services industries worldwide, and to safeguard the operation ability of systemically important banking functions – as well as the protection of at least the covered or preferred class of deposits and savings (EBA-Statement, 2020; European Commission, 2020). Therefore, also the “CRR Quick Fix” was implemented in 2020 (European Commission, 2020b). On October 27, 2021, the first draft versions of the revised CRR were discussed, and it is assumed (as of Mai, 2022) that the CRR III will not be put into formal law before late 2022/2023. Capital requirements are expected to increase for European banks on a

[1] Basel IV is the term used in the financial industry. The official supervisory term used by the Basel Committee on Banking Supervision (BCBS) is “finalization of Basel III”.
[2] CRR denotes the capital requirements regulation and CRD the capital requirements directive. From 06/2019 onwards the CRR II and CRD V. The CRR III was originally planned for Q2/2020 yet rescheduled due to the COVID-19 pandemic in spring 2020 to 2022, drafted versions appeared in late 2021.
[3] FRTB is the fundamental review of the trading book.

weighted average between +6.4% to +8.4% in a long-term perspective (EY, 2021b, p. 4). Furthermore, it is evident that for reasons of avoiding bankruptcy or resolution (acc. to BRRD II[4]), preventing extended supervisory measures, or for mere business continuity management, an institute has to limit its downside risk (Berg, 2019; Berg, 2019b; European Commission, 2019c; Hull, 2015, pp. 33-36). The BRRD is a European act to ensure orderly bank resolution and to avoid taxpayer-funded bail-outs (European Commission, 2019c, preface). Instead, a bank is obliged to use bail-in-able instruments and write-down clauses for so-called senior unsecured non-preferred bonds to ensure it possesses enough capital for loss absorption, recapitalization, and market confidence charges upon recapitalization (WDCC[5], as in § 72 b (2) CRR II). A further objective is the protection of sight and saving deposits of retail clients as far as possible, and a harmonizing of European insolvency ranks (Berg, 2019b; European Commission, 2019c; European Commission, 2019d). Generally, it is therefore essential for banks that risks are modeled precisely, are not underestimated, and a resolution threat is avoided (Berg, 2019; Witzany, 2017). However, there is also an inconvenient upfront "risk", in case a model is too conservative (Gordy, 1998b; Witzany, 2017).

The more precise a credit (portfolio) risk measurement method is and the less unnecessary conservative its buffer unfolds, the higher the margin when debt instruments like loans, credit facilities, or bonds are adequately priced (Bouteillé & Coogan-Pushner, 2021, p. 231; Hull, 2015). Consequently, a bank is more competitive, and less capital is needed to be held as required regulatory capital to absorb risk (Bouteillé & Coogan-Pushner, 2021; Witzany, 2017, pp. 150-151). This is further due to the reason that at least the so-called Pillar 2 (rarely also denoted Pillar II, Pillar two) of the Basel III capital accords allows the use of internal models, Pillar 1 differentiates between so-called standardized and internal approaches (Witzany, 2017, pp. 109, 156). Pillar 2 denotes an additional bank-specific requirement for capital covering the risks which are underestimated, purely internally regarded, or not treated by Pillar 1 (ECB, 2022c). It is regarded later in the thesis in detail. Pillar 1 is the general, formal minimum capital requirement taken from previous Basel accords for market risk, operational risk, and credit risk (Hull, 2015; Witzany, 2017, p. 111). Pillar 3 as the last Pillar aims to discipline

[4] Bank recovery and resolution directive
[5] Write-down and capital conversion

banks by means of enhanced transparency and public disclosure requirements, it is not directly concerning capital calculations (EBA, 2022f). Therefore, for precisely calibrated models there might be a – almost[6] – sole Pillar 1 boundary which implies that not "too much" Pillar 2 credit risk enhancement has to be materially considered (Hull, 2015). If an institute is further allowed to use internal models also for the Pillar 1 calculations – denoted then as an IRB[7] institute – at least the parameter probability of default (PD) is calculated with its own data and approved models (European Commission, 2019, §143 f.; Witzany, 2017, pp. 109-110). For so-called advanced IRB institutes, the loss given default (LGD) and corresponding exposure at default (EaD), potentially including the credit conversion factor CCF and a maturity factor, are expected by law to be calculated as well within an internal model, as will be shown later (BIS, 2005; European Commission, 2019; Resti, 2006, p. 8; Witzany, 2017, pp. 108-115).[8]

Credit risk and associated credit risk-weighted assets (RWAs) constitute the most relevant instance and proportion of RWAs for the majority of banks (EBA, 2021b, p. 46). Historically, the global trend and promotion of using internal models were accelerated through Basel II and III, and supervisory preference toward internal models compared to the standard approach (Witzany, 2017, pp. 12-16, 115). Nowadays, whereas the U.S. institutions and authorities are tending more back toward the standard approach overall, European banks and supervisors on average still prefer internal models (Fitch, 2017; Pugsley, 2017). Half of the European banks' RWAs on average were roughly generated by internal models in the past (Resti, 2006, pp. 8-10). The advantages in calculating more precise and economically valuable results are at forehand. To reduce the risk of banks calibrating the RWAs as to their own needs the ECB performs TRIM[9] tests with their inspecting joint supervisory teams – the JSTs (ECB, 2019; ECB, 2022g). It sets binding as well as recommending (minimum) standards in cooperation with the European

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[6] Mainly excluding the IRRBB based on Pillar 2, where IRRBB: Interest rate risk in the banking book.
[7] IRB(A): Internal ratings-based approach, approach relying on the own, internally calculated and used, permanently risk-adjusted, and monitored ratings – and hence PDs of a bank for its (credit) clients.
[8] An IRB bank is obliged to apply the standard IRB formula for credit risk – using the internally calculated parameters above – in the Pillar 1 framework afterward. Furthermore, the finalization of Basel III sets an overall "output floor" of 72.5 % of the standardized approach after a phase-in period. The output floor was introduced due to excessive volatility in risk-weighted assets (RWAs) among "similar" IRB banks' portfolios.
[9] Targeted review of internal models: A thorough qualitative and quantitative test of the internal models used by banks. It is accompanied by a sector analysis and peer group review. The corresponding "TRIM Guide" (ECB, 2017) was later succeeded by the "ECB Guide on Internal Models" (ECB, 2019).

Banking Authority EBA (EBA, 2018; ECB, 2019; European Commission, 2019b).[10] The measures are accompanied by regular updates of the specific guidelines, acts, and the CRR itself – e.g., sensitivities and risk weights were modified in the past and updated in the credit risk and the counterparty credit risk (CCR) frameworks due to empirical results, and following the findings of quantitative impact studies, the QIS (BIS, 2017; EBA, 2021g; European Commission, 2019; Feridun & Özün, 2020; Witzany, 2017, pp. 12-16). Especially after the great financial crisis (GFC) in 2008 the regulators' approaches became increasingly stricter (Bouteillé & Coogan-Pushner, 2021, pp. 143, 231; Hull, 2015, pp. 368-416; Martin et al., 2014).

The above-mentioned effects and limitations hence floor the quantitative impact of more precise models to some extent within the regulatory universe. Nevertheless, still for regulatory reasons and more importantly for the pricing of bonds and for (internal) risk management purposes these credit portfolio models[11] remain of utmost importance. This fact also holds for loans, which – as regulators stress – should be regarded in the same way as bonds in light of a consistent market-oriented common loan and bond portfolio approach (OCC, 1998; Tapiero, 2004). Precise models allow more competitive pricing and risk controlling, and are therefore ultimately resulting in higher possible returns, in absolute as well as RAROC[12]-based terms, for the institutions (Bouteillé & Coogan-Pushner, 2021, p. 231; Gordy, 1998b; Witzany, 2017, pp. 119, 150-151).

The main aim of the thesis is to introduce a credit portfolio model which is effective (compared to a linear model) and even a better predictor than current credit portfolio models for some common bond portfolios. It should deliver results that may serve as a competitive advantage in certain areas. Such a model would be of practical relevance for banks, and can be utilized to deliver more precise risk calculations, pricings, and hence returns. It contributes to academic research in that important field as it illustrates dependency and portfolio structures in a novel way, and improves current research models.

[10] A further examination of institutions' models and a more direct comparison is achieved through supervisory benchmarking portfolios (SBP).
[11] As common in academic research in the field of credit risk, the terms credit portfolio models and credit portfolio risk models are used interchangeably throughout the thesis.
[12] RAROC: Risk-adjusted return on capital

The first research hypothesis[13] hence states that the SVM regression (SVR) model is an effective model for credit portfolio risk (CPR) measurement of typical bond portfolios. Therefore, i.e., to prove the effectiveness, a comparison of the prediction of SVR calculated bond portfolio prices and losses, after training the SVR model with the portfolio data and calibrating it, with a linear multi-factor model (LM) and on a random sample of real portfolio losses is performed. First, a Kruskal-Wallis rank sum test for nonparametric independent samples is applied to verify if the two models have generated significantly different distributions of outcomes.

In terms of measuring the concrete corresponding "errors", hence the differences in the models' predictions in comparison to the real data in the portfolios, the second research hypothesis claims that SVR's "error"[14] is smaller.

Following this benchmarking, the further goal is a comparison of the two main industrial approaches for measuring credit portfolio risk with SVR. Therefore, the third research hypothesis states that the SVR method may outperform even the classical models for a majority of selected standard bond portfolios.

To achieve the aim of the thesis, which is to verify the research hypotheses and show the usefulness of the SVR approach for the common bond portfolios, the thesis is divided into five chapters. To deal with credit portfolio risk generally in a precise manner, the necessary objects are defined in Chapter 1, which are equity, bank-specific regulatory own funds, and debt as well as alternative asset classes. Characteristics of these asset classes are described, and first (credit) risks appearing in connection with them are mentioned. Furthermore, collections of these assets forming a portfolio or a certain type of fund are described as well as corresponding investment styles and strategies. To measure the success of these strategies, risk and return metrics and ratios are introduced in Chapter 1.

In Chapter 2, risk and the various types of risk appearing in banking – mainly including credit risk – are described in detail, risk management with its principles and requirements will be presented as well as formalized risk measures including moment-based measures like volatility, drawdown measures, and (T)VaR. With these measures and tools, the task of asset pricing and creating optimal portfolios, in terms of risk/return

[13] The formalized null hypothesis in Chapter 5 negates the formulation of the research hypothesis as usual.
[14] Measured in terms of a root mean square error (RMSE) as shown later.

characteristics, is considered. Therefore, the classical modern portfolio theory of Markowitz, the capital asset pricing model, the single-index model, and asset pricing theory are introduced in chronological order (Markowitz, 1952; Markowitz, 1956; Sharpe, 1963; Sharpe, 1964; Lintner, 1965; Ross, 1976a; Ross, 1976b). Innovative ideas such as the incorporation of behavioral components and machine learning techniques for portfolio management are briefly touched on (Thaler, 1993).

A key factor of the APT model is the mentioned PD, a central parameter for credit risk management and directly connected with rating "grades". As a consequence, internal as well as external rating processes and systems and further validation techniques for these systems (models) are illustrated in-depth in Chapter 3. Thereby, quantitative scoring and rating functions based on mathematical-statistical approaches are state-of-the-art for measuring credit risk for single obligors. Finally, quantitative models for the various (other) types of risks in banking, commonly used within academic research and the financial industry in practice, are introduced for a comprehensive model overview in the same chapter.

The focus of the thesis is credit risk, and hence credit portfolio risk models are illustrated in detail. In the academic field of credit risk research, there a mainly two current standard types of schemes for modeling credit portfolio risk. These measure the potential (unexpected) loss of multiple obligors, which are correlated – the default correlation structure thereby presenting the decisive modeling component. The first ones are the so-called Merton models or enterprise-value-based models with their most well-known representatives Credit Metrics® and KMV®. The other ones are actuarial or hazard-rate-based models. These models are occasionally also referred to as Poisson mixture models with its most famous instance being CreditRisk+®. A third econometric macro-factor-based model, CPV – CreditPortfolioView® – is described as well. It can be similarly classified as the first variant. All of these have their inherent strengths and shortcomings. These, as well as underlying ideas and precise definitions of either scheme, will be presented and highlighted as a further aim of the thesis in the following Chapters 4 and 5 in greater depth.

As will be laid out in Chapter 4 method preferences and a detailed comparison of these industrial models referring to a certain class of obligators (debtors) are possible, and some transformations between the two foremost applied model types, as well as equivalences within a unified framework, are shown. A comprehensive literature review was utilized to give an overview of the current state of research in the field. Whereas further improvements and extensions of these models exist and are presented, the development since the inception of these models in the late 1990s and 2000s was generally modest, and only few innovations (as e. g. the ZPP model) succeeded.

Therefore, in Chapter 5 a novel approach to credit risk measurement with the help of support vector regression is presented. Support vector machines are a classifier (SVM) or regressor (SVR) technique from the field of machine learning. They aim to categorize data into two or iteratively more classes or respectively approximate a given function utilizing multi-dimensional (back-)transformation and so-called “kernel functions”. That structure intuitively resembles rating or binary default categories and portfolio risk distributions, and lead to the idea to apply SVR to credit portfolio risk. As a further advantage, SVR is prone to high-dimensionality and nonlinear, nonparametric problems. In recent years, the SVM method was combined with techniques from quantum physics, which will potentially accelerate SVR calculations in the future even further (Dalal et al., 2021).

Having then introduced the SVM approach as a classification as well as regressor technique and as a by-product a technique widely used and of foremost general importance in the field of artificial intelligence, the usability and effectiveness of the SVM in respect to measuring credit portfolio risk as stated in the research hypotheses is shown in the empirical part.

A prerequisite is real, comparable bond portfolio data to derive “real case” Value-at-Risks (VaRs). Principally, the VaR contains a default-based VaR component, and a rating-migration-based VaR part which translates from ratings subsequently into different credit migration spreads, forward curves, and hence present values (exploited by CreditMetrics®). First, the SVM method is however compared with a linear model on randomly partitioned data, including a training (80 %) and test (20 %) set. The superiority and hence usefulness of the SVM is proven by means of a Kruskal-Wallis test and concrete root (of the) mean square error (RMSE) comparison calculations. The aim is to see how well the models’ predictions fit the real data (goodness-of-fit) with RMSE denoting a kind of complementary “fitting error”. The next task is calibrating CreditMetrics®, CreditRisk+® and SVM regression with a sample of that portfolio data.

In the final part of the dissertation, the comparison of the models' predicted VaRs with real out-of-sample data is executed – proving again that the SVR approach is effective, and further showing it is even the most precise credit portfolio risk model.

CHAPTER 1
ASSET CLASSES AND INVESTMENT PORTFOLIO TYPES

1.1 Equity and Debt (Bonds), Regulatory Own Funds

Before credit portfolio management and credit risk measurement techniques are introduced it is necessary to define the objects upon which these systems operate – assets and portfolios themselves.

An asset in this thesis is defined as a financial resource or object one can invest in and from which one expects a future economic benefit as an aim of that investment. In the official international financial reporting standards (IFRS) conceptual framework of the IFRS foundation (IFRSF)[15], the standard-setter for global accounting rules, the term asset is defined in a similar fashion. It is described, a bit more technically, via the terms "economic resource" and "right", as a "present economic resource controlled by the entity as a result of past events. An economic resource is a right that has the potential to produce economic benefits" (IFRS Foundation, 2013, p. 3; IFRS Foundation, 2018, p. 26). The Corporate Finance Institute® (CFI®) as an influential organization for investment professionals defines: "An asset is a resource owned or controlled by an individual, corporation, or government with the expectation that it will generate a positive economic benefit" (CFI, 2022, p. 1).

In terms of accounting, it is then booked in the asset side of a balance sheet or account – hence indicating ownership or possession. From an accounting perspective, an asset further represents the "opposite" of a liability (e.g., debt) and in the financial industry is needed to cover them, e.g., considering asset-liability-management in the banking and insurance sectors (IFRS Foundation, 2018, p. 30)[16]. Assets can be considered and categorized in many respects, e.g., simply according to their prices which do not necessarily reflect or be equal to their inner cash flow as discounted values in the short

[15] Supported by the International Accounting Standards Board (IASB), which is part of the IFRSF.
[16] In terms of banks and their role in the financial industry, further transformations size-, risk-, term-, and maturity-wise between different assets and liabilities take place (Hicks, 1946; Ho & Saunders, 1981; IMF, 2018).

term[17] and naturally corresponding scarcity (Damodaran, 2014). They can be classified according to their usage or to physical appearance, their convertibility, liquidity, and fungibility (CFI, 2022). Fungibility means how comfortably assets can be sold or traded, and it therefore influences the capital structure of a company (Viswanath & Frierman, 1995). Physical appearance might be viewed in regard to the assets' material (e.g., hard metal versus soft commodities or even digital assets) and hence also storability, however more general, the usual differentiation between tangible and intangible assets is considered. Tangible or fixed assets are assets with physical existence, mostly longer-term used, and with the ability to be touched, felt, or seen – intangible ones not, e.g., legal rights, or royalties (Ciumag, 2012, p. 48; Van der Lei et al., 2012).

From a risk management perspective an asset‘s intrinsic idiosyncratic as well as systematic risk, its maturity profile, and related cash flows as interest or coupon payments, dividends, bullet payments, or similar payouts could be considered (Gregoriou, 2006, pp. 107-131; McNeil et al., 2015). Especially the differentiation between systematic risk and idiosyncratic one is introduced in Chapter 2. Furthermore, regulatory and legal aspects can categorize certain assets, like ownership status, special rights connected to those assets, taxes, and restrictions as well as optional and underwriting components (Basile & Ferrari, 2016; IFRS Foundation, 2003, pp. 7-8). While treating a variety of these aspects within the following chapters, the main focus of the thesis deals expectably with the price and corresponding risk of assets.

In the financial industry, assets are connected with investments, where investment is seen as an outlay of money to reach a certain purpose, normally a profit or income in the future (Cambridge University Press, 2011; Stewart, 2006). The usual macro-economic relation to savings (as deferred consumption) and immediate consumption also holds true for investments in the financial industry (Mankiw, 2019).

An investment (generally) can include a range of assets such as

- equity (e.g., stocks)
- debt (e.g., bonds)

[17] According to Eugen Fama's efficient market hypothesis in the middle or long run and when markets avoid or are not subject to frictions like transfer controls, tariffs/bans, illiquidity, or high transaction costs the price will normally tend towards the inner, "true" value as will be seen later (e.g., in terms of arbitrage-freeness or discounted cash flows (DCF) and depending on the price measure in use).

- currencies and related interest rates as EURIBOR, SOFR, ESTR, and formerly also the LIBOR[18]
- commodities
- real estate, real estate investment trusts (REITs)
- precious metals
- other alternative investments (hedge funds, private equity, private debt, or certain certificates and derivatives for instance)

These asset classes, namely collections of all assets of the same or a very similar type in a class, can be further devised (IMF, 2009, p. 4; Maginn et al., 2007; SEC, 2008; SNA, 2008):

Equity instruments are instruments that incorporate an ownership interest in an entity, and common shares define the most basic ownership interest herein, they illustrate "the residual corporate interest" once liabilities are subtracted from assets (EY, 2021, p. 110). Besides common stock, equity can also appear as preferred stock or in special forms known as ADRs, GDRs, or SDRs (IMF, 2009, pp. 6-7). A GDR, denoting a global depositary receipt, is a special instrument which is representing shares in a foreign company (IMF, 2009, pp. 6-7). An American depositary receipt is an instrument that illustrates shares in a foreign stock, taken for trading on an American stock exchange (SEC, 2012b). Special drawing rights are defined by the International Monetary Fund (IMF) as "an international reserve asset, created by the IMF in 1969 to supplement its member countries' official reserves" (IMF, 2021, p. 1). Especially with the increased circulation of the not free-convertible Chinese yuan ("renminbi") and shifted global economic weights toward a more multilateral global economy SDRs became a useful tool for many countries (IMF, 2021).

Common Stock ought to be regarded as direct, "normal" ownership in a company's stocks including for instance voting rights whereas for preferred stocks voting rights are normally excluded, but on the other hand, this special form guarantees preferred

[18] The LIBOR and the other rates like EURIBOR, SOFR, and ESTR are interbank rates, i.e., the rates for which banks lend to each other short-time (Hull, 2015, pp. 215-217; McNally, 2021). Due to manipulated ("rigged") London interbank offered rates (LIBOR), the LIBOR system and other rates were replaced by the SOFR (Secured overnight financing rate) in the US, SONIA (Sterling overnight index average) in Great Britain, and ESTR (Euro short-term rate) in continental Europe. The New EURIBOR (Euro interbank offered rate) replaced EURIBOR. See also (IFRS REG IASB, 2018; EIOPA, 2020).

treatment in terms of (the amount of) dividends and distributions receivable (EY, 2021; Kaye, 2005). Other special forms of ownerships (e.g., SPACs[19], popularized in recent years) and legal constructions such as LLPs (limited liability partnerships) exist, are however not in the scope of the thesis (Dowling, 2007; Fabozzi et al., 2008b). At that point, economical or accounted equity is considered, which has to be differentiated from regulatory requirements as will be shown later in this chapter.

Debt securities are usually classified according to their originating party as sovereign bonds (like U.S. treasury bonds, German bunds[20]), municipal bonds, agency bonds, corporate bonds, or special money market paper, as commercial paper or certificates of deposit (Hünseler, 2013; IMF, 2009, pp. 24, 31; SEC, 2008b; SEC, 2008c; SEC, 2012).

As indicated by the name, sovereign or state bonds are issued by a certain country and hence have the taxing-right-backed financial power of a state underlying its debt service. Municipal bonds are issued by regional or municipal government authorities and are generally (in case slightly) more risky than national government debt (Bouteillé & Coogan-Pushner, 2021, pp. 145-147, 152-160; IMF, 2003; IMF, 2009). Depending on the federal structure and financial market development of a country municipal bonds are more widespread (e.g., in the United States) or less (Hildreth, 2006). Whereas national debt is regularly issued by only one country,[21] for municipal bonds it is also common that a group of (neighboring) regions issue bonds collectively (SEC, 2012; Maliqi, 2012). Agency bonds are issued by government-sponsored enterprises (GSE) or state agencies (SEC, 2012). Corporate bonds are finally debt issued by private corporations via different possible vehicles as will be seen later, similar also short-term money market instruments (SEC, 2008c).

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[19] Special purpose acquisition companies. A SPAC is just a shell company when it starts its own initial public offering (IPO) in contrast to normal IPOs of operating companies. After the IPO – possibly even years after – the SPAC management (its sponsors) look to acquire or merge with lucrative operating companies, the initial business combination (IBC). See, e.g., (Klausner et al., 2022) for a recent study on SPACs or the U.S. Securities and Exchange Commission (SEC) for basic information (SEC-Statement, 2021).

[20] For instance, Poland even had its own Ministry of Treasury which was also responsible for state debt and bond issuing, alongside the ministry of finance, established during the Polish administrative reform of 1996. However, it was dissolved in 2017 and included in the Ministry of Finance. In 2019 a (type of limited) treasury ministry was created again to deal with state-owned companies and assets, see (Roca, 2019).

[21] In Europe and the eurozone there were discussions about common "eurobonds" on several occasions, e.g., again during the COVID-19 pandemic in 2020 (Giavazzi & Tabellini, 2020). However, the final European recovery package included pro-rata (proportional) liabilities and hence no "eurobonds" in a real, strict sense, cf. (European Commission, 2020). Other historic monetary unions as the Latin monetary union had no further common liability structures, see (Theurl, 1992).

An important aspect of corporate bonds is the tradability in financial markets and thus easier risk transfer compared to ordinary loans, which are generally held in the banking book of a financial institute up to maturity (Schiffmacher & Humlach, 2009; Witzany, 2017). The term structure and maturity profile of a bond is a decisive aspect, as they relate to cash flows and (default) risk over the time horizon, and a bond is hence normally rewarded with a higher premium the higher the duration of the (otherwise equivalent) bond is (IMF, 2009, pp. 26, 39).[22]

One differentiates along the maturity line between short-term bills (e.g., the American "T-bills", German "Schatzanweisungen") then obligations, medium-term notes (MTNs) or T-notes, and finally long(er)-term bonds as different types of notes or so-called fixed income instruments (Fabozzi, 2020; Maliqi, 2012; SEC 2008).

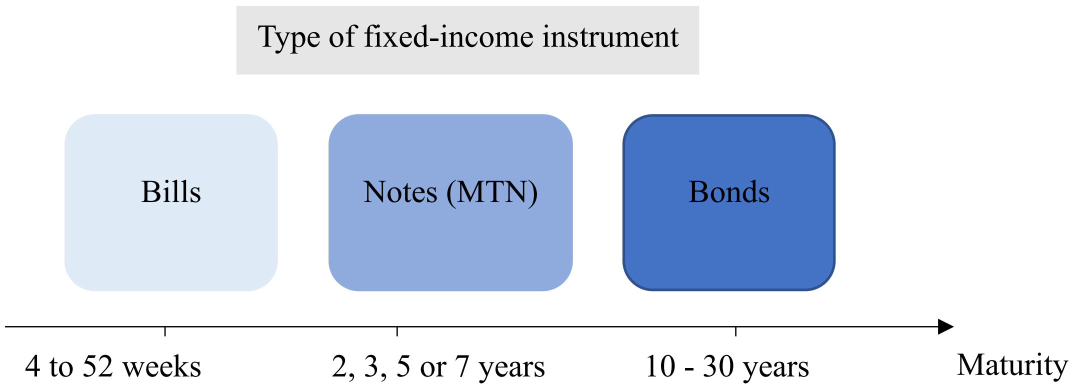

Figure 1 Fixed-income instruments, sorted along with their original maturity.

Source: Own illustration

Depending on tradability, placement type (public or private), and ownership status bonds are further classified (Fabozzi, 2020). Private placements are issuances that are only available for a pre-selected limited group of investors, in some cases even only one investor with very specific requirements for the note, whereas public placements are placements for the general investing public with equal conditions and mainly executed via auctions or public offerings, e.g., on a stock exchange or exchange platform (Fabozzi, 2020). Registered bonds or registered note loans are placed for a certain, pre-known

[22] This is however not always the case, especially preceding recessions and in times of financial distress an inverting yield curve is rather common, cf. (CFI, 2022b).

owner and the rights of the owner are connected to that entity, similarly, promissory note loans (often referred to by their German origin as “Schuldscheindarlehen”, SSD) as described in (Reichling et al., 2005, pp. 170-177). Consequently, they are not traded, and risk concentration is higher (Reichling et al., 2005).

Additionally, order bonds are existing, which award the owner the right to hand it to certain other entities in a pre-defined endorsement fashion (assignment rights[23]), see (Fabozzi, 2020). They are however less frequently issued and utilized in practice compared to registered notes or promissory note loans (Reichling et al., 2005).

The remaining types of bonds are known as bearer bonds, which are publicly listed with a defined unique international securities identification number (ISIN) and are freely traded, possibly in an anonymous fashion, e.g., on a public exchange (Fabozzi, 2020; SEC, 2008; Schiffmacher & Humlach, 2009).

The type and number of issued securities have to be registered by a registrar holder, for bearer bonds often large banks or functional settlement instances (e.g., Clearstream Banking International in Luxembourg) are responsible for carrying out that duty whereas for named, “personal” notes[24] usually the emitting bank itself is executing that process (Clearstream, 2022).

All of these securities illustrate debt and are hence exposed to credit risk. They are treated within a unified credit risk framework from Chapter 2 onwards. Furthermore, debt securities like bonds can be classified by the type of their underlying collateral (“secured-ness”) into unsecured or secured respectively collateralized instruments (IMF, 2015; SEC, 2008).

Instances for secured bonds are covered bonds based on real estate (e.g., mortgages), public emitted loans by state or municipal agencies, ships, or even airplanes as collateral (Packer et al., 2007). The most prominent and original versions, besides Anglo-American covered bonds, are the German “Pfandbriefe” and the Luxembourgish, Swiss, or French “Lettres de gage”[25] (Packer et al., 2007). Secured debt decreases credit risk in a natural fashion, as it reduces the possible loss amount given default of the counterparty by the sell-off value (possibly with a “haircut”) of the collateral (Bouteillé & Coogan-Pushner, 2021; de Laurentis et al., 2010, pp. 20-21).

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[23] German (e.g., for order bonds, SSDs): “Zessionsrechte”.
[24] Like the SSDs and registered bonds described.

Further examples of secured finance are repos (repurchase agreements), which are collateralized by a security (e.g., a stock or bond) and contain the promise of the lender of the security (for which it received money) to buy it back including interest or fees (Bouteillé & Coogan-Pushner, 2021, p. 219; Schindler & Hindelang, 2016). Hence, repos are an opportunity to receive short-term liquidity in a rapid way; or gain the possession of a security for a short time in case one is positioned on the counterparty side of the transaction (Schindler & Hindelang, 2016). Depending on the side of the transaction an entity resides, one is consequently referring to that instrument as repo or reverse repo respectively.

Furthermore, the insolvency rank of the creditors can be differentiated, where junior (or even first-loss peace) debt is ranking low, and is hence served late, mezzanine debt is in between, and senior debt is ranking high hence served first (Nijs, 2013). Therefore, different credit risk profiles can be represented by different ranks (and tranches). One should stress at that point that mezzanine capital as a hybrid form can appear on one hand in the form of mezzanine equity (like silent partnerships, German "Genussscheine", or for instance participation rights with equity kicker) or on the other hand as mezzanine debt (Nijs, 2013). Technically, the latter is a special form of subordinated debt (Fabozzi et al., 2008b). Additionally, there are also other forms of hybrid capital existing, yielding characteristics of equity capital as well as characteristics of debt capital (EY, 2021; Fabozzi et al., 2008b).

Examples are nonstandard participation rights with specific triggers or special convertible bonds as contingent convertible bonds issued by banks, often abbreviated and referred to as "CoCos", with a regulatory call option executed once the bank's equity falls under a pre-defined threshold and would therefore breach regulatory requirements (Albul et al., 2010; Avdjiev et al., 2013; EY, 2021; Frank, 2014). Contingent convertibles can serve as useful tools for banks to safeguard that the institutes fulfill their regulatory capital requirements.

In the financial world debt securities with a maturity of less than roughly a year (conventionally 397 days) are usually considered money market instruments and abbreviated as MMIs (IMF, 2015, p. 10). They are called commercial papers, e.g., in the case of short-term promissory note loans issued by a company, or in case they have a tradable and a deposit-like component with a certain named owner the instruments are labeled as certificates of deposit (Choudhry, 2011; IMF, 2003; IMF, 2015, p. 65).

Having illustrated the types of equity and debt in the financial markets the next step is to consider the capital of banks more precisely, as this is the foundation of most regulatory purposes like the common reporting framework COREP[26] with its own funds requirements and the basis to cover (potential) losses hence materialized risks (Hull, 2015). While debt and third-party equity (holdings) thus bear risks like credit risk, own funds on the other side shall safeguard against the risks.

When regarding the banking sector in a more detailed fashion one has to be especially aware of the tier structure of banks' regulatory capital, the so-called regulatory "own funds" (BIS, 2019). The question of sufficient regulatory capital, which can absorb (potential) losses and ensures the survival of a bank, is the central one of regulatory capital requirements and is dealt with in detail in this chapter (Rutkowski & Tarca, 2015; Witzany, 2017, p. 2).

Many regulatory changes and capital charges in that area were due to financial crises and turmoil in the past (Hull, 2015, pp. 368-416). In terms of laws and regulatory requirements for banks, the thesis concentrates on the European Union. While most of them are derived from the (international) Basel accords, some details and exact requirement quantities differ compared to other jurisdictions like the United States as shown in (Sabel, 2013; Witzany, 2017, pp. 11-16).

The most important capital for a bank is CET 1[27], common equity tier 1, consisting of own shares/equity, retained earnings, funds for general banking risk, and accumulated other comprehensive income – AOCI (Bundesbank, 2022; CFA, 2022c; European Commission, 2019, §26). The decisive part is formed by own shares and retained earnings (Hull, 2015). A precise list of which instruments are eligible as CET 1 is updated every few years by the European Banking Authority (EBA, 2021e). It is the core capital of a bank and the "hardest", i.e., safest and directly available source of capital (Bundesbank, 2022; Hull, 2015).

Second to it is AT 1, additional tier 1, which is mainly hybrid capital like participation rights or silent partnerships meeting further requirements as perpetuity, instruments have to be fully paid up, the acquisition of ownership is not (indirectly) funded by the entity itself, and no early redemption is incentivized for the institution

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[26] See the European Banking Authority (EBA) on the Common Reporting Framework (COREP) in (EBA, 2021d) or for the British case in (FCA, 2021).

[27] See, e.g., the Bundesbank on CET 1 and own funds for definitions and a broad overview (Bundesbank, 2022). Sometimes also denoted CET1 or CET-1.

(European Commission, 2019, §52). It is also highly valuable for regulatory purposes. The combination of these, *CET 1* and *AT 1*, is called tier 1 capital (*T 1*) implicating (BIS, 2019):

$$T\,1 = CET\,1 + AT\,1 \qquad (1)$$

Whereas before the great financial crisis there were three distinct kinds of tier capital existing, in the aftermath, following thorough reviews of its causes and assessments of the optimal capital structure of banks, a more conservative approach was followed and tier 3 capital abolished (BIS, 2019; Hull, 2015, pp. 152-164). Consequently, besides tier 1 capital there only exists tier 2 capital (T 2) in the realm of regulatory own funds (Bundesbank, 2022). The T 2 category contains mainly subordinated bonds – apart from national-specific instruments, most however were ceased after a transitional phase in 2021/2022 – with a remaining maturity of at least five years (European Commission, 2019). In case the bond with such an original maturity bears a current maturity within a shorter period of time, this remaining maturity is split pro-rata and with x years of time-to-maturity (TTM) as x/5 of the carrying amount – however normally calculated day-wise (EBA, 2013).[28] This means x/5 of its book value and accrued interest[29] are considered own funds T 2 eligible, whereas (5–x)/5 of the carrying amount are considered subordinated T 2 in phase-out and hence not (applicable) T 2 (EBA, 2013). Subordination by law, contractual subordination, or statutory subordination are eligibility characteristics as stated in the CRR III, e.g., a contractually subordinated own funds eligible plain-vanilla bearer bond with TTM of seven years is a regulatory recognized part of a bank's T 2 capital. The sum of tier 1 (*T 1*) and tier 2 (*T 2*) capital is known as total capital (*TC*); hence it can be written as (BIS, 2019; European Commission, 2019):

$$TC = CET\,1 + AT\,1 + T\,2 \qquad (2)$$

The *CET 1*, *AT 1,* and *T 2* capital components, also labeled Pillar 1 capital, are essential for a bank, and have to account for at least 8 % of its risk-weighted assets (RWA) as a regulatory requirement (BIS, 2019; Hull, 2015, p. 373).[30][31] As will be shown later,

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[28] Calculation of outstanding tier 2 (T 2), which is succeeding the pre-payment of amounts that have been amortized or "phased-out" (EBA, 2013, p. 1). For a calculation example refer also to (EBA, 2013).
[29] Book value (*BV*) and accrued interest (*AI*) are summed up and called carrying amount (*CA*): *CA = BV + AI*.
[30] For these assets exposure on-balance-sheet, as well as off-balance-sheet and derivative exposure, has to be included - this is then part of the so-called Cooke ratio (Hull, 2015, p. 370).
[31] These 8 % are also sometimes denoted as capital adequacy ratio – CAR (Witzany, 2017, pp. 15, 108).

when considering the banks' assets, regulators are mainly interested by the "risky-ness of assets" not by the pure nominal amount of them, hence the exposure of assets is not taken nominally but attached with a certain risk weight – RW (Hull, 2015). In the early days of the Basel accords (Basel I) these weights were mainly 0 % (e.g., for cash, state debt), 20 % (OECD issues, etc.), 50 % (e.g., secured mortgages) or 100 % (corporates, more risky debt, etc., extended by a special 150 % class) as Hull or Witzany describe (Hull, 2015, pp. 373-374; Witzany, 2017, p. 15). Later the weights in the so-called standardized approach – and even more regarding the method known as internal models, the alternative way for suitable banks to calculate risk weights (on their own) described in detail in the following chapters – were calibrated in a more sophistical manner. As will be seen in the course of this thesis the mentioned 8 % ratio, comprising of a compulsory 4.5 % CET 1, 1.5 % AT 1, and 2 % T 2, appears throughout many pieces of legislation (Hull, 2015, p. 405).

The idea behind the 8 % figure is grounded in the empirical fact that the unexpected losses (UL) and the RWA relate to expected losses (EL) (Witzany, 2017, p. 111). The factor is roughly three to four in total, i.e., total/stressed losses (TL) are that times higher when for instance a 99.9 % confidence interval is used, so (Witzany, 2017, p. 111):

$$UL = TL - EL \tag{3}$$

Therefore, if one takes an average portfolio (of for instance BB+ to B- rated companies), the historic default rate is roughly 5 %, the historic loss rate that occurs (subtracting the recovery amount one can receive) is roughly 60 % for senior unsecured debt hence the expected loss is roughly 5 % × 60 % = 3 % (Witzany, 2017, pp. 111-112). Now 3 % × 3 – 3 % = 6 % and 3 % × 4 – 3 % = 9 %, the mean is 7.5 % and hence rounded 8 %.

Many formulas in the European implementation of the finalization of Basel III, the capital requirements regulation CRR III, contain the 8 % figure or equivalents thereof, like a factor of 12.5 with which many absolute sums of risk covering capital are then multiplied (European Commission, 2019; Hull, 2015, p. 380; Witzany, 2017). Equities in the standardized case are then equipped with a credit risk weight (RW) of normally 300 % for publicly traded and 400 % for other entities, and may reach 1.250 % (hence with factor 12.5 the complete value as capital backup is required) for "high-risk" cases, as intuitive (BCBS, 2019, pp. 10-12).

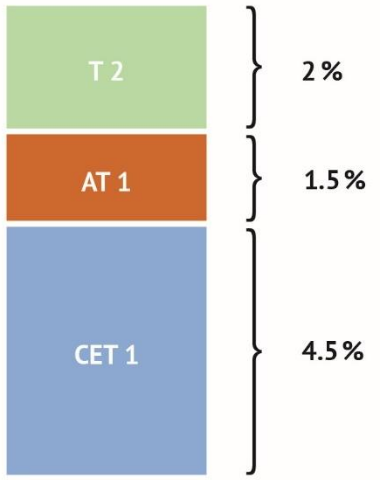

Figure 2 Breakdown of the Pillar 1 (P1) requirements for own funds according to the CRR III.

Source: Own illustration.

The supervisory bodies extended their requirements in terms of a bank's solvency by adding a second pillar of capital[32] for own funds as, e.g., explained by the ECB (ECB, 2022c). It is denoted as Pillar 2 requirement (P2R), which is individually set by the supervisory authorities in the supervisory review and evaluation process (SREP) as laid down in (EBA, 2018; EBA, 2018b). In the course of that process, the regulators are checking the solvency capital and liquidity of a bank in a detailed fashion, and also validate the bank's internal capital adequacy assessment procedure (abbreviated as ICAAP for capital and ILAAP for liquidity) in terms of quantitative and qualitative aspects (Buchmüller & Igl, 2019; Hull, 2015).[33] The P2R then contains the results of those findings, special risks or internally calculated excesses of main types of risk as well as risks not covered in Pillar 1 as the interest rate risk in the banking book (BIS, 2019c; EBA, 2018; Hull, 2015, pp. 212-234).

The EBA updates its guidance for the SREP process regularly, the latest was a slight modification in 2022 (EBA, 2022).

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[32] Definition of Pillar 2 by the ECB as in (ECB, 2022c).
[33] Also cf. the Bank of England on ICAAP, SREP, and Pillar 2 (BoE, 2021).

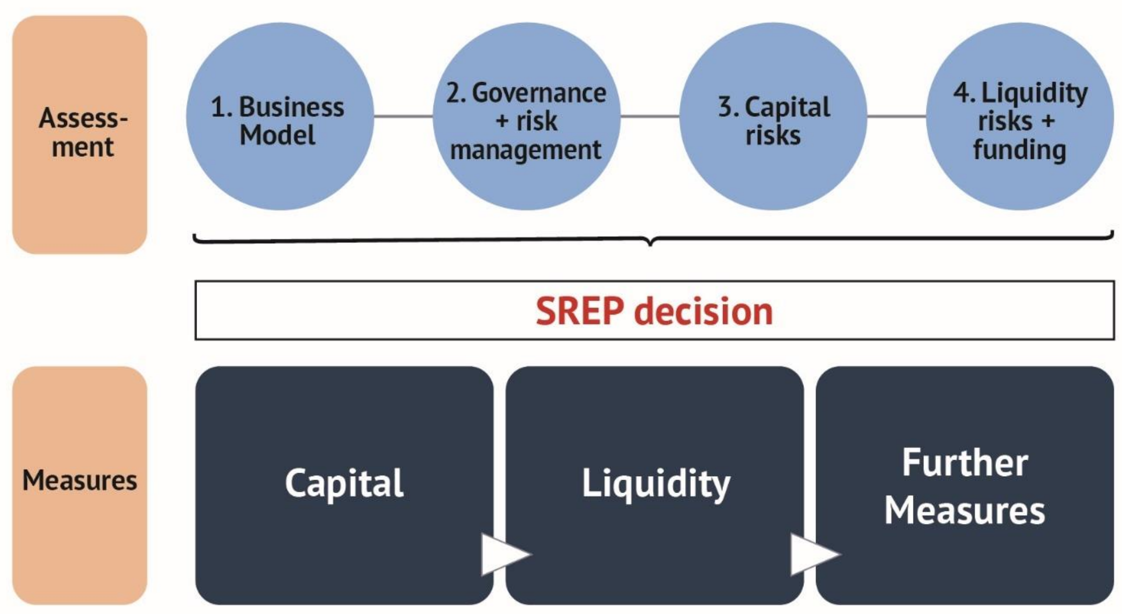

Figure 3 SREP – the process, assessment, and measures.

Source: Own illustration, according to the EBA (EBA, 2018).

Qualitative results of the EBA's yearly stress test for significant institutes[34] (SIs) or the nationally supervised LSI stress test (formerly known as "NZU"/LIS[35]) for less significant banks are also included in the P2R in euro area countries (ECB, 2022c; Farmer et al., 2022). Finally, a marking in four descending categories of interest within the scale of one to four, according to the complexity, type and size of business, risk awareness and appetite, governance and control measures, systemic importance and business strategies of a bank, takes place (Buchmüller & Igl, 2019; EBA, 2018; EBA, 2018b, p. 11). Having set a specific requirement for P2R aligned with a certain range of that mark, hence an absolute add-up to the former 8 % of, e.g., another 2 %, one derives at the TSCR, the total SREP capital requirement (EBA, 2022).

Since the year 2021, the EBA and ECB have also gradually introduced climate stress tests accompanied by given methodologies and guidelines for banks (ECB, 2022). In 2022, the first overall climate stress test – stressing the banks' available capital and consisting of three modules with short-term and long-term scenarios for transitional and

[34] The European Banking Authority (EBA) uses the term (financial) institute in official documents as a general one for banks, savings and loans associations, etc., throughout this thesis the term institute is used interchangeably with the term banks. A current list of institutes that are deemed significant by the European Central Bank (ECB) and hence directly under ECB (and EBA) supervision is available here (ECB, 2022d).
[35] "Niedrigzinsumfrage"/Low-interest survey

physical risks – was executed in practice (ECB, 2022). During that test, institutes are obliged to break down and categorize their business counterparties and clients by their amounts of CO2 emissions and their efforts to scale back on these emissions in a precise manner (ECB, 2022). In a medium-term perspective, the results are expected to become parts of ICAAP and the P2R (Deloitte, 2020).

If one then adds the combined buffer requirement for banks (CBR) to the TSCR the result is the OCR, the overall capital requirement (EBA, 2022). As indicated by the name the combined buffers serve as additionally available capital buffers. The buffers consist of individual components as well as general or systemic ones (Witzany, 2017, p. 12). Thus three sub-buffers are defined. A national counter-cyclical buffer (CCyB[36]) to mitigate the credit cycle and avoid exuberances, which is set rather similarly within the euro area, then a capital conservation buffer (CCB[37], of 2.5 %) and finally certain systemic buffers, if an institute is relevant for the entire national, European, or global financial system – and hence coined a systemically important bank, a SIB (Behn et al., 2022; EBA, 2020; Hull, 2015, pp. 405-406).

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[36] The countercyclical buffer (CCyB) is described in more detail here: (ESRB, 2022).
[37] The capital conservation buffer (CCB) is described in more detail here: (ESRB, 2022b).

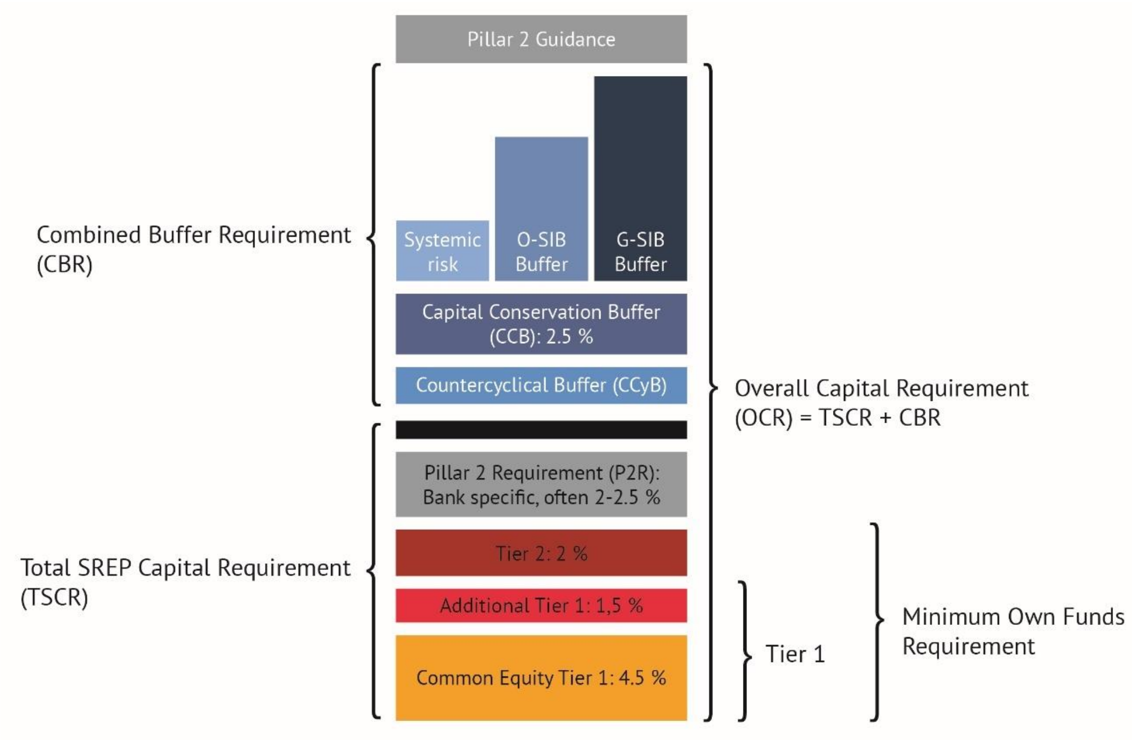

Figure 4 Overall capital requirement (OCR) for banks.

Source: Own illustration, in accordance with EBA regulation as in (EBA, 2018; EBA, 2022).

As the "interconnectedness of banks" severely amplified the financial crisis in 2008/9, a prudential view of these connections became a strong focus of supervision (Freixas & Rochet, 2011; Hellwig, 2008). Regulators tried to avoid a situation where institutes are "too interconnected to fail" beside their exposure sizes (known as "too big to fail", abbreviated TBTF[38]) and could thus cause the necessity of a public bail-out instead of an ordered resolution (Freixas & Rochet, 2011; Hellwig, 2008; Hull, 2015, pp. 152-164). As a consequence, the corresponding systemic capital buffer requirements were set (Behn et al., 2022).

While the bank lending and capital borrower's balance sheet channel was regarded rather early in the academic literature, summarized for example by Bernanke in 1995, the bank capital channel only became a serious research topic with the Basel accords and increasingly after the GFC (Bernanke & Gertler, 1995; Drumond, 2009; Gordy & Howells, 2006). Drumond gives a comprehensive overview of the pro-cyclical effects of the Basel accord's bank capital regulation, and as a consequence of these studies, the

[38] See, e.g., the FINMA as Swiss banking supervision authority, comparable in other jurisdictions (FINMA, 2014).

countercyclical buffer (CCyB) was introduced to mitigate amplifying effects (Drumond, 2009, table 1). Therefore, capital buffers are allowed and even encouraged to be used during economic downturns like the COVID-19 pandemic recession – as opposed to a breach of the TSCR which is never allowed – yet they should be fully filled in "normal" economic times (Behn et al., 2022; Couaillier et al., 2022). Moreover, the OCR figure is a certain threshold (among others) for the distribution of an institute's returns. If it is not fulfilled distributions like dividends are not allowed to be fully paid out (European Commission, 2019; Hull, 2015).

The final component of a bank's required own funds is the P2G (Pillar 2 Guidance) as it is denoted by the regulators (EBA, 2017). This is a guidance target on top of the bank's OCR, which ought to be fulfilled in normal times. It includes a spare amount for potentially occurring times of financial distress in the future and unusual events. THE P2G comprises mainly the quantitative result of the mentioned EBA stress tests[39] (EBA, 2022). The OCR plus P2G sums up the "total capital"[40], which is required by a bank under normal circumstances (EBA, 2017; Hull, 2015). It is important to note that, apart from the pre-defined Pillar 1 requirements, where CET 1 is 4.5 percentage points hence 56.25 % of the first pillar, banks P2G as well as (pro-rata) P2R and the combined buffers have to consist of CET 1 capital (European Commission, 2019). Average available CET 1 of banks per Q4/2021 was 15.4 % of risk-weighted assets on a fully-loaded basis, and TC was 19.6 % of risk-weighted assets – all on a size-weighted average – hence enough excess capital at the moment (EBA, 2021b, pp. 3, 13). In times of crisis like the COVID-19 pandemic or adverse scenarios, a bank is allowed to also make use of the P2G, besides its buffers (EBA-Statement, 2020).

Regarding the bank's regulatory own funds, it is eminent that some subordinated T 2 debt capital is counted as regulatory capital, more precisely "own funds capital", and is not considered as (full) debt capital in the regulatory sense (European Commission, 2019). That differs from and is unequal to equity in the balance sheet according to accounting standards like IFRS or the (national) generally accepted accounting principles, nGAAP (IFRS, 2018). In either of them, T 2 would be counted as debt, not as equity

[39] For smaller banks in the EU similar LSI stress tests exist. In Germany, in that context, the P2G is sometimes referred to as EZK, "Eigenmittelzielkennziffer", which might be translated as own funds target (figure).

[40] In an intuitive sense, considering that is the capital needed to fulfill the total aggregated requirements mentioned. Not to be confused with the official term total capital in Pillar 1 which is just the sum of T 1 and T 2.

(IFRS, 2018). In recent years, due to its ranking in insolvency and its use in cases of bail-in-scenarios, some debt is classified as minimum requirement of own funds and eligible liabilities (MREL) capital or in case of global systemically important banks, the so-called GSIBs, denoted as TLAC[41]-eligible (European Commission, 2019c). This bank resolution connected term MREL refers to a certain kind of debt, with a remaining maturity of at least one year, fulfilling regulatory clauses in the respective contracts (bail-in clauses and others, listed in article 44 of the BRRD II[42], article 12 of the SRMR II[43]). This special debt is bearing no structure, e.g., can not be a structured note, while conventional floating rate notes that are based plainly on standard interbank offered rates (IBORs) like LIBOR[44] in GB, EURIBOR and EONIA in the EU, or their successor rates SONIA (GB), new EURIBOR, €STR (EU), or SOFR (US) are allowed (EIOPA, 2020; ESMA, 2021; Hull, 2015, pp. 215-217; IFRS REG IASB, 2018). The aim is to keep that capacity facile enough to avoid obstacles to resolution as a too narrow time frame (hence the obliged residual maturity of at least one year) or too complex products which are hard to dissolve (hence no structured or derivative components except the IBOR-related ones are allowed) (Deloitte, 2016).

Furthermore, there is no possibility of containing derivative components, and no creditor calling rights within that year's timeframe are permitted by the BRRD (European Commission, 2019c). Especially subordinated – according to the BRRD II definition in the preamble[45] – MREL-eligible debt securities, forming the class of non-preferred (MREL-eligible) senior unsecured debt instruments, are of high regulatory importance and hence issued regularly by banks (SRB, 2022).[46]

1.2 Alternative Assets and Investment Funds

Apart from a single asset perspective on debt instruments and equity or own funds instruments – thus including capital required from banks by regulatory authorities to cover risks – one can group certain assets of possibly different asset classes together. The

[41] Total loss absorbing capacity
[42] The bank recovery and resolution directive II, consolidated version of 2019 (BRRD I was of 2014): (European Commission, 2019c).
[43] The single resolution mechanism regulation I, original version: (European Commission, 2014), amendment for SRMR II in (European Commission, 2019d).
[44] See the SEC's statement on LIBOR transition to new IBOR rates (SEC, 2021b).
[45] The BRRD II and its preamble can be found here: (European Commission, 2019).
[46] See, e.g., Reinwald in a consulting whitepaper (Reinwald, 2018).

various assets might be forming a wider collection multiple investors are interested in – an (investment) fund[47] (SEC, 2007).

An investment fund frequently just called a fund, for the scope of this thesis is pooled capital belonging to numerous investors (European Commission, 2022). The fund manager uses this capital to purchase assets on behalf of the investors, while each investor retains ownership of his (own) parts or shares (Terraza & Razafitombo, 2013). By law, depending on the type of fund and the circumstance if the originator of the fund (which legally equals an investment PLC or LLP by Anglo-American law[48]) is also managing it internally or externally through a management company the so-called fund administrator and depending on the country's further investment laws, also the assets have to be kept separated,[49] specially protected by law, and stored by an external custodian or depositor (Gogarn, 2012).

In the investment industry funds including just stocks and bonds (and cash) are called mutual funds or (rarely) mixed funds (Harkopf, 2016; SEC, 2007). However, there are further alternative assets and opportunities to invest money. They are also prone to (credit) risk as will be seen later.

Alternative investments are then defined as investments in nontraditional assets and furthermore not in classical banking products (SEC, 2007; Zetzsche, 2020). Within that context stocks, bonds, and holding cash are considered traditional assets and sight, term, or saving deposits, and also life insurance contracts being traditional financial products (Chamber et al., 2020). An example of an alternative investment is a private equity investment or a stake in a wind or solar park. Funds that also contain alternative investments are coined multi-asset funds (Chambers et al., 2020; Zetzsche, 2020).

[47] The term fund here must not be confused with the term fund(s) in the context of own funds requirement, where it refers to the funding (re-financing) of a company's assets.
[48] PLC: Public limited company (also denoted plc), LLP: Limited liability partnership (also denoted llp)
[49] As described by law in (European Commission, 2014b).

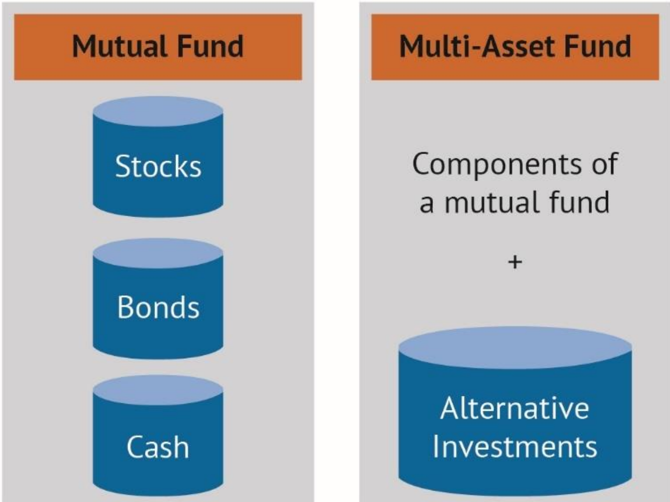

Figure 5 Mutual fund and multi-asset fund.

Source: Own illustration

A further classification of (sub-)assets in the realm of alternative investments can be done as in the following paragraphs. Comparable ways are found in documents of the industry professionals' association AIMA® – the Alternative Investment Management Association® – or the industry's standard institute CAIA®[50] (Chambers et al., 2020). However, one should mention at this point, that in some terminologies and classification schemes currencies, commodities and also real estate holdings are not considered alternative investments, but asset classes in their own right, as in (Fabozzi et al., 2008; Fabozzi & Markowitz, 2011; Fabozzi et al., 2020).

A typical alternative investment asset class is currencies (Chambers et al., 2020; Ponsi, 2007). Currencies can be disassembled further as a complete portfolio currency (e.g., a "U.S. dollar portfolio"), a share of a portfolio or they can appear in a hedged way (e.g., labeled "Quanto-Fund", "EUR hedged tranche").

An investment portfolio (also just referred to as a portfolio) is moreover coined in this thesis as a collection of certain investable assets which are allocated to achieve an investment aim (Aby & Vaughn, 1995; Fabozzi & Markowitz, 2011; PMI, 2017). Such a portfolio groups (sometimes but not necessarily similar) assets together but does not have to be investable by different investors – it can also belong to just a single individual or be

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[50] Chartered Alternative Investment Analyst Association®

artificially constructed for certain investment structuring aims and lacks strict legal definitions – as opposed to a fund (Fabozzi & Markowitz, 2011). A fund hence ought to be considered as a special legal and technical form of an investment portfolio with various fund investors (SEC, 2007). Apart from a few risk-decreasing legal requirements – as minimal diversification requirements like the "5-10-40"[51] - concentration rule in Europe or regarding fund leveraging utilizing derivatives or, e.g., special bond investment grade quality requirements for insurances and pension funds – funds and portfolios are regarded in an equivalent fashion from a risk perspective as a collection of correlated individual assets with their inherent risk characteristics.

In the context of currencies, multi-currency baskets are trading baskets containing various different currencies and are frequently used for currency speculation (Garner, 2012). Corresponding swaps (swap lines) for the currencies – allowing to exchange one currency to another at some timeframe in the future for a certain amount and price (the FX rate) – can be considered as well (Chambers et al., 2020; Ponsi, 2007). Especially multinational companies are in need of such instruments by the nature of their business.

For many companies which operate in different countries thus currency management along the whole supply chain is essential. Derivatives on currencies like options and forward contracts, optionally allowing them (or force them in the case of forwards) to receive a certain amount of a currency at a certain time (period) in the future for a certain price to hedge against a disadvantageous price movement of a currency, like a sudden unwanted decline of the euro-dollar exchange rate, form an indispensable tool (Garner, 2012; Ponsi, 2007).

A rapidly evolving class of alternative investments is one of crypto-assets,[52] including cryptocurrencies, which are generally based on the distributed ledger technology (DLT) and increasingly regarded by the media and public as an alternative to gold or other currencies (Curran, 2021; Feyen et al., 2022; Feyen et al., 2022b; Sandner et al., 2020; Valuewalk, 2021). The regulatory environment is only set up in a rather rudimentary way so far, and some details are still discussed (BIS, 2019b; Giudici et al.,

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[51] Funds according to the UCITS directive (alternative ones according to the AIFM directive and hence, e.g., a closed alternative fund not) are not allowed to invest more than 10 % of the fund's assets in one single counterparty. Furthermore, all assets with more than 5 % of the fund combined must not surpass 40 % of the fund's assets. Hence, a fund needs to hold at least 16 assets of different counterparties.

[52] Opinion of the European Central Bank "on a proposal for a regulation on Markets in Crypto-assets ("MICA") and amending Directive (EU) 2019/1937" (ECB, 2021).

2020). Famous proponents of "cryptos" are Bitcoin®, Ripple®, Solarna®, Cardano®, Ethereum®, or Tether® (Nakamoto, 2008; Sandner et al., 2020).

These share some of the properties of currencies as tradability, exchange possibilities, acting as means of payment and partially possess storage functionalities, however not all (Chiu & Koeppel, 2017; Feyen et al., 2022). Especially, they are generally neither issued nor backed by central banks yet – though the Chinese central bank offered a crypto-renminbi prototype some time ago, and the western central banks work on an E-euro and E-U.S.-dollar respectively as well.[53] If they are not issued by central banks cryptocurrencies are no official, public means of payment or "money" (deVries, 2016). In case a central bank issues such an official digital currency one speaks of a central bank digital currency, a CBDC (Harvey et al., 2021). Only in particular cases cryptocurrencies are backed by assets and are then denoted as stable coins or stable cryptos like diem, formerly known as libra, issued by meta® (formerly facebook®). However, recently meta® decided to sell diem to Silvergate®, and its future "survival probability" seems to be low (FT, 2022). Some researchers and also the ECB generally question the stability of these stable coins, especially if, e.g., bank runs appear in times of a crisis (Panetta, 2021).[54] In reminiscence of the issuance of new stocks for a young company initially listed on an exchange platform, known as an initial public offering (IPO), the issuance of new coins is termed initial coin offering (ICO) as in (Sandner et al., 2020). The volatility for most cryptocurrencies like Bitcoin® or Ethereum® is remarkably high yet, which makes them unsuitable for, e.g., regular wage paychecks at the moment (deVries, 2016). The market risk might be thus seen as high.

However, they are increasingly used as speculative investments especially among young groups of investors using online and app-based investment vehicles like the Robinhood® platform combined with Reddit®[55], sometimes applied as a portfolio diversification component with a low overall share in the whole portfolio (Welch, 2022). Crypto-exchanges as FTX®, founded just three years ago, are meanwhile valued higher

[53] Central bank digital currencies (CBDC) or crypto-money are expected to get launched during the next years – like the mentioned E-euro, as described in (Sandner et al., 2020).
[54] In the process of the last finalization of this thesis a similar scenario even happened, when the TerraUSD stable-coin, swap-related to the cryptocurrency Luna, crashed. See, e.g., the Wall Street Journal article "Crash of TerraUSD Shakes Crypto. 'There Was a Run on the Bank.'" in (Osipovich & Ostroff, 2022).
[55] Coming to fame during the 2021 short squeeze of the game stop stock where young crowd retail investors made (temporarily) high profits for collectively agreeing to buy and bet against certain short-positioned hedge funds. Popularly illustrated in (Yun, 2021). See also for more details (Welch, 2022).

than, e.g., Deutsche Bank® as of Mai 2022.[56] In smaller countries cryptocurrencies are even used as a means of exchange and alternative for the weak local currency or the dollar dependence – as in El Salvador[57], Nigeria, Guatemala, or the Central African Republic[58] (Sandner et al., 2020). Increasingly other tokenized and encrypted forms of assets appear – combined with smart and verifiable contracts, and labeled as level II cryptos – like crypto-SSD-IOs[59] (Harvey et al., 2021). Meanwhile, level II networks and platforms as Lightning® overcome weaknesses of the “old” level-I assets as Bitcoin® (Arcane, 2022; Harvey et al., 2021). Companies like Tesla® or Visa card® as payment providers also increasingly accept cryptocurrencies as means of payment (Lee, 2021; New York Times, 2021). One should however keep in mind that while the (credit) risk of default for states as currency providers is very low and sometimes negligible, though possible as known from the cases of, e.g., Argentina, Russia, or Greece, the one from private cryptocurrency issuers is supposed to be higher in most circumstances (Blustein, 2005; Feldstein, 2002; Sandner et al., 2020). Proper due diligence concerning the (credit) risk of the issuer is thus recommended.

As currency and cryptocurrency trades are executed around the clock and form a trillion-dollar market yearly, currencies are considered an essential alternative asset class (Feyen et al., 2022b; Giudici et al., 2020; Harvey et al., 2021; Newbery, 2022; Statista, 2022; Statista, 2022b).

Besides currencies, major alternative investment instruments are commodities (Harasheh, 2021). Commodities are commonly divided into “soft-commodities” like grain, wheat, soya, cacao, coffee, sugar cane or pig halves and “hard-commodities” like copper, aluminum, or steel, and the special treated commodities gas and oil (James, 2017). The most-traded sorts of (crude) oil are the light ones west Texas intermediate (WTI) and Brent oil, e.g., from the North Sea (CFI, 2022d; ICE, 2020). In several contexts energy is regarded – as in the traded case on power exchanges or also in the area of project finance (cf. below) – as another form of a commodity in its own right, and in the literature

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[56] See FTX® on its website (https://ftx.com) and for a comparison with, e.g., Deutsche Bank® use the calculator for market caps, retrieved Mai 15, 2022, from https://companiesmarketcap.com/deutsche-bank/marketcap/#:~:text=As%20of%20May%202022%20Deutsche%20Bank%20has%20a,company%20by%20market%20cap%20according%20to%20our%20data.?msclkid=33859087d02f11ecb4bde67d38ed8e82

[57] As illustrated in the New York Times on (New York Times, 2021b).

[58] In late April 2022 (BBC, 2022).

[59] Schuldscheindarlehen-Initial Offering, SSD-IO, with SSD being the German version of a promissory note loan.

sometimes even “weather” and weather derivatives are seen as a form of a commodity (Kulkarni, 2004). Weather derivatives are used for hedging agricultural exposures as well as for speculative objectives (Cao et al., 2007).

Having a basic economic function as wheat for nutrition or steel for buildings, gold for jewelry and its accoupling physical delivery and settlement properties as well as a speculative, future-oriented character, commodities might be employed for various purposes (James, 2017). Market participants however in any case need a deep understanding of the fundamentals and market sentiment – acting then, e.g., as commodity trading advisors (CTAs) in investment banks or brokerage firms – to be successful (Fabozzi et al., 2008). Regarding commodities, credit risk appears in the form of pre-payments or delivery in advance in the timeframe where the commodity is paid for but is not delivered by the counterparty yet or vice versa (Bouteillé & Coogan-Pushner, 2021). It is sometimes also denoted as delivery risk, or in the case of a settlement or clearing intermediary labeled settlement risk and regarded as a special subcategory (Bouteillé & Coogan-Pushner, 2021). Once one has obtained the commodity and possesses it oneself[60] the credit risk component has evidently disappeared, other than for instance market risk.

Special commodity exchanges exist for commodity trading in the United States like the Chicago Mercantile Exchange (CME®), Chicago Board of Trade (CBOT®), Chicago Climate Exchange (CCX®), and U.S. Futures Exchange (USFE®) in Chicago, the Intercontinental Exchange (ICE®) in Atlanta, the Kansas City Board of Trade (KCBT®), or the New York Mercantile Exchange (NYME®), see (Commodity, 2020). In Great Britain for instance the London Metals Exchange (LME®) and London Commodity Exchange (LCE®) are operating, whereas in continental Europe the European Energy Exchange (EEX®) in Leipzig or the Climate Exchange (CLIMEX®) in Amsterdam are important, as well as for example the commodity department of the European Exchange EUREX® and Euronext® (Commodity, 2020). While increasingly also so-called exchange-traded funds (ETFs[61]) on commodity shares, certificates on commodities like exchange-traded commodities (ETCs), or derivatives[62] are offered to

[60] Otherwise if it is held in a depository the named risks (delivery and depositary) arise again.
[61] For a definition see the SEC in (SEC, 2017).
[62] Derivatives can be forwards (which have to be exercised and fulfilled by both sides), options (which can be exercised when the option holder chooses to decide so), exchange-traded forward contracts called futures, or swaps (where an exchange, a “swapping”, of assets and/or money takes place). Furthermore, combinations of these like swaptions - which is an option on a swap - exist.

retail investors, the commodity market still remains largely in the hand of institutional investors, banks, and large global companies (Fabozzi et al., 2008; Schaeffer, 2008). This asset class is further exposed to enormous geostrategic and political risk besides the normal market forces, as lately seen in the case of the Russian invasion of Ukraine in 2022, which spurred an enormous rise in oil and gas prices in the western world (Caldara & Iacoviello, 2018; Corden, 1984; ECB, 2022b; Murray, 2018; WEF, 2022; Weizhen, 2022).

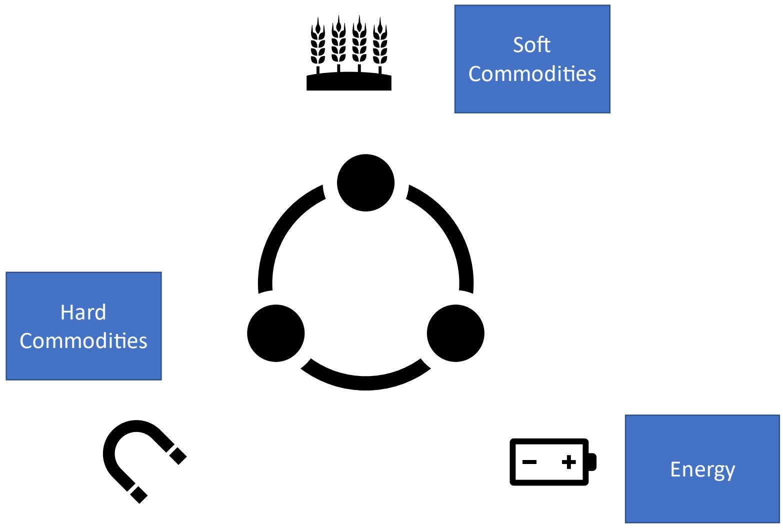

Figure 6 Categories of commodities.

Source: Own illustration.

Real estate, the next class of alternative investments, is usually separated into the residential real estate and mortgage sector on one hand and the commercial (and industrial) real estate side on the other (Haight & Singer, 2004; Goddard & Marcum, 2012). Often, they are abbreviated as RRE and CRE (NAREIT, 2021). Residential real estate can be further divided into apartments (flats) or houses, multi-family or single-family homes, and special forms like detached houses or semi-detached (duplex) houses (Haight & Singer, 2004).

For (semi[63]-) professional investors especially residential building complexes and the rent or commercial sell-off of certain units are interesting, whereas the area of private house buying or home construction is not considered (Haight & Singer, 2004). Yet, as seen, these can be utilized as well as collateral for the loans banks are reaching out, therefore reducing the banks' unsecured credit risk exposure or collateral (covered) pool for covered bonds. Regarding real estate and real assets in general, credit risk may arise in a natural fashion, if an estate is funded by a loan or bond, or it may arise again due to pre-payment before the estate is completely constructed (construction risk) and ready-to-use (Haight & Singer, 2004; Hull, 2015). Afterward, risk may arise similarly through (in most cases temporary) material, quality, or operational problems, when the promised amount (like rent or energy) cannot be delivered due to quality problems – subsumed as operational risk, which is introduced later. Credit risk may again arise in the form of default risk when pre-payments or acquisition of shares of an estate or real asset have taken place, but no cash flow is received because the provider or its users (tenants, energy consumers) default (Bouteillé & Coogan-Pushner, 2021).

In terms of commercial real estate, the sectors which are currently of the highest demand investor-wise are office space, logistic real estate, retailers, and hotels (NAREIT, 2021). Common types of portfolios or funds operating within the realm of real estate are either "closed (real estate) funds", which have certain strict conditions on minimum investable amounts, withdrawal restrictions, and tax treatment, and are mainly used by (semi-) professional investors as investors become stakeholders in the business as liability partners, or "open-ended real estate" funds with fewer restrictions, as the possibility to only one year-ahead selling announcements in the EU and shorter minimum holding periods (Goddard & Marcum, 2012; SEC 2016, p. 5). As indicated, in the real estate investment sector one thus has to differentiate between direct investments or holdings, e.g., the direct purchase of a building complex, indirect fund investments, e.g., in "normal" open undertakings for collective investments in transferable Securities (UCITS) and stakeholder/ownerships as in the case of (closed) alternative investment funds, known as AIFs (Gogarn, 2012).[64] Furthermore, a common investment vehicle are real estate investment trusts, REITs, which are single real estate companies or grouped indices of

[63] Semi-professional investors are defined as very experienced investors with a minimum commitment of 200 000 EUROs (respectively 100 000 EUROs for the European venture capital fund regulation), cf. (ICLG, 2021).
[64] Definitions by the European Commission in (European Commission, 2022).

companies in the real estate sector and hence represent an investment in (real estate) stocks not directly in real estate like building complexes (Sotelo & McGreal, 2013). Additionally, in many cases of real estate investments – in the case of infrastructure or renewables investments like in wind parks this fact holds as well – one considers not only already available, completely constructed properties but the area of project finance with the whole life cycle of real estate including planning, building, and then selling or renting (leasing) to others, and even managing the estates is regarded (Goddard & Marcum, 2012). In project finance, the development and construction phase have a substantially different risk and capital leverage[65] profile and often even different sponsors[66] than the phase of running the project (Joseph, 2013). Thus, investments in new greenfield projects vs. in existing brownfield projects, in which case the estate is already completely built, are differentiated (Adams & Watkins, 2002; Preqin, 2022).

Similar project (or object) finance sponsored forms of investment is infrastructure investments. They inherit their own risk and return characteristics (Lewin et al., 2015). A special case of them, which is frequently referred to as an own asset class, is the sub-category of renewable assets ("renewables") like solar and wind power plants or parks, but also including energy forms like water power, geothermal energy and bio power (Mohamadi, 2021; WEF 2016).

In contrast to that, classical infrastructure projects are, e.g., toll collect highways or other infrastructure typically build within a PPP, a public-private partnership (APEC, 2018). A sub-classification of infrastructure investments including renewables is achieved by closer monitoring and assessment of the underlying risk and return profiles. Low-risk, "income type" investments are labeled as core infrastructure investments like social infrastructure or guaranteed PPPs (Gourntis, 2017; Mercer, 2021). Credit risk during the operating phase is generally rather low because of the PPP design, often quasi-monopolistic well-rated providers, state guarantees for loans and long-term contracts (Mercer, 2021; Waters et al., 2015). The next stage core plus (Core+, CP) describes rather similar but slightly more risky projects, frequently implemented with the help of large oligopoly companies (Mercer, 2021). Finally, the terms "value add(ed)" with a balanced risk profile and "opportunistic" with higher volatility and expected returns are used to

[65] Leverage is defined as the equity to debt ratio in financing a project (and on the other hand fund leverage of the equity/debt ratio within a fund, including derivative instruments).
[66] Synonym for certain classes of investors in the context of project finance acting as main underwriter or promoter.

define infrastructure projects. Usually, the expected returns on that different so-called infrastructure investment strategies are ranging incrementally from 4-5 % in the core case to 14-15 % in the opportunistic one. As usually the higher expected returns are linked to a higher counterparty (credit) risk and less experienced, economically fewer stable providers (Waters et al., 2015).

Increasingly electronic infrastructure such as 5G-networks or cloud and data centers are becoming part of alternative investments in infrastructure (Mercer, 2021; Preqin, 2022).

Many of the infrastructure linked vehicles – similarly as in the case of securitization transactions with ABS[67] or CDO/CDS[68] papers – are specially set up for a project (object) financing and risk transfer purpose and are hence called special purpose vehicles or special purpose companies, SPCs (Bouteillé & Coogan-Pushner, 2021, p. 317). Available data in reference to such projects is sometimes sparse compared to liquid time series[69], and due to survivorship and appraisal bias it is less reliable or not representative for the entire class (Preqin, 2022). Yet, there are specialized providers like Cambridge Associates®, the self-proclaimed market leader Preqin®,[70] EDHEC®, or the Cambridge university with its center for alternative finance (CCAF[71]) tackling these issues and providing increasingly better data. Modeling possibilities for these types of assets and available "proxy solutions" in case of sparse data are mentioned later on.

In some cases, real assets combine not solely real estate but any type of investment bringing potentially "real" income, like the "natural assets" farmland – more generally the acquisition, development, and construction of land as described by the Basel and CRR III ADC exposure class – or timberland and waters (Demaria, 2020; Villanacci et al., 2002). In addition, the asset class "machines, plant, and property" like fabrics, machinery or automotive vehicles are counted as real assets (Demaria, 2020; Villanacci et al., 2002).

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[67] Asset-backed securities, securities in which coupon payments are (re-)financed by pooled, securitized underlying assets like auto loans, credit card loans, or mortgages. In the latter case, then called mortgage-backed securities – MBS.
[68] Collateralized debt obligations, obligations that are securitized, pooled together to receive a higher rating due to – often questionable – diversification effects, divided into tranches, and then sold to investors. Similar to ABS, cf. (Bouteillé & Coogan-Pushner, 2021; Martin et al., 2014).
[69] Though there are also liquid alternatives like certain listed hedge funds and listed infrastructure available.
[70] As stated on the Preqin® website, retrieved Mai 16, 2022, from https://www.preqin.com/about/who-we-are
[71] Cambridge Research Center for Alternative Finance at Cambridge university, retrieved Mai 16, 2022, from https://www.jbs.cam.ac.uk/faculty-research/centres/alternative-finance/?msclkid=07eb6f6db03811ecb9b497e791258b93

These types of assets are commonly associated with special forms of lending as well as finance/sale or operating leasing (Villanacci et al., 2002).

Moreover, commodities and the real estate sector as well as real assets have proven to be valuable sources of diversification for a portfolio (Johnson & Jensen, 2001). Credit risk naturally may arise during the acquisition period in the form of non-delivery caused by the bankruptcy of the counterparty (Hull, 2015). It can arise during the development and construction period due to the default of developers, construction companies, or providers or during the operating period caused by the default of an external operating company or servicers/providers (Bouteillé & Coogan-Pushner, 2021; Demaria, 2010). In rare cases, as these investments normally form a wide, rather diversified cohort, credit risk may appear also (indirectly) in terms of (defaults of) consumers, users, or tenants/leasers (Bouteillé & Coogan-Pushner, 2021; Demaria, 2020; European Commission, 2019; Hull, 2015). Though these phases and cases might appear rather different, they can be treated again within a similar modeling framework as will be seen in Chapter 2.

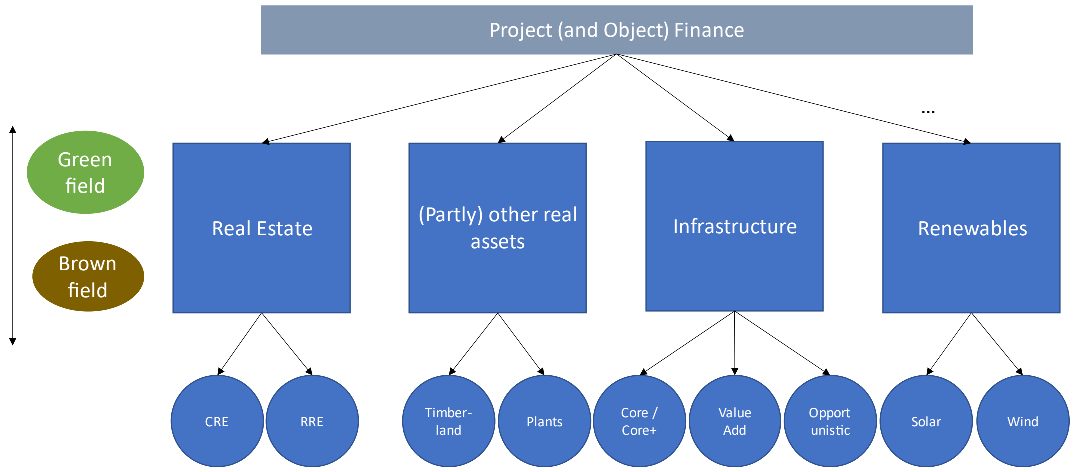

Figure 7 Project finance with the forms of real estate and real assets, classical infrastructure, and renewables.

Source: Own illustration.

Real assets are increasingly popular as they outperform other classes during times of financial distress and high inflation (Blackrock, 2021; Demaria, 2020). This is possible as the income side can often be adapted through enforcing higher nominal prices, rents, or fees and increased demand for the goods such as energy which became relatively scarce, while the cost and expense side may be capped by "expense pass-throughs", "beneficial leverage", or higher replacement costs (Blackrock, 2021, pp. 4-6; Mayer, 2022). Inflation, as commonly known in economics and, e.g., shown by Friedman, is mainly a monetary phenomenon (Friedman, 1992; Lothian, 2014). High state debt is hence, especially when financed by quantitative easing, intricately linked with higher inflation (Reinwald, 2022a). Therefore, real assets are demanded in times of inflation, they serve as a natural hedge and diversification component in that regard (Demaria, 2020). A similar function and perception of inner value are accredited to precious metals (Darst, 2013).

Precious metals are, e.g., gold, platinum, and silver which also serve as special asset classes (Darst, 2013). Especially gold has a thousands of years long perception as storage of high value, coin money, inflation protection, means of exchange, backup currency, or an underlying asset (Buranelli, 1979; Durant, 1954). In the US until President Richard Nixon's 1971 decoupling of the dollar from the gold binding, established in the Bretton Woods agreement of 1944, the currency (US-$) was convertible into ounces of gold (Mayer, 2022; NMA, 2003). Apart from gold, which is besides jewelry and the luxury goods industry also used as effective conducting metal in the area of microchips and semiconductors, the chip sector enormously relies on rare metals – also referred to as "rare earths" – like Yttrium, Scandium, Samarium, or Thulium (Mosley, 2012). These are also considered alternative investment metals, though – contrary to their name – they are not rare around the world (Mosley, 2012). However, production sides are very expensive to build, and China is currently acting as a nearly monopolistic provider (Mosley, 2012). The geopolitical risk mentioned above, combined with the problem of undiversified supply chains, can be therefore seen in that respect as well.

There is a wide range of other alternative investment opportunities, like hedge funds (Cumming et. al, 2021). Originally, this vehicle was set up for hedging ("protecting") against market risk and building up risk-avoiding, averse positions in relation to its underlying ("base investment"), e.g., when holding (long) a certain stock in a broader portfolio and then simultaneously short-selling it, i.e., betting with the means of derivative options for a price decline of this stock, or equivalently eliminating its individual, idiosyncratic risk when considered in a portfolio risk context (Cumming et al., 2021; Holler, 2012). Since the 1990s however, hedge funds are generally considered investment vehicles that are free in their (dis-)investments and longer-term strategic as well as short-term tactical asset allocations, when opportunistic (Cumming et al., 2021). Hence, they intend to make profits in all market phases and business cycle periods, also falling markets (Holler, 2012). Hedge funds are divided by their applied investment styles and strategies, e.g., a "Long-/Short strategy", "market neutral strategy", "directional or relative value strategy", "global macro strategy", "event-driven strategy", or "managed futures/CTA strategy" (Baker & Filbeck, 2017). Derivatives such as forwards, futures, options and corresponding options' strategies (as the "collar" or "butterfly") are heavily utilized by hedge funds, and the overarching alternative investment class is commonly referred to as hedge funds & strategies (Baker & Filbeck, 2017). As hedge funds are directly related to their investments and usually have few operational business and risks besides, when investing in a hedge fund or fund in general the risk of the fund roughly equals the risk of its investments, and acquired holdings (Baker & Filbeck, 2017; European Commission, 2019). Hence, regulators envision a "look-through approach" (European Commission, 2019). For more details concerning hedge fund strategies, which are not in the further scope of this thesis, see for instance Cumming, Satchell, or Kiesel[72] (Bingham & Kiesel, 2004; Cumming et al., 2021; Satchell, 2015). Often, they are also categorized as the sub-class absolute return funds (Holler, 2012).

A further, closely linked asset class is private equity, where the investors directly allocate their money into a fund that is investing in private (unlisted) companies with the means of equity and owner-type instruments (Demaria, 2013; Pöllath, 2018).

Depending on the investment targets, i.e., the acquired companies' age structures or "vintages", positions in the market, and the phases of enterprise growth one differentiates between the styles of venture capital, buyout, growth, and distressed debt (Demaria, 2013). Venture capital is commonly denoted as VC (Demaria, 2013). In terms of VC, it is a widespread practice to even separate between the seed stage and startup stage in the early phase of an investment, the succeeding growth or expansion stage, the

[72] Bingham and Kiesel authored a standard book on risk-neutral pricing and valuation including common derivative hedge fund strategies (as bull-spread, bear-spread, wrangle, butterfly) - cf. (Bingham & Kiesel, 2004).

following pre-IPO stage, and finally the multi- or later-stage when regarding corresponding funds (Braun, 2018, p. 4).

Buyout investments are further classified by the type of initiator(s) of the buying-out of (parts of) a company (Gleissner & Schaller, 2008). Management buyouts are initiated by one or more current managers or employees of a company who acquire the majority of stocks and buy themselves (parts of) the company making them the new owners (Demaria, 2013). If the management team is external (before the transaction) and acquires the company one speaks of a management buy-in, MBI (Braun, 2018, pp. 7-8). Leveraged buyouts and so-called squeezes[73] with debt capital are the most popular form of buyouts, where private equity funds use (foreign) debt capital and hence a high debt/equity leverage ratio to acquire (parts of) a company (Demaria, 2013; Gleissner & Schaller, 2008). This is evidently a method to minimize the use of their own equity and to receive a higher return on equity. In practice, LBOs are often combined with MBOs, and managers receive equity incentives to align interests for the transaction (Braun, 2018). Another form of buyout transaction, where the PE investor (institution) directly acquires the company from the vendor is denoted institutional buyout, IBO (Braun, 2018, p. 7). Finally, owners' buyouts are buyouts, where some of the (part) owners of a company buy up the whole (or majority) of a company, e.g., the rest of the free-floating stocks to gain full control (Demaria, 2013). This might be a method to gain control and hence change the (strategic) direction of a company when (parts of) the management of a company disagrees with the current owners on major decisions or the future strategy of the company and are convinced they have better strategies and can monetarize on them as new owners. The forms of buyouts are abbreviated as MBO (MBI), IBO, LBO, and OBO respectively (Braun, 2018, pp. 6-8).

Growth style is the term for private equity investments in rapidly growing or already maturing companies (Demaria, 2013). Regarding the underlying risk and return structure, growth strategies are more similar to buyout strategies than to venture capital, which incorporates a considerably individual characteristic and vice versa (Braun, 2018, p. 4; Canderle, 2020; Pöllath, 2018; Ritchie, 2017). In the case of VC, the probability of default of an equity funded company is much higher, many startups do not survive the first years, hence a large cohort and well-diversified portfolio is essential for business

[73] Respectively called squeeze-outs.

investors to reduce credit risk exposure (Canderle, 2020; Demaria, 2013). For buyouts of established companies the equity (credit) risk is often comparable to one of “normal” peer-group companies (Braun, 2018). The overall risk, however, can be substantially higher for private equity investments and transactions, when buyouts are highly leveraged and funded by external debt (Demaria, 2013; Pöllath, 2018; Witzany, 2017). In all cases, proper due diligence of companies and a profound understanding of the market sector is required to be successful, and most investors are again (semi-) professional ones (Demaria, 2013).

There are special forms of private equity (PE) existing like mezzanine PE, which exploits hybrid capital like participation rights or subordinated debt options or also turnaround/distressed debt strategies (Gleissner & Schaller, 2008). These commonly involve highly activist (sometimes disrespectfully labeled as “rogue”) investors, often motivated as “event-driven”, i.e., incentivized by certain corporate actions like dividend payments, interest step-ups, the issuing of new stocks and hence diluting of old ones or events like rating downgrades, hostile takeovers, or mergers and acquisitions (M & A) of companies (Braun, 2018; Pöllath, 2018). Private equity funds further possess a different incentive and reimbursement structure regarding their management and general owners, with carried interest[74] being an essential part of the cash flow structure (Demaria, 2013; Demaria, 2015).

[74] Apart from only the usual so-called high-level watermark and performance fees for fund managers awarding them with higher payments, bonuses or normally direct participation rates of, e.g., 20 % of the on-top performance once they reach a certain performance threshold (Demaria, 2015).

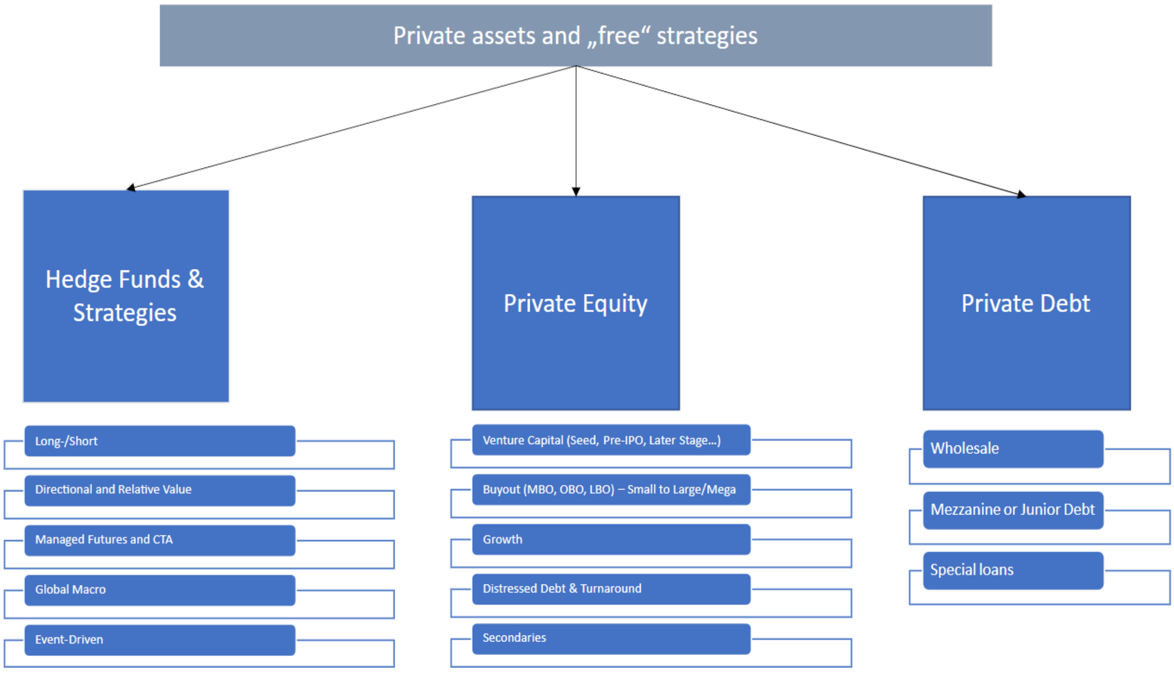

Figure 8 Hedge funds and strategies, private equity, and private debt.

Source: Own illustration.

The natural pendant of private equity capital structure-wise is private debt (PD[75]). In the case of private debt investors or collective funds, as again this type of investment can be set up in a direct or indirect manner via funds and fund ownerships, reach out loans to companies (Ritchie, 2017). This fund instrument is often used when the companies were not able to arrange bank loans or directly issue traditional bonds or promissory note loans (Grün, 2021; Nesbitt, 2018). The underlying credit risks regarding the loans are hence often higher in comparison to standard bank loans (Nesbitt, 2018). As with registered bonds and promissory note loans ("SSDs"), investors are frequently institutional investors, insurance companies, and large pension funds (Nesbitt, 2018). In the area of private debt, one classifies mainly in terms of the attributes and characteristics of the given loans taking into account the seniority, i.e., at what level an investor is positioned within the creditor ranking, in rare cases collateralization of the debt, tranches available, and maturity as well as interest options and types (Grün, 2021). The latest feature refers to the type of interest, i.e., if the debt interest is paid via fixed or floating rates (coupons), in which frequency (i.e., semi-annual or annual) and if it is containing

[75] Not to confuse with PD as an abbreviation for the term probability of default as mainly used throughout this thesis.

options (special payments regarding certain pre-defined events) or not. This is similar to the case of standard debt securities (IMF, 2009). As debt types in these funds, one separates between whole loan, mezzanine, or equity debt types (Grün, 2021; Nesbitt, 2018). Regarding many characteristics, the (debt-funded) real estate project finance described before and private debt often serve each other as available proxies and are, e.g., occasionally interchangeable used for validating the formal underlying risk models as will be mentioned later (Joseph, 2013).

Finally, rather exotic investments are commonly coined under the phrase "alternatives". These are collectibles like noble wines (e.g., the Château Pétrus), extremely rare stamps (like the "Blue Mauritius"), music instruments (like a Stradivari violin), or other exotic investments like fine art, so-called electronic non-fungible tokens (NFTs) like special digital pictures, sports teams, or vintage cars (Mirabile, 2021). Credit risk appears mainly in the acquisition phase caused by the time difference between payment and delivery (settlement), later-on the risk associated with collectibles is due to conservation of the quality of the good and market demand for the scarce asset (Mirabile, 2021).

Figure 9 List of alternative investment classes.

Source: Own illustration.

As proven in the academic literature many of these asset classes are most of the time rarely correlated or even negatively correlated to each other (on average, e.g., equities and bonds or hedge funds and cash/currencies), the various assets therein therefore form are a valuable source of diversification for a portfolio (Nguyen et al., 2020; Pojarliev & Levich, 2012; Webb et al., 1988; Yan & Garcia, 2017; You & Daigler, 2013; Ziobrowski & Ziobrowski, 1997).[76]

	Positive	Negative
High	0.7-1.0	(0.7)-(1.0)
Moderate	0.4-0.7	(0.4)-(0.7)
Low	0.0-0.4	(0.0)-(0.4)

	Investment Grade Bonds	Cash	Commodities	Currencies	Equity Market Neutral	Event Driven	Global	Hedge Funds	International Equity	Long/Short Equity	Managed Futures	REITs	S&P 500®
Investment Grade Bonds	1.00												
Cash	0.17	1.00											
Commodities	(0.19)	(0.15)	1.00										
Currencies	(0.09)	(0.04)	(0.41)	1.00									
Equity Market Neutral	0.07	(0.17)	0.32	(0.54)	1.00								
Event Driven	(0.04)	(0.32)	0.65	(0.26)	0.35	1.00							
Global	0.01	(0.21)	0.57	(0.43)	0.37	0.80	1.00						
Hedge Funds	0.10	(0.27)	0.59	(0.30)	0.48	0.89	0.82	1.00					
International Equity	0.02	(0.19)	0.58	(0.52)	0.41	0.76	0.94	0.79	1.00				
Long/Short Equity	(0.01)	(0.25)	0.54	(0.37)	0.51	0.80	0.87	0.91	0.83	1.00			
Managed Futures	0.43	0.01	(0.08)	0.10	0.11	0.07	0.13	0.40	0.08	0.24	1.00		
REITs	0.39	(0.18)	0.31	(0.16)	0.27	0.56	0.67	0.60	0.60	0.58	0.28	1.00	
S&P 500®	(0.02)	(0.20)	0.51	(0.33)	0.32	0.76	0.97	0.78	0.85	0.84	0.15	0.68	1.00

Figure 10 Historical correlations between asset classes from 01/2011-12/2021.

Source and copyright: Guggenheim® investors. Retrieved Mai 28, 2022 from https://www.guggenheiminvestments.com/mutual-funds/resources/interactive-tools/asset-class-correlation-map?msclkid=fcea45f6cfcd11ec8c5fbf8

Further valuable reasons, as their different profiles, maturity and tenor structures, ownership rights, and various inherent risk components were presented.

[76] Updatable correlations between common indices on shares, bonds, commodities, real estate and, e.g., gold can be shown here: https://www.portfoliovisualizer.com/asset-class-correlations (Retrieved Mai 16, 2022).

1.3 Investment Strategies and Funds

Having described the available forms of assets, asset classes, inherent (credit) risks and investment opportunities and first associated types of funds the next step is to define investment strategies and further funds. Recall that various assets from potentially different asset classes with their own characteristics regarding, e.g., returns and (credit) risks can be grouped together to form a portfolio or an investable fund.

An investment fund (or also an investment portfolio) is set up with a certain investment aim. This aim can be a certain return[77], also denoted as performance, over a specified time horizon (i.e., 5 % p. a.), it might be more broadly defined as, e.g., "the conservation of capital" or it may be a certain risk/reward profile or even tax and regulatory purposes. Depending on the aim and possible target group – e.g., a public fund for retail clients and associated tranches versus one for institutional clients or trusts – an accompanying investment strategy is derived (Aby & Vaughn, 1995; Gregoriou, 2006). The centerpiece for its implementation is the selection of assets, the asset allocation (Fabozzi & Markowitz, 2011; Ibbotson, 2010; Gregoriou, 2006).

Considering the asset allocation and the management of a fund or portfolio one can then differentiate between the overall, long-term allocation strategy, which is known as the strategic asset allocation (SAA) and the short-term allocation the tactical asset allocation (TAA) (Fabozzi & Markowitz, 2011; Gregoriou, 2006; Galoppo, 2021).

The TAA allows for over- or underweighting of assets in pre-defined ranges according to short-term investment opportunities. This part of the allocation is also often accompanied by a temporary "derivative overlay" for hedging or leveraging purposes (Galoppo, 2021).

[77] A return r over a time horizon t_0 to t_1 with corresponding asset prices P_0 and P_1 is defined in the thesis as $(P_1\text{-}P_0)/P_0$ which equals $P_1/P_0 - 1$ and used within this thesis as such. Sometimes a log-return or return defined as $ln(P_1/P_0)$ is considered and rather equivalent for small deviations and nonskewed normal distributed data.

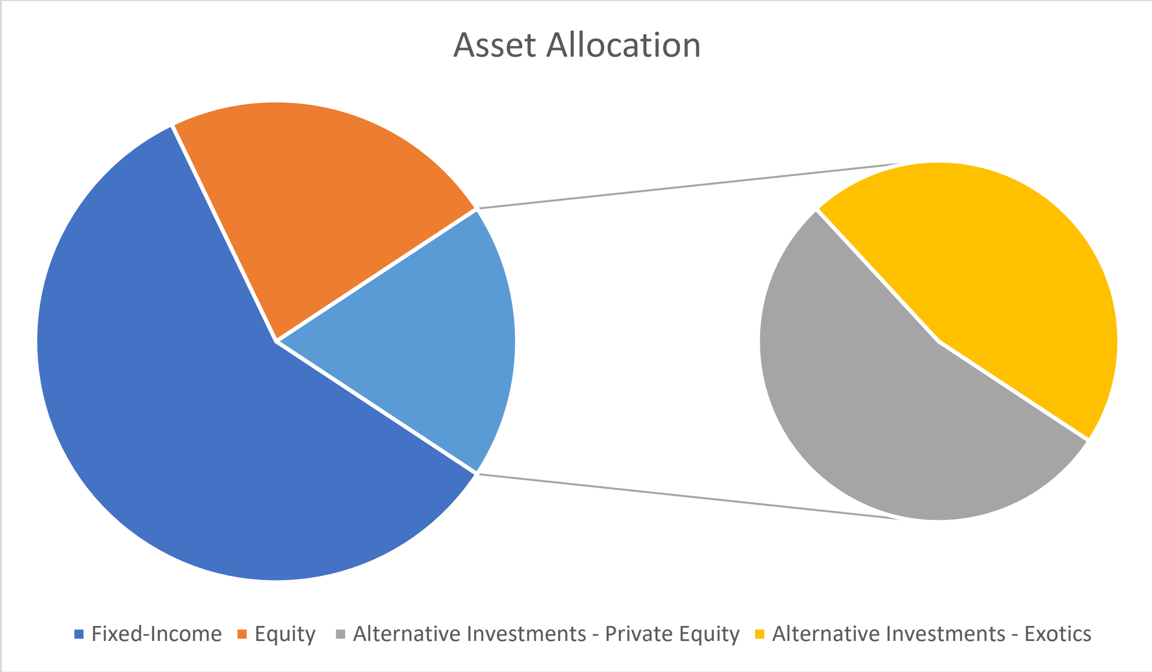

Figure 11 Asset allocation example of a multi-asset fund.

Source: Own illustration

Empirically, studies show that the SAA accounts for most parts of the performance compared to the TAA, even though the percentage varies and the formerly assumed "90%" is definitely overhauled, and recent studies – like Davis' meta-study – tend more toward "60%" or "more than half" of the returns (Campbell & Viceira, 2002; Davis et al., 2007; Fabozzi & Markowitz, 2011; Ibbotson, 2010; Ibbotson, 2010b; Kinlaw et al., 2021).

In any way the strategic asset allocation and strategy behind it is crucial for the performance (aim) and risk of any portfolio or fund. An example strategy is the aim of capital conservation with the means of a fund consisting of 60 % bonds, mainly consisting of European corporate bonds in the associated iBOXX® Corporate Bond Index, combined with 40 % large cap stocks, listed on European stock exchanges and their issued indices as the EURO STOXX 50 ®. Hence, a 60 / 40 allocation is the SAA. In times of higher volatility, it might be secured by derivatives and ratios for instance adjusted to 70 / 30 – as TAA.

Furthermore, a company might also just advise or portfolio-manage such a fund, which itself is administered and often risk-managed by another management company and even invests in a mixture of fund managers (in a fund of funds structure) – these funds are known as "white-label funds" (Galoppo, 2021). That kind of construction is often exploited when (several) small fund managers or boutiques shall manage the portfolio composition and selection ("picking") of assets and exposure to managers shall be diversified, whereas large investment companies like Universal Investment® provide the fund administration, settlement, and depositary tasks, further post-trade executions, documentation, and risk management reports. Additionally, also payments and occurring bills can be externally managed by so-called paying agents and the registration of assets is done by registrar holders or agents (Broby, 2010). National numbering agencies (NNAs) add and register the ISIN and stock exchanges and platforms execute the issuing process (Galoppo, 2021). The form of management and degree of outsourcing (organizational features) should be derived from or supporting the investment aim(s) and strategies, the same holds true for the setup and specific type of fund to be selected (Broby, 2010; Terraza & Razafitombo, 2013).

As briefly mentioned before, considering the overall setup, two kinds of funds are allowed in the EU and (mainly) the US. These are general UCITS funds (also known as OGAWs[78]), which are the "normal", open[79] funds and so-called Alternative Investment Funds (AIFs), constituting by law the rest of funds (SEC, 2016, p. 5). Alternative Investment Funds often being closed[80] "special funds" – and hence only open for investment for semi-professional investors as opposed to "public funds" that are available for every private investor (SEC, 2016; Terraza & Razafitombo, 2013).[81]

Funds have further characteristics and distinguishing properties. They are either listed, which means registered and traded on a daily basis on an exchange, like an ETF or on the other hand not listed (Meziani, 2016; SEC, 2016, p. 6). A fund may be composed of sub-funds and in that case, it is called a fund of funds, acting as a "roof" for its sub-funds (Bookbinder & Strachman, 2009). Furthermore, a fund can be fed by the money of other funds. In this case, the "feeder funds" invest in the "master fund" (Bookbinder & Strachman, 2009).

Modern funds are often also referring to their investment paradigms and "black-list" forbidden companies to not invest in, like coal energy suppliers, or on the contrary in a stricter manner only "white-list" – explicitly allowed or desired – companies. These

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[78] Known as "Organismen für Gemeinsame Anlagen in Wertpapiere" in Germany and the "DACH-region". In that context the term DACH stands for Germany (D), Austria (A), and Switzerland (CH) as abbreviated in the EU.
[79] It is possible to buy and sell shares of that fund at all times.
[80] Closed means just open for a certain period of time for investors and normally also only to a certain group of special investors before being soft closed or hard (completely) closed.
[81] A good comparison, e.g., for typical Luxembourgish constructs, can be found on (Chevalier & Sciales, 2022).

fund approaches are called ethical ("blue") or sustainable ("green") funds (Terraza & Razafitombo, 2013).

The exact breakdown of a portfolio or fund is usually, apart from the types of invested asset classes, further achieved by a decomposition of invested regions and geography, branches and industry sectors, currencies, and often the top ten largest holdings (Ang, 2014; Satchell, 2016). Therefore, funds depending on their asset holdings are classified and also named as (pure) bond or debt funds, money market funds[82], or equity funds, when only investing in this sole class of assets (Harkopf, 2016; IMF, 2009; SEC, 2016). Otherwise, as described before, they are labeled as traditional mutual funds or multi-asset funds when containing various asset classes. Exceptional cases are balanced or hybrid funds – when containing stocks and bonds but additionally with the aim of a balancing strategy or a capital plan, e.g., for pensioners, like a monthly income plan (MIP), as illustrated in another possible graph below (Harkopf, 2016; IMF, 2009; SEC, 2016).

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[82] A money market fund (MMF) is "an open-ended mutual fund that invests in highly liquid short-term financial instruments" (IMF, 2009, p. 10). Money market funds might be set up with a "constant net asset value", which is the inventory value of a fund as its total volume divided through the number of shares (CNAV), where the net asset value NAV is "a constant $1 per share, or with variable NAV (VNAV) where the NAV can fluctuate" (IMF, 2009, p. 10). Generally, money market funds are possible be installed as either UCITS or AIF fund cases (IMF, 2009).

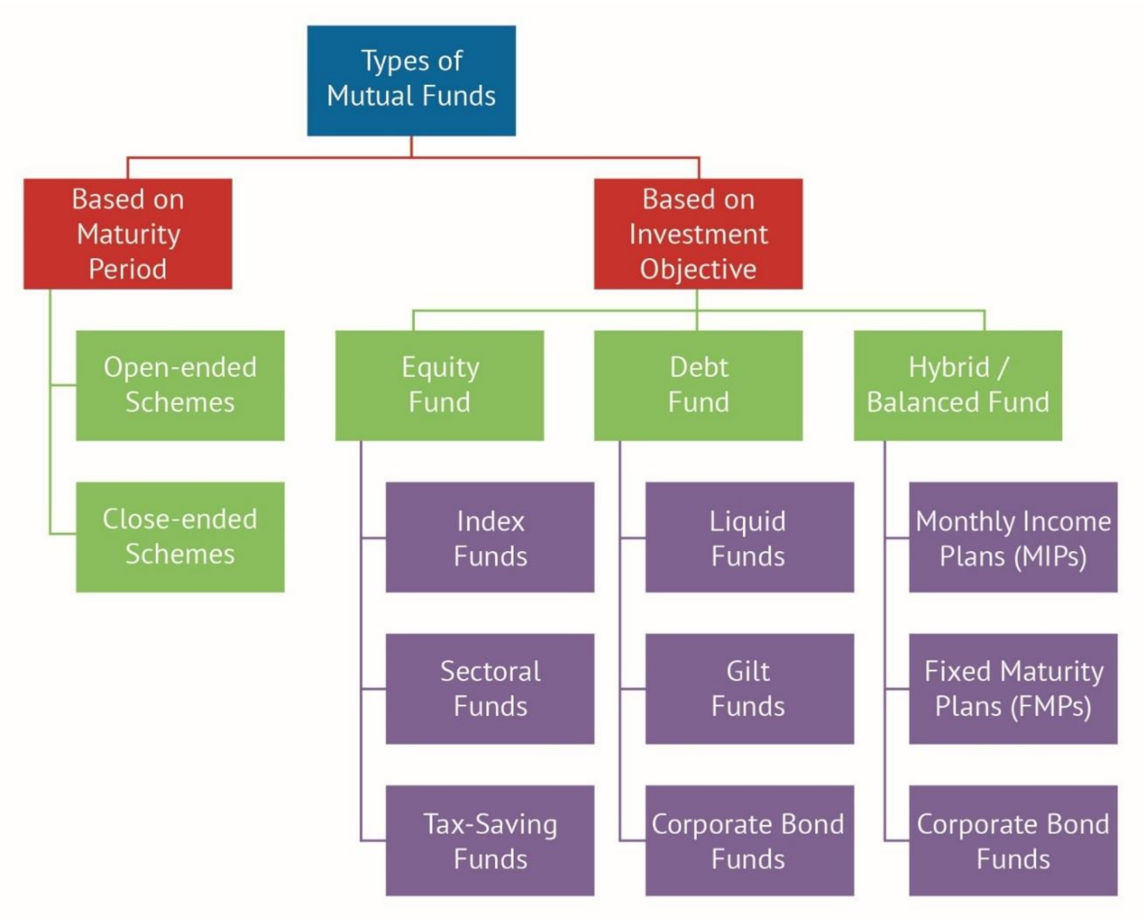

Figure 12 Types of mutual funds.

Source: Own illustration, in accordance with (Harkopf, 2016).

According to the geographical locations of their investments funds can be labeled as emerging markets funds, composed of assets located mainly in the emerging countries of the world, as developed countries funds, which are composed of assets located mainly in developed nations, or frontier markets funds that are composed of assets located mainly in countries on the brink to a developing status (MSCI, 2021; MSCI 2021b). In the fund industry, depending on the regional focus of the invested assets, one speaks of U.S. funds, Asian funds, European funds, often EMEA funds – funds only investing in countries within Europe and Middle or East Asia – or ROW funds, which focus on the rest of the world except Europa, the US, and Asia (Harkopf, 2016).

Funds can be further categorized according to the branches or sectors they invest in, like technology funds, i.e., funds where stocks are picked from the technology sector or a technology index (Satchell, 2016). Another example is mining funds, in which case only mining companies are constituents of the funds.

In case funds are classified according to the market capitalization (“size”) of the purchased shares of companies they are labeled small cap funds when just containing small or very small (micro) companies whose market capitalization, the market value of its outstanding shares amounts to less than three hundred million or up to two billion U.S. dollars, depending on the jurisdiction-specific definition (Farlex, 2012). Similarly, mid cap, and large cap funds are defined.

Funds can be further classified according to their management activity as active or passive funds. An important factor and reason for many investors to prefer so-called passive investment funds like ETFs, where often just an index like the S & P 500 ® (U.S.) is replicated and the assets are rarely rebalanced, over active ones, which are actively managed and the constituents regularly changed by fund managers, is the factor costs or fees (Meziani, 2016; SEC, 2016, pp. 24, 26, 41). The less one needs to trade the less the trading and management costs are – including costs for research, risk, strategy, execution, and post-trade documentation – and the better the ceterus paribus net performance, hence the performance after costs is (Barclay et al., 1998; SEC, 2016, p. 26). For proponents of Eugen Fama’s efficient market hypothesis (EMH), which states that markets are efficient in the longer-run and irrational exuberances and exaggerations occur only, at maximum, very short-timed and that it is not possible to outperform (“beat”) the market in the long-run, passive investments are the selection of choice as active fund managers statistically underperform against the (overall) market (Fama, 1970; Harvey & Liu, 2022). In the long “battle” between proponents of (more) active funds as Kosowski and of passive funds as, inter alia as seen, Fama and French, the researchers Harvey and Liu recently examined both of the methods used in an overarching study and confirmed older findings by showing that the evidence points, with some restrictions/softening of the original statement, toward the Fama-French conclusion (Fama & French, 2010; Harvey & Liu, 2022; Kosowski et al., 2006). Hence, only very few active fund managers are able (by their skill instead of luck) to outperform the market (Harvey & Liu, 2022).

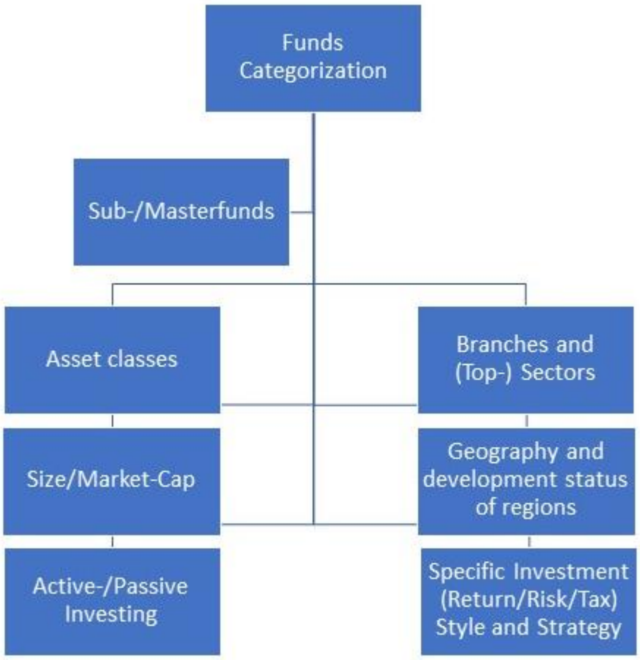

Figure 13 Breakdown and classification possibilities of funds.

Source: Own illustration.

Generally one differentiates between costs on the side of the fund or asset manager, e.g., for the administration, bureaucracy, and the deposit process[83], which are included in the total cost of ownership[84] figure (TCO) and (total) fund costs. More precisely these are called – because also including trading costs and performance fees, e.g., a performance share when outperforming over a certain threshold, denoted as a high-level-mark – the all-in-one fee (SEC 2016). In contrast, the total expense ratio (TER) is only the annual cost of holding an ETF or other fund (SEC, 2016). Finally, an investor has costs on the buyer's side like deposit costs at his depository bank, upfront payments, or emission fees, which cover the sales costs of the investment company. Empirical research finds that fund costs are one of the most essential factors when considering net performance (Barber & Odean, 2000; Barclay et al., 1998; Fama & French, 2010; Fischer & Gerhardt, 2007; Koestner et al., 2017).

[83] Costs for the depositor, custodian and possibly paying agent containing.
[84] In a comprehensible fashion explained in (Riedl, 2021).

Nowadays, funds and funds of funds are frequently labeled according to their investment style or precise (allocation) strategy like value or growth funds, smart beta or low vola funds, large-buyout funds, or event-driven funds (Barberis & Shleifer, 2003, pp. 161-199; Cerniglia & Fabozzi, 2018; Chincarini & Kim, 2022; Harkopf, 2016; SEC, 2016). [85] Value funds aim to find assets, mainly shares, which are underpriced in comparison to their perceived inner value – generally calculated by a discounted cash flow or equity method of their free or owner's cash flow – and therefore appear to be a chance for investment when considering that security margin (Graham, 1949). The security margin hence is the difference of the (traded) share price to the share's inner value or "fair price". Warren Buffet and Charlie Munger are probably the most famous proponents of that investment style, which is dating back to Graham and his book "The intelligent investor" (Graham, 1949). The approach aims to focus on solid companies in regard to their (low) debt ratios, with high and constant income, profits, revenue, and liquidity ratios, often being market leaders in their branches with high entry barriers and a conservative management with a proven track record (Frazzini et al., 2019; Greenwald, 2001). Research proved that strategy as being successful during the "normal", moderate phasis of a business cycle (Basu, 1977; Frazzini et al., 2019; Otuteye & Siddiquee, 2019). Growth funds tend to invest in companies with growth opportunities, often these companies are less established, more risky, and reside for instance in the tech sector. The style growth can be a successful strategy under certain circumstances as well (Asness et al., 2000). Smart beta funds select their assets with regard to the beta-factor of potential constituents, e.g., by requiring a low beta of less than 0.5 (Chincarini & Kim, 2022). The beta-factor roughly indicates how strong a certain single asset (stock) correlates with the overall market and how much price movements are amplified – multiplied by the beta-factor (Chincarini & Kim, 2022). Low volatility funds choose assets that have a ceterus paribus low volatility compared with the overall market volatility or in pre-defined absolute terms, while volatility is defined by financial markets here as the historical standard deviation (hence the square root of the variance) of (daily) returns of that asset over a certain period, e.g., the last five years[86] (Cerniglia & Fabozzi, 2018; Chincarini & Kim, 2022; Gao & Guo, 2018). Another type, large-buyout funds are e .g. funds utilizing

[85] See (Morningstar, 2004).

[86] In the financial sector a year normally consists by definition of 250 trading days (on average excluding weekends and bank holidays).

the buyout strategy in the area of private equity as described before while focusing on large companies, i.e., with a market capitalization of more than five hundred million dollars. Event-driven funds are funds that are set up with the aim of buying (or selling) companies or parts of their shares in expectation of a certain corporate action or credit-influencing event to appear soon, which impacts the share price – Jorion is discussing proper risk management for that style (Jorion, 2008).

It is common that multiple combinations of the mentioned types are taken into account and funds labeled according to it for instance in the case of so-called multi-strategy balanced funds (SEC, 2016). In either case, consistency and "sticking to the style" seems to be a critical issue (Brown et al., 2002).

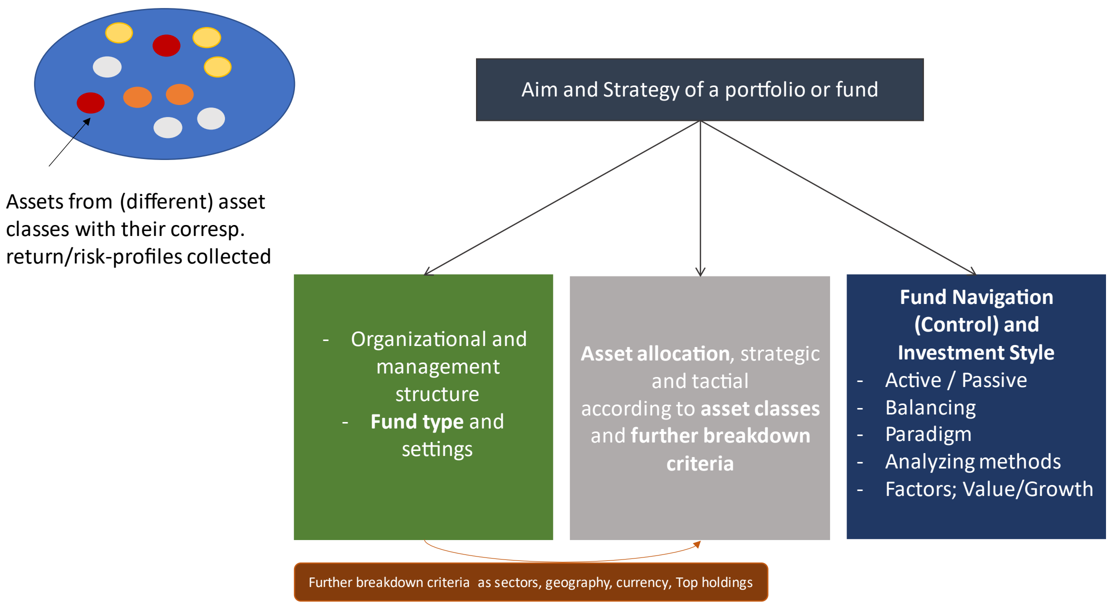

Figure 14 Aim and strategy of a portfolio or fund, influencing the fund type and settings chosen, the asset allocation based on asset classes and further breakdown criteria, and the fund navigation and investment style.

Source: Own illustration.

Depending on different investment aims and economic circumstances, as the state of the business cycle, most of the various portfolio types and investment strategies have their legitimacy and empirical foundations though some of them like "value-momentum"-

strategies or – a bit less clear – small cap funds seem to outperform[87] others more often (Asness 1997; Asness et al., 2013; Cerniglia & Fabozzi, 2018; Ehsani & Linnainmaa, 2022; Geczy & Samonov, 2017; Meyer & Mrozik, 2014; Switzer 2010).

Common investment mistakes and biases were investigated by Barber, Odean, and Campbell, more recently by Loos et al. from the University of Frankfurt for individual investors, and by Ahmad et al. for institutional investors (Ahmad et al., 2017; Barber & Odean, 2011; Campbell, 2006; Firth, 2015; Loos et al., 2014). Overall, the expected biases home bias, overconfidence bias, etc., the disposition and anchoring effect as well as the assumption of too less diversified portfolios are confirmed (Barber & Odean, 2000; Barber & Odean, 2001; Barber & Odean, 2011; Fischer & Gerhardt, 2007, pp. 10, 12, 13, 15; Koestner et al., 2017; Loos et al., 2014; Shefrin & Statman, 1985). Annaert et al. and Fischer together with Gerhardt have shown that financial advice and guidance based on research findings is able to significantly improve investment returns (Annaert et al., 2005; Fischer & Gerhardt, 2007).

The academic literature provides multiple well-known measurements of "performance"[88] and "risk", e.g., the α, β-factors for performance; volatility and maximum drawdown, i.e., the maximum amount an asset lost in its price during a certain period of time as a year, for risk or industry typical combined ratios like RARO(RA)C, Sharpe ratio, Calmar (Sterling) ratio, Sortino ratio, or information ratio as also defined by Leach or in the standard book "Investment Performance, Analytics, and Risk - Glossary of Terms" (Leach, 2010; J. P. Morgan, 2008; Sortino & Price, 1994). These ratios like the risk-adjusted return on risk-adjusted capital (RARORAC) put return and risk in relation, i.e., the return figure in the nominator and the risk figure in the denominator of the ratio (Leach, 2010; Schierenbeck et al., 2014). Risk-adjusted return means that a risk-free (market) rate like the federal funds rate or the rate of ten-year (assumed asymptotically risk-free) treasury bonds are subtracted from the measured asset return (sometimes also further costs) and the risk-adjusted capital, in this case, is the so-called Value-at-Risk[89] figure, defined later in this thesis (Schierenbeck et al., 2014). It roughly states the loss

[87] According to the definition of returns mentioned before with return at the point in time t as $r(t)= P(t)/P(t-1)-1$

[88] The term performance is used equivalently used to return, sometimes surplus return over a risk-free rate. Outperforming means having higher (net) return rates.

[89] Sometimes also denoted Value at Risk.

amount of money that was never exceeded with a certain probability (like 95 %) in a certain period of time in the past, like a year.

The Sharpe ratio uses the volatility (σ) instead of the Value-at-Risk in the denominator, the Calmar ratio employs the maximum drawdown (*MaxDD*), the Sortino ratio a similar downside risk (σ_d) also defined later in a detailed fashion and the information ratio is used by index trackers or passive ETFs, when the goal is to replicate an index, here the denominator is the so-called tracking error (*TE*) measuring the deviation of the tracker's returns from the original one (J. P. Morgan, 2008). The formula[90] for $RARORAC$ might be written as:

$$RARORAC = \frac{Risk - Adjusted\ Return\ (RAR)}{Risk - Adjusted\ Capital\ (RAC)} \tag{4}$$

Example:

$$RARORAC^1 = \frac{R_p - R_f + Fees - Expected\ Loss - Cost\ of\ Capital - Cost\ of\ Operations}{VaR} \tag{5}$$

with R_p denoting the portfolio return and R_f denoting the risk-free rate, hence the difference is the excess return (spread) of the portfolio over the risk-free rate (de Laurentis et al., 2010, p. 31). *VaR* is the Value-at-Risk, a common risk measure in finance, which is defined later.

By using the Value-at-Risk instead of the volatility σ in the RARORAC formula (occasionally also then defined as RARORAC as opposed to RAROC in case of just capital used in the nominator else or in the nominator without costs/fees) normality of returns is not needed anymore and hence Markowitz' theory of optimal portfolios can be also applied to bonds and loans with nonnormal credit return distributions and not only to equity as seen in Chapter 2 – often then denoted RARO(RA)C 2020 (J. P. Morgan, 1997, pp. 6, 12; Witzany, 2017, p. 119).

It would be also possible to use Jensen's alpha in the nominator, with

$$\alpha = R_p - \mathrm{E}[R_p] \tag{6}$$

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[90] A note on formulas formatting: According to APA7 and common standards, important formulas on a new line are formatted utilizing a formula editor and generally (apart from e. g. cumulative functions as the probability P, cdf F, expectation E, standard function (max, min, log, etc., except the e-function as also the number e exists) and apart from Greek letters) are written in italics. When formulas or variables or assumptions/extensions appear within a text and are in connection with a formula they are coherently written in that style, otherwise nonitalic.

and $E[R_p]$ denoting the expected portfolio “market” return of the capital asset pricing model (CAPM) as will be seen later. Therefore, Jensen’s alpha is denoting the individual “excess return” over the market return. For the denominator, one could imagine other risk measures, as the commonly used volatility σ or the ß-factor of the CAPM.

Then one can define return per risk ratios:

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma} \tag{7}$$

with volatility σ

$$Treynor\ ratio = \frac{R_p - R_f}{ß} \tag{8}$$

with CAPM market beta-factor ß

$$Calmar\ ratio^{91} = \frac{R_p - R_f}{MaxDD} \tag{9}$$

with maximum drawdown *MaxDD*

$$Sortino\ ratio = \frac{R_p - R_f}{\sigma_d} \tag{10}$$

with downside volatility σ_d

$$Information\ ratio = \frac{R_p - R_f}{TE} \tag{11}$$

with tracking error *TE*

Having defined assets, assets classes, investment in funds and portfolios and their characteristics, investment returns and empirical “success factors” as well as briefly mentioning return and risk ratios, the precise definition of risk measures – as this is a cornerstone of the thesis – followed by a theory of portfolios and optimal portfolios (in terms of return and risk measurement, hence applying the definitions to the portfolios) seems convenient.

To follow the historical timeline, the Markowitz model including portfolios labeled as effective portfolios, the capital asset pricing model (CAPM), and asset pricing theory (APT) are presented as models for portfolio management and optimal allocation in that order in the following chapter. For the historically interested reader on CAPM, the portfolio theory of Markowitz and the APT including valuation techniques one can

[91] It is important to note that, in contrast to the other measures and as opposed to the so-called Sterling ratio which is similar to the Calmar ratio but considers also a whole year, the Calmar ratio operates on monthly returns (Lee et al., 2010).

recommend the reader to the original works of Markowitz, Sharpe, and Fama, or for an alternative comprehensive introduction refer to the excellent "Handbook of Quantitative Finance and Risk management" by C. F. Lee, A. C. Lee, and J. Lee (Fama & MacBeth, 1973; Lee et al., 2010; Markwoitz, 1959; Sharpe, 1964). The handbook also covers further interesting principles of finance, which are not in the scope of this thesis such as equivalence principles, arbitrage-free-ness and the so-called two main (fundamental) theorems of asset pricing. These theorems define under which circumstances the price of an asset is possible to determine and under which in a unique way.

Roughly speaking under the condition of no-arbitrage it is generally possible to find a statistical measure (in that context a so-called martingale price process[92]) to determine the price of an asset, and in terms of an additionally complete market that price is unique (Björk, 2009, pp. 33, 36; Lee et al., 2010). Arbitrage occurs when the same asset is bought (and then sold) in different (parts of) markets to take advantage of a price difference in its quotes, it formally and more generally means having the chance of a profit with a probability greater than zero without having any risk – a risk-free chance of a profit (Bingham & Kiesel, 2004; Björk, 2009, pp. 7, 9, 33). In practice in a developed, liquid market one expects it to be rather arbitrage-free as if that would be not the case, agents known as arbitrageurs would directly take the chance to make a profit hence moving the price in the direction of the arbitrage-free price until it is reached (soon). A complete market is one in which every (possible) claim (i.e., every asset and its corresponding price movement) can be replicated in a certain portfolio and hence also be hedged (Björk, 2009, pp. 10, 18, 36; Lee et al., 2010). After the theorem was first proved by Harrison and Pliska for finite probability spaces, it was further extended by Kreps in the same year (1981) for more general settings (and infinite spaces) and by Delbaen and Schachermayer[93]– assuming a similar yet more strict concept as arbitrage, labeled "no free lunch with vanishing risk" – NFLVR (Harrison & Pliska, 1981; Kreps, 1981; Delbaen & Schachermayer, 1994; Delbaen & Schachermayer, 1998). The concepts of NA, NFLVR, and completeness and their mechanisms are regarded for pricing and therefore finally risk determination. Equivalence principles in finance just refer to identities and conditions under which two sorts and thus sides of present values (or future values) of discounted

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[92] For a precise definition see the annex as well as for hazard rate models and credit risk see (Bielecki & Rutkowski, 2004, pp. 166, 222).
[93] Even for unbounded processes when allowing for sigma-martingales instead of martingales, see (Schachermayer, 2008, p. 8).

cash flows are the same, e.g., premium payments of annuities and the value of a future insurance rent or cash flow payout. These principles can be found in connection with derivative pricing and for instance the calculation of CDS legs.

The current thesis, as it is focused on credit risk, is mainly concentrating on debt portfolios consisting of different bonds at a later stage. Ultimately, four different types of portfolios depending on the parameters geography (e.g., the US, euro area) and risk structure ("volatility risk") of obligators are selected. Obligors denote the companies[94] which have to pay back their debt and issued the bonds, they are also called debtors, debt portfolio companies, or constituents of the debt portfolio.

[94] The term company is used interchangeably with enterprise or corporation in the following chapters of the thesis. In the same way client, customer, and occasionally in the context of credit risk, derivative or transaction counterparty are used interchangeably.

CHAPTER 2

INVESTMENT AND CREDIT RISK MANAGEMENT: DEFINITIONS AND TERMINOLOGY

2.1 Risk, Types of Risk Appearing in the Financial Industry and Risk Management

In this chapter, the definitions and terminology for measuring credit risk which are commonly used within the area of financial (investment) risk management are presented as well as current portfolio theory (Markowitz-Theory, CAPM, and APT).

In finance, risk is defined as the "lack of (definite) predictability of outcomes" or "uncertainty of outcomes" (Gericke, 2018; Hull, 2015, pp. 20, 21; Zopounidis et al., 2021). Risk can have a two-sided component for upturn risk or success, often referred to as "chance" as well as for downturn risk or failure. In parts of the literature, the concentration is on the downside potential, and it is thus therein referred to that element as "risk". Generally, the fact is considered that losses can occur because of nondeterministic, value-impacting variables which are labeled risk factors (Hull, 2015, pp. 36, 62).

Risk is measured in terms of fluctuation of prices or returns[95], when considering again that the return at some discrete point t in time with prices $P(t)$ and $P(t\text{-}1)$, $t\text{-}1$ denoting one point in time before like the day before, is defined as

$$r(t) = \frac{P(t) - P(t-1)}{P(t-1)} = \frac{P(t)}{P(t-1)} - 1 \tag{12}$$

or sometimes

$$r(t) = \log\left(\frac{P(t)}{P(t-1)}\right) \tag{13}$$

and for continuous-time naturally as an infinitesimal (stochastic integral) limit thereof. The "fluctuation" or risk can be measured in ways of deviations of the returns of a certain mean, maximal drawdowns/losses or just considering the returns with the highest probabilities in some interval, i.e., the most frequent appearance in the past or in some simulation of possible future outcomes (Hull, 2015, p. 20; Zopounidis et al., 2021).

[95] More precisely the (often historic but - even more general - also stochastic present or future) price or return process.

All of these approaches are precisely defined in this chapter later. It is presented in which area risk is appearing in banking and which types of risks generally exist in the financial industry. In the area of financial risk, one can find different types or classifications of risk. The most common types of risks relevant to the banking sector are the following ones, cf. (Hull, 2015, pp. 62-63, 184, 430, 535, 557).

Credit risk or address risk, which is composed of the two components default risk (essentially the risk of "bankruptcy" for a company) and migration (creditworthiness) risk, is treated first (Bielecki & Rutkowski, 2004, p. 3; Hull, 2015, pp. 430-499). This type of risk is the central one in the thesis and describes the risk that an entity (counterparty, client) is not able or willing to fully pay back its obligations due on time and the contractually agreed conditions (Bouteillé & Coogan-Pushner, 2021, p. 3; Hull, 2015, pp. 430-515).

Default risk is defined in article 178 of the CRR (European Commission, 2019). A company defaults as one possibility when it either declares bankruptcy or is forced to declare it by the state or regulatory authorities, for example when a company seeks the protection under chapter eleven of the U.S. bankruptcy code (Bouteillé & Coogan-Pushner, 2021, p. 3). That is for instance normally the case when a company (will be soon in or) faces the situation that it cannot repay its debt because the amount of debt D exceeds its value of assets A, and the equity is hence negative, which is called insolvency. The second possibility is that a company breaches the days past due criterion for default identification, i.e., its payments are already due for (more than) ninety days and were not paid back yet (European Commission, 2019, §178). This case of inability or unwillingness to pay can be due to insolvency when the debt exceeds the remaining assets or due to (temporarily) illiquidity (Bouteillé & Coogan-Pushner, 2021, p. 3). In both cases, a company does not pay back its obligations on time. In practice, many debt issuances contain covenants with cross-default definitions, stating that once a company defaults on, e.g., one of its other bonds, the named issuance is declared defaulted as well (Bielecki & Rutkowski, 2004, pp. 10-11). The third possibility is required bank-specified indicators of unlikeliness to pay (UTP), which are further defined in the corresponding EBA guidelines, as the "Guidelines on the application of the definition of default under article 178 of Regulation (EU) No 575/2013", see (EBA, 2016; European Commission, 2019). Among UTP criteria can be, e.g., the default of a parent company or affiliates, a cross-default, an extreme internally induced business event (like fraud, as in the case of

Wirecard® in Germany 2020)[96], or an externally triggered one, e.g., all assets are nationalized by a socialist government or a natural disaster destroys the plants and equipment (EBA, 2016; EY, 2019, pp. 5, 18; Makri et al., 2014). Furthermore, the breach of certain credit covenants or the downgrade of a company to a (near) default grade by an accredited rating agency like Standard & Poor's ® are regarded as common UTP flags (EY, 2019, p. 18). The various unlikeliness to pay criteria form the third possibility of default (European Commission, 2019). The determinants of unlikeliness to pay and non-performing[97] loans in general, as well as the relation between the UTP (NPL) ratio and a bank's capitalization and risk management, were researched by Cucinelli et al., Makri et al., or Louzis et al. (Cucinelli et al., 2018; Louzis et al., 2011; Makri et al., 2014). As expected, macroeconomic factors such as high unemployment rates and debt ratios are positively correlated with UTP and NPL ratios, and GDP growth for instance negatively correlated (Louzis et al., 2011, pp. 12, 26; Makri et al., 2014). The main findings concerning banks were that there is a positive correlation between the UTP ratio and a bank's capital (yet procyclicality for NPLs) and a negative one between the UTP ratio and a bank's proper organizational structure and strict risk treatment, showing the importance of a well-set risk and dedicated UTP management unit for lowering UTP and NPL ratios (Cucinelli et al., 2018, pp. 27-29; ECB, 2017b; Louzis et al., 2011, pp. 26-27). On top of UTP criteria, the regulators require that also conditions for the return to a non-defaulted status and "a treatment of the definition of default in externally provided data" have to be defined by banks (EBA, 2016, preface, p. 1; European Commission, 2019; EY, 2019, pp. 5-6; Makri et al., 2014).

One has to keep in mind that besides default also cure can happen after a restructuring process, generally, company debt can either mature (and be fully paid back), a company can default before the maturity date or it can be restructured but cured (after or without a default) – therefore, a cure is a competing form to default (Tong et al. 2012; Wycinka, 2015).

The thesis by its nature concentrates on the economic triggers of default, as the criterion that the debt exceeds the assets available for a company, an extreme business cycle induced failure or other common exogenous factors (Bouteillé & Coogan-Pushner,

[96] Cf. as an example (FT, 2020).

[97] Denoted also, e.g., by the IMF, as nonperforming (American English version). For the scope of the thesis, as European regulation is the main focus, the EBA term non-performing (British English version) is used.

2021; Hull, 2015). The past due criterion is not modeled, as this is a purely formal criterion and a directly tracked indicator by a bank. A (formal) declaration of bankruptcy is also not further treated, as it is normally the consequence of the modeled economic factors before. Further possibilities as personal fraud, jurisdictional issues or other operational risk-related events – as defined later in this chapter – are not a part of original credit risk. Neither are rare operational risk events like natural catastrophes, even though they all might be even already included in past historical default data when such events happened in the past and were subsequently leading to defaults. The rating agency case is included in the considerations directly when using ratings for a credit risk model, or indirectly, when the ratings are a for instance a consequence of negative equity, as a model output by a credit risk model (de Laurentis et al., 2010; Witzany, 2017). Hence, the approach used in the thesis seems appropriate and comprehensive.

Having defined the default risk component one ought to consider credit migration risk, the latter one being the risk that the creditworthiness of a debtor is changing (without directly causing default), e.g., in case a downgrade or negative outlook by a rating agency occurs due to worsening financial fundamentals or negative news, concerning a certain company and its ability to repay its debt in the same manner and under the same circumstances as assumed before (Bielecki & Rutkowski, 2004, pp. 28, 407; Witzany, 2017, pp. 88-93).

As a special case credit risk is referred to as counterparty credit risk (CCR) for the default possibility of a derivatives counterparty and the migration risk, in this case, is incorporated in the term credit (debt) value adjustment, in short form the CVA (DVA)[98] – risk (Bouteillé & Coogan-Pushner, 2021, p. 284; Hull, 2015, pp. 480-490; Witzany, 2017, pp. 9-10, 217-220, 231-237; Zopounidis et al., 2021). If one considers a quoted bond of a company or a defaultable[99] interest rate claim or curve the difference between the risk-free rate (of a treasury bond) and a defaultable bond yield curve, e.g., a corporate yield curve, in a perfect liquid and rational market without market risk is then called credit spread (Hull, 2015). The risk associated with that is hence denoted as credit spread risk and includes default and migration risks (Bielecki & Rutkowski, 2004, p. 264; Giesecke & Goldberg, 2005). It is important to note that even for same-rated bonds the credit spreads can differentiate (Giesecke & Goldberg, 2005). This is due to the fact that rates

[98] CVA: Credit value adjustment, DVA: Debt value adjustment
[99] A defaultable claim is a claim that bears credit risk and can therefore default (and credit migrate).

are more precise than rating grades (the former are “finer grided”, more granular) and can therefore differentiate between, e.g., an “average” BBB+ grade and a “better” (i.e., within the highest 5 % percentile of BBB+s, but still not BB-) BBB+ grade (Hull, 2015; Witzany, 2017). An even more granular differentiation in default or migration risk and its changes can hence be included in credit spread rates. Furthermore, when credit risk relevant information is published or publicly available[100] generally credit spreads (as well as credit default swaps) react fast, whereas ratings are published just once in a while and are therefore lagging in time, while also constituting an “average through the cycle” view (Witzany, 2017, pp. 93-94, 167). This market observation (“stylized fact”) is illustrated in more detail at the end of Chapter 3.

It is crucial to keep in mind that the interest rate of a loan should include not only the credit spread or risk premium component[101] above the risk-free rate but also the internal costs of funding of a bank (“own credit/funding spread”), the administrative costs of the bank, the cost of embedded options if existing and finally a profit margin as the bank normally intends to make a profit with the loan or bond (de Laurentis et al., 2010, pp. 29-30; Witzany, 2017, p. 93).[102] One can swiftly derive, in terms of simplified assumptions and normalized *EAD* unit, the following correspondence between the credit risk premium *RP* and the probability of default *PD* of a loan by assuming no-arbitrage or equivalently that the premium payments compensate exactly for potential default with probability *PD* (Witzany, 2017, p. 94):

$$RP = \frac{EL}{1 - PD} \tag{14}$$

As

$$\begin{aligned}(1 + r) &= (1 + r + RP)(1 - PD) + PD(1 - LGD) \\ &= 1 + r + RP - PD(1 + r + RP - 1 + LGD)\end{aligned} \tag{15}$$

because with no-arbitrage one should receive the same amount when investing in a risk-free bond of unit one and return $1 + r$ (the left-hand side of the equation) as when investing in a risky bond of unit 1 with return ($1 + r + RP$) after one period, hence the

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[100] The use of private information is excluded here as it is forbidden by law to trade with such information, referenced to as insider trading.

[101] That is including at least the expected loss – due to default and migration, incorporated in the standard costs of credit concept.

[102] Though sometimes, especially in highly competitive markets, loans are cross-financed and used as “door-opener” for a future lucrative client relationship and are not (all of them) making profit regarded isolated on their own (Bouteillé & Coogan-Pushner, 2021; Witzany, 2017, pp. 93-94).

risk is compensated through the risk premium *RP*. As the bond is however risky it has a probability that it is defaulting, the *PD*. If it is not defaulting with probability (1 - *PD*), the return (1 + *r* + *RP*) is earned, if it is defaulting with probability *PD* then just the part not included in the loss given default (which equals the recovery rate, 1 - *LGD*) is earned.

These two options are constituting the right-hand side of the equation and overall, after re-ordering, the equation above is obtained (Witzany, 2017, p. 94). The formula then further simplifies to

$$RP = PD \cdot RP + r \cdot PD + PD \cdot LGD \tag{16}$$

and with unit *EAD* the expected loss is defined as

$$EAD \cdot PD \cdot LGD = PD \cdot LG = EL \tag{17}$$

hence

$$RP - RP \cdot PD = r \cdot PD + EL \tag{18}$$

so generally

$$RP = \frac{EL + r \cdot PD}{1 - PD} \tag{19}$$

If *LGD* = 1 (complete 100% loss) is assumed, then

$$RP = \frac{(1 + r)\,PD}{1 - PD} \tag{20}$$

illustrating the risk premium in terms of a product of the risk-free rate and the (*PD*) odds. Sticking to an arbitrary *LGD* and the general formula above, however, regarding the special case of a risk-free rate *r* of zero, the formula

$$RP = \frac{EL}{1 - PD} \tag{21}$$

is finally derived. The remaining excess (or liquidity) spread risk is also commonly considered a further component and is due to illiquidity and sentiment like anxiousness in the market (Bielecki & Rutkowski, 2004; Downing & Covitz, 2007; Witzany, 2017, p. 236). It indicates the further risk premium and hence interest rate add-on one needs to pay compared to the same (or a very similar) bond with the same default and migration probability in a more liquid and timid market time (Bielecki & Rutkowski, 2004, p. 7; Bouteillé & Coogan-Pushner, 2021; Downing & Covitz, 2007).

A type of risk that is sometimes classified as a class in its own right, yet possibly appears for all types of risks under certain unfavorable circumstances, is concentration risk. However, especially in the area of credit risk credit, concentration risk ought to be observed very closely (Ávila et al., 2012; BIS, 2006; Bouteillé & Coogan-Pushner, 2021, pp. 37, 251; Kozak, 2015). It has to be reported in the large exposure (LE) and million loans reporting of banks in the EU and surveilled on a daily basis (European Commission, 2019b). The concentration in regard to the loan portfolio of a bank may appear in terms of concentration among very few counterparties and hence large exposures, in terms of geographical concentration, sector-wise or country-wise concentration, (foreign) currency concentration, or maturity concentration, which might cause further problems for the asset-liability-management (Bouteillé & Coogan-Pushner, 2021). Moreover, empirically, the contagion risk in times of crisis generally increases with high concentration risk in a portfolio (Ávila et al., 2012; Badreddine, 2016; BIS, 2006; Brunnermeier, 2008; Schiavone, 2018).

As most banks and lenders rely on the task of providing credit (loans) as their primary business objective and loans constitute a major share of their assets, the credit risk components form the overwhelming part of the risks the institutes face (EBA, 2021b, p. 46). A thorough credit risk assessment of a credit client is hence crucial in determining if an institute believes in a debtor's ability to serve the credit – the name credit stemming from the Latin word "credere", which means to trust or to believe (in the ability to repay the debt), indicates that already (Bouteillé & Coogan-Pushner, 2021, pp. 47, 81, 107; Park & Greenberg, 2017).

Credit risk can appear, when a loan is given to another entity or equivalently a bond issued by another entity is bought (Hull, 2015). This illustrates the main use case for banks and is therefore mainly covered in this thesis. Credit risk can also appear in leasing (or renting) contracts when the lessee cannot pay the leasing fees. Another important example is receivables, which might be not paid back by the trading partner who already received a good or service upfront, e.g., in trade finance (Bouteillé & Coogan-Pushner, 2021, p. 7). A similar case is prepayments on goods and services, which inherit the risk that the other side of the transaction does not deliver or not on time – or vice versa when goods are delivered first, the counterparty does not pay (Bouteillé & Coogan-Pushner, 2021; Hull, 2015). Generally, whenever there is a time gap between the delivery and the payment of goods and services credit risk arises. This is nearly always the case when no guarantor (or insurance, etc.) or central counterparty, with time-adjusted collateral, exist in a transaction (Witzany, 2017, p. 217). Credit risk also appears in the area of project finance, as there might be a substantial time delay between the construction

phases and payments, and hence severe credit risk can arise, as illustrated in (Joseph, 2013).
Another example is deposits or depository goods. There is always a risk that the deposited goods or money ("deposits") are gone or not fully covered in case of a default of the counterparty (Bouteillé & Coogan-Pushner, 2021; IADI, 2020). For instance, bank deposits in Europe are generally only covered up to 100 000 EUR and in the US by the FDIC[103] only up to 250 000 $ by law. The last category involves (contingent) claims, guarantees or derivative contracts, in which the counterparty is not able (or willing) to fulfill its obligations and does not pay back the owned money (or assets) fully on time regarding the underlying contract (Bouteillé & Coogan-Pushner, 2021, p. 7).

To completely understand the real economic credit risk of a client, a bank needs to take its obliged "know your customer" process (KYC) and its due diligence very serious, as often a counterparty consists of borrower units and connected clients, complicated holding structures or shell companies with "hidden" beneficial owners or special purpose vehicles (SPVs) in case of derivative transactions (Bouteillé & Coogan-Pushner, 2021).

Furthermore, banks need to build loan reserves (provisions) for expected losses and adjust them regularly, also on-time provisions and write-offs (at a period's end) for already (partly) defaulted exposures and hence incurred losses (generally on a single base and in rare cases of homogeneous, larger portfolios on an equivalent portfolio base) – normally the IFRS method is used in that regard by international banks (Witzany, 2017, p. 6). Assets are in the latter case accounted for as impaired assets (Witzany, 2017). An early and prudent provision is crucial concerning credit risk (Burroni et al. 2009; European Commission, 2019). According to Witzany, it is "one of the most important items scrutinized by external auditors during an annual review" (Witzany, 2017, p. 100). A comprehensive overview of the requirements of the Basel accords for credit risk management is given in the corresponding original BCBS papers or by Berg, Burroni, Hull, or Witzany, all relevant parts for the thesis are introduced later on (BCBS, 1999a; BCBS, 2000b; BCBS, 2022; Berg, 2019; Berg, 2019b; Burroni et al., 2009; Hull, 2015; Witzany, 2017, pp. 11-16).

[103] Federal Deposit Insurance Company, the U.S. deposit insurance agency.

The possible approaches for credit risk in (Pillar 1) regulatory terms are, as mentioned, the standardized approach (SA) as well as the internal ratings-based approach (IRBA) – where banks determine the PD (and also exposure and loss at / given default in the case of the advanced IRBA) by an internal model (Witzany, 2017, pp. 108-115). Credit portfolio models are then used to determine the credit risk amount for Pillar 2 and internal measurement and controlling purposes as will be shown later.

While credit risk constitutes the most important risk category for the vast majority of banks, with an average of 82.8 % of RWAs as of Q4/2021 and a stable average of roughly +80 % over the years, there are also further types of risks banks are exposed to (EBA, 2021b, p. 46).

Market Risk describes the risk which is connected with (downward) market price movements (FED, 2022; Gericke, 2018; Hull, 2015, pp. 184 - 341). From a supervisory point of view, it "stems from all the positions included in banks' trading book[sic] as well as from commodity and foreign exchange risk positions in the whole balance sheet" (Bundesbank, 2022c; EBA, 2022e, p. 1). Position risk especially includes the categories of share (price) risk and debt security (price) risk in market trading, which is visible in the prices of stocks respectively bonds (Auer, 2002, pp. 97-110; Zopounidis et al., 2021). They can be due to changed expected financial fundamentals of a company, overall market or business cycle changes prompting worsening profit outlooks, due to the market liquidity situation, or changed market sentiment like increased anxiousness (Fama & French, 1993; Fama & French, 2014; Hull, 2015; Szylar, 2013).

Generally, that risk, therefore, illustrates a change in the economic outlook and in the expected prospects of a company. Along with that price risk for (mainly) bonds and stocks, known as position risk as seen, an important risk type is interest rates risk (Milanova, 2010, p. 396). Interest rate risk in that regard describes the risk of a (sudden) change in interest rates (Milanova, 2010, pp. 396-398). Interest rate risk mainly determines the price of a bond, however, it is also influenced by other factors constituting the position risk as for instance market expectations about future interest rates and monetary policy as well as liquidity concerns (Bai et al., 2019; Hull, 2015). It can be further specified, as the interest rate curve might be divided into a risk-free base rate as the fed funds rate[104], which is a common or general interest rate risk, and the risk premium

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[104] Or normally the rate of a certain treasury bond of the US or as another common possibility short-term swap rates like the overnight-indexed (average) swap (OIS) as in (Hull, 2015, pp. 215-217).

rate that is due to common market or sector factors as well as individual funding characteristics and spreads - it implicates the banking and trading book of a bank, even in potentially different manners (Hull, 2015, pp. 212-234; Tapiero, 2004; Zopounidis et al., 2021). To separate it from trading book risks, regulators coined the term Interest Rates Risk in the Banking Book (IRRBB), when referring to the risk of interest-sensitive assets held in the (longer-term) banking book of an institute (BIS, 2019c).

While price risk and accompanied spread and volatility risk appear for all assets traded in financial markets there are certain further risks associated solely with specific assets (Gericke, 2018).

When considering foreign exchange (FX, Forex), interest rates and currencies among different geographic regions and countries, a (foreign) currency risk appears as an additional risk component (Hull, 2015). Regulators require its consideration for all on- and off-balance-sheet items of a bank, thus including the trading and banking book (EBA, 2022e). Commodity risk, e.g., due to certain production or mining issues, due to storage, availability of facilities, reserves, and transportation as well as global demand, is a further asset-specific risk type which ought to be considered in the realm of market risk (EBA, 2022e; Hull, 2015).[105] Real estate risk, which is connected with the determining factors of the overall housing market situation, socio-demographic developments as well as the individual location, construction issues or for example vacancy rates, is another important category, especially in the recent years and essential for mortgage lenders or real estate funds (Haight & Singer, 2004). Some other market-related risks, which were often overlooked or treated not precisely enough by banks in the past, are option risk, base (rate) risk, and hedging risk in the area of financial derivatives, funding risk for a bank itself or conduit (or SPV[106]) risk in relation to securitizations (Hull, 2015).

Market prices are often very volatile and have normally a high frequency of available quotes with, e.g., daily or intraday prices (Hull, 2015, pp. 185, 248; Milanova, 2010, pp. 399, 405, 410). Sometimes even "real-time quotes" published by stock exchanges are accessible – hence the prices and returns can be observed and backtested rather conveniently (Hull, 2015; Milanova, 2010; Scandizzo, 2016). Internal approaches,

[105] Often again accompanied by political risk when the commodities and natural resources are exploited in or imported from politically unstable countries. Commodity risk has to be considered again for the trading as well as the banking book (EBA, 2022e).
[106] Special purpose vehicle, a legal vehicle set up for certain securitizations, e.g., for asset-backed securities (ABS).

the so-called internal models approach (IMA) for market risk as well as the internal models method (IMM) especially for derivative CCR/CVA models, are allowed to be used to determine the RWA – as well as (again) regulatory standardized approaches like the SA - FRTB[107] or the so-called simplified standard FRTB for RWA calculation (Hull, 2015; Witzany, 2017).

The average RWAs in terms of market risk are roughly 4 % over the last years in the European Union (3.6 % as of Q4/2021), considerably reduced since the great financial crisis (EBA, 2021b, p. 46). In many cases, however, especially for investment banks and trading-oriented banks, market risk naturally also forms a rather important part of the overall risk assessment and required risk capital (Gericke, 2018).

Operational risk, the third most important type, describes risks appearing in the (daily) operation of a bank and the execution of its business (Gericke, 2018; Hull, 2015, pp. 535-556). It contains risks evolving from human misconduct, human errors like a "fat-finger" in trading, management mistakes, or even fraud (Bundesbank, 2022d; OCC, 2019). Secondly, it includes risks arising from natural disasters, pandemics such as the SARS-2-pandemia in the early 2000s or the COVID-19-pandemia in 2020 or climate change, which is generally spoken an environmental risk (Kaiser, 2021; Ryder, 2022). The third part stems from systems like the IT infrastructure a bank has in use or other technologies applied and forms an increasingly important risk an institute faces (Gericke, 2018; Ryder, 2022). As mentioned, regulators abolished the possibility to utilize the internal AMA[108] and with the finalization of Basel III and the introduction of the CRR III in Europe solely the standardized measurement approach (SMA) remains left (European Commission, 2019).

Generally, the risk areas and three topics human, environment / exogenous factors, and systems are hence sub-categorized in operational risk scenarios. However, especially reputational issues such as unethical behavior, fraud, or unsustainable business models, but also cyber risks are evolving as ever more important risk factors banks are dealing with (Gericke, 2018). As a consequence, the related divisions should receive more risk-weighted capital and further human resources to control them. As in Q4/2021 operational risk amounted to an average of 10 % of a bank's RWAs (EBA, 2021b, p. 46).

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[107] FRTB: Fundamental review of the trading book, denoting the market risk framework of the finalized Basel III accords.
[108] Advanced measurement approach

Liquidity risk[109], the fourth important category, is the risk of becoming illiquid and being not able to cover a certain outflow (draw) of assets during times of financial stress (Brunnermeier, 2008; Gericke 2018; Hull, 2015, pp. 557-575; Zopounidis et al., 2021). Examples are large cash withdrawals by bank customers in times of crisis ("bank runs"), the "freezing" of the interbank market or securities which cannot be sold anymore due to market illiquidity, see e. g. (Brunnermeier, 2008; Takemura, 2012). For measurement purposes, there is Liquidity-at-Risk (LaR) on one hand, which includes a certain 30-days-stress imposed measure with deposit withdrawal scenarios and countermeasures through the use of bank-owned high-quality liquid assets (HQLAs), as in the LCR regulations of the CRR (European Commission, 2019; Hull, 2015, pp. 557-575; Zopounidis et al., 2021). Liquidity Value-at-Risk (LVaR) on one hand, has to be differentiated from LaR on the other, with LVaR denoting the "normal" Value-at-Risk measure extended by "settlement costs", hence adopted for liquidity purposes but without hypothetical stress scenarios (Hull, 2015, pp. 563-565).

Besides regulatory reporting like LCR/NSFR[110], ALMM reporting and stress testing in the area of liquidity risk, a (precise) daily surveillance of internal limits is essential for liquidity risk controlling and the treasury department of a bank.

An even more granular classification of risks might be employed in some cases of banks, adding, e.g., political risk, regulatory risk, business (strategy) risk, participation (ownership) risk, or concentration and contagion risk to a banks risk inventory (Bouteillé & Coogan-Pushner, 2021, p. 4; Zopounidis et al., 2021). All of these risks have to be precisely described in the risk handbook and framework of a bank, some are however hard to quantify like political risk or regulatory risk and are in practice mainly dealt with by certain internal monitoring processes, proper training of the staff, lobbying and participation in respective bank associations' sessions (Hull, 2015; OCC, 2019).

[109] The terms liquidity and liquidity risk are used in four different settings and senses, and therefore one has to differentiate between them thoroughly (Hull, 2015). The first one is referring to the absence of short-term illiquidity, i.e., the institute is able to cover cash outflows (and drawbacks) in a stress situation with its own highly liquid assets. A fundamental regulatory indicator for that scenario is the liquidity coverage ratio (LCR). The second one is the ability of an institute to fund itself middle- to long-term, also maturity transformation wise, and that its assets and the corresponding required stable funding are matched by the right available liabilities. The corresponding regulatory indicator is called the net stable funding ratio (NSFR). The third one refers to the fungibility and tradability of assets – considering the time it takes to sell them in a potential illiquid market. The fourth one then refers to the amount of money (like M1) which is in the market, referring to the liquidity controlled and brought to the markets through central banks and subsequently commercial banks in a fractional reserve banking system (Hull, 2015, pp. 570, 572).

[110] Again LCR denotes the liquidity coverage ratio, NSFR is the net stable funding ratio and ALMM the additional liquidity monitoring metrics (Hull, 2015, pp. 572-574).

In the area of operational risk, one might especially add and consider conduct risk, reputational risk, legal, tax and litigation risk, cyber/ IT risk, and model risk itself as subcategories (Buraschi & Corielle, 2005; Hull, 2015, pp. 587- 605).

Figure 15 The main risk types appearing in banks as illustrated in any standard textbook, for example (Hull, 2015): Credit risk, market risk, operational risk, and liquidity risk.

Source: Own illustration.

For the described types of risks in a bank, all sorts can further appear in terms of concentration risks, when aggregated in an undiversified fashion.

The classification of types of risk also depends on the business strategy, therefore, linking business and risk strategy, reporting, and documentation closely together – as also required by regulatory laws, in combination with the "risk appetite" and overall risk controlling philosophy of a financial institute (Bouteillé & Coogan-Pushner, 2021, pp. 21-29; Martens & Rittenberg, 2020, pp. 1-3). The risk appetite, similar as described in ISO31000, is written down in a risk appetite framework (RAF) and ought to be documented thoroughly (Gericke, 2018; Hull, 2015; Martens & Rittenberg, 2020). The risk tolerance thereby is the maximum bearable amount of risk (losses).

Risk is dealt with in several divisions and levels of an institute. The organization structure is hence a crucial component for proper risk management, with the risk management department normally residing close to the finance department (i.e., accounting, reporting, treasury, and business controlling) of a bank, yet clearly separated from the business and investment banking division (Bouteillé & Coogan-Pushner, 2021, p. 32; Witzany, 2017, pp. 4-6). The chief risk officer (CRO) as "top risk officer", meanwhile an industry standard, is sitting in the management board of the bank (Witzany, 2017, pp. 4-6).

Generally, to defend against threats from risks, banks apply a 3-level-system, labeled as three “lines of defense” (Bouteillé & Coogan-Pushner, 2021, p. 22; Gericke, 2018). The first line of defense is the market division, which has the first look at upcoming risks and the duty to report and restrict them. The second line of defense is the risk control unit, often further divided, e.g., containing a credit risk control unit (CRCU) with the sole purpose of credit risk controlling (Bouteillé & Coogan-Pushner, 2021; de Laurentis et al., 2010). Quantitative analysts commonly labeled as “quants” and (separated) model assembling and validation experts are part of that line of defense. It is essential that the employees in the CRC unit possess the required skills, techniques, quantitative education, and tools to assess credit risk in a proper way and at the same time, it is crucial that they operate in an independent and unbiased manner (Witzany, 2017, pp. 4-5). Often the CRO as risk controlling head also directly reports to the shareholders’ supervisory board (Witzany, 2017, p. 5).

As the third line of defense the internal auditors sometimes denoted internal revision in Europe, are installed, who back-check the adequacy of the control measures in place on a regular and ad-hoc-event case base (Gericke, 2018). The risk control and especially the internal audit unit have to be separated and independent, can require access to (nearly) all documents, and report directly to the C-level of a company (Witzany, 2017, p. 4). Hence, risk management has to be a key task for senior management and lived by “tone from the top” (Bouteillé & Coogan-Pushner, 2021, p. 33; Gericke, 2018).

In the area of regulatory affairs and the Basel accords (finalized Basel III as of 12/2018), there is a clear differentiation between the strictly regulatory, CRR-defined, and rather formal Pillar 1 risks and the more qualitative, CRD encrypted, and often nationally preconized Pillar 2 risks (European Commission, 2019; Witzany, 2017, pp. 11-16). Whereas the core risks credit, market, and operational risk have to be quantified in both Pillars, thus including Pillar 1, the interest rate risk in the banking book (IRRBB) is a prominent representative of a pure Pillar 2 risk, utilizing quantification methods like net interest income (NII) for income and P & L purposes and economic value of equity (EVE) for economic valuations (European Commission, 2019; Hull, 2015, pp. 212-234).

Sometimes one disassembles a risk further into a systematic part belonging to the country risk, to a certain industry sector, or just “the market” and a pure individual, idiosyncratic risk component (Gregoriou, 2006, pp. 107-131, Witzany, 2017). Eventually, when dealing with different countries or currency areas, one should supplement the transfer risk (Aldasoro & Ehlers, 2017). That means the risk of transferring an asset from a foreign currency and country to a local one, especially when there are capital (flow) controls or tariffs on capital as will be seen again later (Aldasoro & Ehlers, 2017; Hull, 2015).

An increasingly more important risk is model risk, i.e., the risk of using models as simplified versions of reality and a certain type of model in its own right (Abasto & Kust, 2014; Behm et al., 2013; Breinich-Schilly, 2021; Buraschi & Corielle, 2005, p. 2884; Glasserman & Xu, 2014; Hull & Suo, 2002; Hull, 2015, pp. 587-605; Limas et al., 2015; Rösch & Scheule, 2010). Due to a growing inventory of (internal) risk models, the "mathematization" of risk, and the rise of artificial-intelligence-based methods the impact of this risk type might be enormous for banks (Breinich-Schilly, 2021; Hull, 2015). Henceforth a strict model risk policy and model risk framework have to be implemented (Farkas et al., 2021; Reinwald, 2022b). Farkas et al. even proved that capital requirements which are adjusted for model risk are as conservative as the finalized Basel III ones, but less volatile and more efficient (Farkas et al., 2021).

Investment risk is a sub-category of banking risk[111], while the former contains the risk of investing, i.e., credit risk, market risk, and possibly further liquidity risk, the latter subsidies all sorts of risks which can occur in the banking or financial sector, also including operational risks (like cyber risk) in banks (Hull, 2015).

The standard definition of investment risk also does not include risks that are indirectly involved when investing into assets, like settlement and clearing risks or depositary risks, etc., these are consequently summarized as opRisks (operational risks) here as well (Hull, 2015). The thesis operates within the class of investment risk, more specifically credit risk.

To deal with the phenomenon of risk it generally has to be managed, hence the following is defined for the purpose of the thesis.

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[111] Sometimes banking risk (or financial entity risk) that includes all risks described before which appear in the banking business, especially credit risk, market, and operational risk, is then divided into (hence differently defined) financial risk comprising of credit and market risk and nonfinancial risk which means operational risk. Furthermore, sometimes banking risk and investment risk are differentiated in the sense that investment risk can only appear in the process of investing in an asset, hence, e.g., credit risk related to a bank loan is regarded as banking risk and financial risk, but not as investment risk. In the thesis, investment risk is used in a comprehensive sense and hence interchangeably with financial risk.

Risk Management is the management and controlling of the resources and commitments of a company in such a manner "as to maximize its value, taking into account the impact that unpredictable outcomes or events can have on firm performance" or existence (Gericke 2018; Hull, 2015; Bansal et al., 1991, p. 1).

The guidelines by supervisory authorities as the EBA, FED, and the NCAs, the laws and experts stress the importance of risk management as a continuing process, as integrated in the overall bank controlling and management, which has to be very well documented and reported (EBA, 2018; Witzany, 2017, pp. 4-5, 11-16). This is usually implemented in the form of regular risk reports, risk dashboards with key risk indicators, known as KRIs, accompanied by early warning signals and triggers, risk-bearing capacity calculations and defining the risk tolerance of the institute (IOR, 2010; SDW, 2022).

The risk culture has to be lived by the top management as well as every single individual in the financial firm – "every employee is a risk-aware employee" (Alexander & Sheedy, 2004; Witzany, 2017). To foster that, the bank needs to implement concrete measures, e.g., setting up whistle-blowing processes to create a means to report suspicious or unethical behavior without the fear of reporting employees of being punished and making sure that the remuneration and bonus system of the bank is risk-adapted (Alexander & Sheedy, 2004; Angeli & Gitay, 2015; Gericke, 2018).[112][113] The graph below summarizes the risk universe and its sub-sets, as in (EBA, 2018; Hürlimann, 2018).

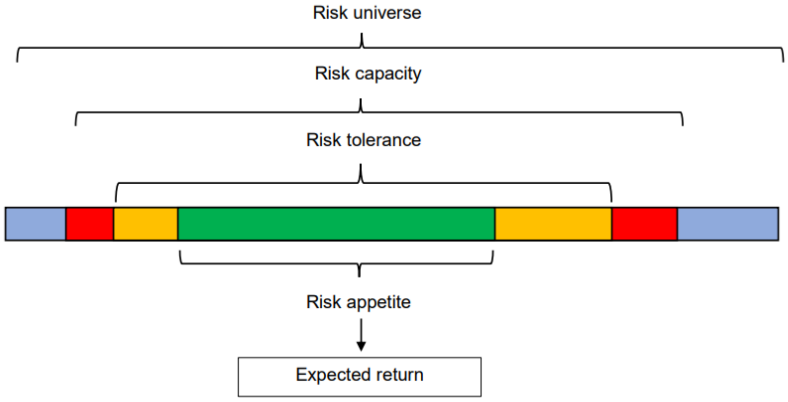

Figure 16 Risk universe and sub-sets.

Source: According to (Hürlimann, 2018).

[112] For more details on regulatory requirements concerning remuneration, see on (EBA, 2022b).
[113] Explained further in "The Professional Risk Manager's Handbook" from the editors C. Alexander and E. Sheedy, published by The Professional Risk Managers' International Association ® in 2004.

Whereas risk (bearing) capacity (RBC) means the maximal capital available to absorb losses, the risk tolerance is a limit below that number as it should be avoided that distributions like dividend payouts are forbidden by regulators (when the MDA threshold is breached), extreme losses occur and have to be reported or regulatory fines have to be paid (EBA, 2018; Hull, 2015). The tolerance hence can be regarded as the maximal limit under minimal standards or practical considerations. Within that business tolerance limit a bank now decides how "offensive" or "defensive" it is taking on risks, time-adjusted also to economic conditions and opportunities – this is the mentioned risk appetite and the corresponding framework RAF (Hull, 2015; Martens & Rittenberg, 2020, pp. 1-3; Witzany, 2017).

As indicated before, an overarching risk management framework and governance system including the RAF and risk strategy as well as the entire management process and step-by-step risk management handbook have to be available for the respective employees in the risk controlling department in a bank (Bouteillé & Coogan-Pushner, 2021, pp. 21-33; Martens & Rittenberg, 2020).

The entire risk management process is considered as the risk inventory, analysis, general valuation (including quantification) and implementation, monitoring processes, control and reporting requirements of all material risks, and mitigating solutions. Risk management hence sequentially involves the following steps, after setting up the risk strategy, as in (Alexander & Sheedy, 2004; BCBS, 2000b; Bouteillé & Coogan-Pushner, 2021; Gericke 2018; Hull, 2015; Witzany, 2017):

(1) An exhaustive identification and classification of the risks that can occur (also known as risk inventory), including also rare or at first nonmeasurable/ nonmodelable risks (NMRs[114]).

(2) A thorough analysis of risks, followed by precise measurement and valuation of the risks in terms of likelihood of their occurrence and the impact or magnitude of losses arising from the risks. Furthermore, the correlations and interconnections of the risks ought to be considered, and finally one compares the quantified risks with the available (economic) risk capital or risk-bearing capacity as well as the risk tolerance.

[114] Sometimes also denoted as risks not in VaR (RNIV).

(3) A permanent controlling and reporting, which includes formal documentation of the risks, has to be ensured.

(4) An implementation of the actions required to control business risks within certain acceptable bounds has to take place.

Conclusions and potential improvements derived from (4) are then again implemented in a lessons-learned process for adapting the strategy and also the steps (1)-(3) in form of a “Do-Plan-Act”-Circle.

Figure 17 Risk management process cycle.

Source: Own illustration, similar as in standard risk management textbooks, e.g., (Hull, 2015).

Furthermore, risk mitigation is commonly following step 4. The actions can include avoiding risk, insuring risk, mitigating risk, transferring/selling/securitizing risk, dividing it, and controlling for it (Bouteillé & Coogan-Pushner, 2021, p. 289).

In recent years, especially following the great financial crisis, institutes also have to test their risk management framework by applying suitable stress tests (Farmer et al., 2022). Some of them are obligatory and pre-defined by the supervisory authorities like the EBA stress test in the European Union or the comprehensive capital analysis and review (CCAR) and Dodd-Frank act stress test (DFAST) in the United States – the latter ones introduced in 2011 respectively 2013 (EBA, 2022c; FED, 2021). Often stress tests make use of various base and adverse scenarios, e.g., historical ones like the great financial crisis (GFC) in 2009 or hypothetical ones like “a sudden 50 % drop in housing prices” or they are simulation-based like Monte Carlo simulations - also hybrid forms exist (Bellini, 2016; EBA, 2018c; Farmer et al., 2022). Overall, risk management is a (circle) process, which has to be challenged constantly.

It is important to stress that credit risk management is roughly consisting of four steps. Credit origination, then (single obligor) credit assessment and analysis, portfolio valuation and management, and finally risk transfer and mitigation (BCBS, 2000b; Bouteillé & Coogan-Pushner, 2021; Witzany, 2017). In the circle above – illustrating the process as most actors in finance regard it – these elements are contained, only the credit (or risk-bearing item in general) is “already there” and one hence starts with the inventory, followed roughly by the other three steps. However, a precise risk manager should already regard the respective risk management process at an earlier stage – at the origination or even before, at the negotiation point of a deal (Witzany, 2017, p. 6). Consequently, in many cases a very bad deal in terms of a risk management viewpoint might be practically uncontrollable or an asset unsellable later and if a “bad risk” can be avoided at the beginning it should be – and the prospective deal then neglected or re-arranged (Witzany, 2017, p. 6). From that perspective, some risk managers even regard the origination process as the most crucial step in risk management (Bouteillé & Coogan-Pushner, 2021, p. 21). Risk managers should always accompany this origination process closely and in a detailed fashion or at least – in the case of much less risky or standardized origination contracts – provide checklist-style approvement schemes and limits to the size of originations as well as “black-lists” for certain countries, branches, or clients, which appear enormously unfavorable in risk terms (Bouteillé & Coogan-Pushner, 2021, p. 34; Witzany, 2017, p. 7).

General credit portfolio risk management guidelines and advice might be gained from the International Association of Credit Portfolio Managers® (IACPM®), a leading industry association in the world (IACPM, 2005; IACPM, 2017). The institute also offers regular surveys among its members and extracts working objectives and changing priorities (trends) thereof, i.e., toward an active balance sheet management including further return enhancements or wholistic approaches within a bank, in the community (IACPM, 2017). Another valuable source for professional risk managers, in general, is the (membership) organization Global Association of Risk Management Professionals® (GARP®) with chapters in every major country.[115]

The steps of credit risk analysis, valuation, and portfolio considerations are dealt with in detail in the thesis. Risk mitigation and transfer is not the scope of the thesis, the main possibilities in this area are, as mentioned, credit sales (i.e., transferring the risk by selling it), insurance techniques (either private single loan credit risk insurance – PCRI – or with the means of credit default swaps, CDS), indirect insurance techniques (e.g., by so-called credit linked notes, CLN, or insurance-linked-notes, ILN), or portfolio swaps synthetic securitization for risk mitigation, SRT, as shown in (IACPM, 2022).

In addition to a general risk management process, various specific handbooks and concepts, in case of necessity also sub-frameworks for the individual risks and their proper management, have to be derived, e.g., for market risk and credit risk (Bouteillé & Coogan-Pushner, 2021; Witzany, 2017; Zopounidis et al., 2021). Furthermore, real up-to-date risk reports have to be submitted at least on a monthly basis to the management of an institute, with daily surveillance and weekly meetings within the smaller risk controlling groups and units (Gericke, 2018). For instance, market risks and large exposure (credit) risks have to be calculated and monitored daily as obliged by the law (European Commission, 2019b; Zopounidis et al., 2021). A quarterly report has to be submitted to the regulators as well as – often presented by the CRO himself – to the risk committee in the supervisory board[116] of the bank.

Finally, for certain risks and their corresponding risk models a validation framework, concept, and validation handbook have to exist. Therein the adequacy of the model including its methodological foundation, assumptions, data input, calculations, and output are challenged (Reinwald, 2022b).

At least yearly, as well as "on special occurrences", like when substantial model changes happened, restructurings or a crisis occurred, a validation process has to take place (Satchell & Christodoulakis; 2008; Tasche, 2006). The results are written in the validation result report (Reinwald, 2022b). Also, the models themselves have to be put into an inventory of models and have to be categorized and valued ("scored") by their importance as well as their potential risk impact and shortcomings, i.e., the severity of

[115] See http://www.garp.org (Retrieved Mai 17, 2022).
[116] In the US – where a single management body system is standard – the board of directors with the supervisory and oversight committee, in most European states as Germany the supervisory board ("Aufsichtsrat") and Switzerland the administration board ("Verwaltungsrat") fulfill that responsibility.

risks (Glasserman & Xu, 2014; Satchell & Christodoulakis, 2008; Scandizzo, 2016). This is usually done with the help of scorecards and heatmaps. Abasto and Kust introduced a promising method to measure incremental model changes by means of weighting Monte Carlo paths under constrained optimization to receive lower and upper bounds (Abasto & Kust, 2014). Relying on categorization and valuation, a conservative capital charge, the model risk buffer (MRB), is finally calculated and assigned to the model (Reinwald, 2022b; Rösch & Scheule, 2010; Scandizzo, 2016).

The legal requirements for risk reporting were set out by the Basel Committee on Banking Supervision in the document "Basel III: A global regulatory framework for more resilient banks and banking systems - revised version June 2011", for risk data management in BCBS 239 in 2013, in BCBS d328 in 2015, furthermore in december 2017 in the finalization of Basel III, and followed by the European authorities as for instance in CRD IV article 74, 76 ff. which had to be nationally implemented (BIS, 2011; BIS, 2013; BIS, 2015; BIS, 2017; European Commission, 2019b; Witzany, 2017, pp. 11-16). For instance, in Germany, the national CRD implementation was done in the "normeninterpretierende Verwaltungsvorschrift MaRisk", based on the modification of the national article 25a. KWG (BaFin, 2021). The latest version, the 7^{th} revision of it, appeared in the fall of 2021 and includes current developments concerning IT risks and outsourcing as well as the handling of critical, "non-performing" loans, further also risks stemming from an environmental, social, or governance induced context known as ESG risks (BaFin, 2021; Hannemann et al., 2022).

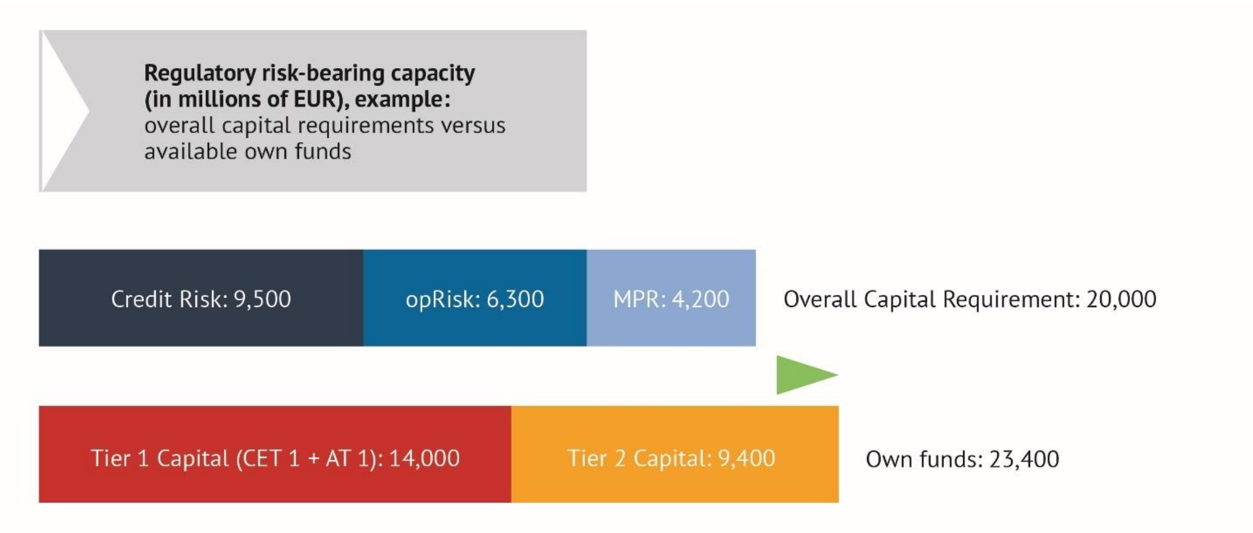

Figure 18 Example of (regulatory) risk capital requirements vs. the risk-bearing capacity of a bank.

Source: Own fictive example and illustration.

As stated in these acts, a bank has to be able to clearly (dis-)aggregate the quantified risks, e.g., measured as VaR, throughout the single risk factors and types, processes and departments (BaFin, 2021; EBA, 2018; Hull, 2015, pp. 36, 618-622). This requirement is including correlations between risks in the institute (BaFin, 2021). Often this is done by means of a “large matrix summation”, denoted as variance-covariance matrix summation, of the different risk types (Hull, 2015, pp. 618-622; Li et al., 2015).

Such a (hybrid) summation is shown, e.g., by Rosenberg and Schuermann who also describe typical shapes of market risk, credit risk and operational risk distributions as normal or *t*-student ones, Gamma ones and Generalized Pareto ones (Rosenberg & Schuermann, 2004).

More sophisticated methods such as copulas, introduced later in the thesis, are discussed for instance by Li et al. (Li et al., 2015; Witzany, 2017, pp. 188-202). The result of these calculations is a total aggregated risk and corresponding VaR exposure of an institute (Hull, 2015, pp. 618-622).[117]

Funds as well as (risk) capital – the one really used, as well as the one which is planned to be available in the future under certain conditions – can be allocated and assigned to certain functions and departments of a company (Figueiredo, 2021). That

[117] See in Germany (BaFin, 2021).

method is commonly known as (risk) capital aggregation or funds transfer pricing (FTP) and can be seen as a kind of "banking within a bank" (Figueiredo, 2021).

Furthermore, an escalating and risk-corresponding limit system has to be implemented, constantly monitored, and updated regularly, as well as occasionally, when induced by "major changes" (Bouteillé & Coogan-Pushner, 2021; Hull, 2015). Literally every new product, business deal and client have to be equipped with a certain limit, and every individual involved in such a deal has to be aware of it (Hannemann et al., 2022). Especially also a global risk limit for the entire institute has to be set (Hannemann et al., 2022). It is common to include the Basel "traffic light" approaches with "red" (which means out of limit, usually from ten limit breaches onwards for a 99 % quantile), "yellow" (which means still in the limit, five onwards), and "green" (meaning the limit is rarely used or not used at all, e.g., three) color codes for reasons of feasibility (BCBS, 1996; Guericke, 2018).

2.2 Risk Measures

In order to measure risk, one needs to define, in a subjective or objective way, the range of possible outcomes (events) and the assigned probabilities, i.e., through a probability distribution, that these outcomes will occur (Albrecht, 2004; Van Deventer et al., 2013). For scientific value, one needs to consider objective or principally at least strong intersubjective measures.

A risk measure for the purpose of this thesis is a statistical measure allowing to describe the uncertainty of an event in a quantitative way (Albrecht, 2004; Bielecki & Rutkowski, 2004; Björk, 2009; Van Deventer et al., 2013; Hull, 2015). It is an operation or map which assigns a certain value to a risk. Such a measure is intuitively necessary to quantify and "measure" risk and calculate risk amounts. To proceed, therefore, a few terms are defined mathematically on the following pages.

A σ-algebra on a set *S* is a collection *A* of subsets of *S*, where

1. $S \in A$ (*A* includes *S* itself) (22)
2. $X \in A \Rightarrow X^c \in A$ (*A* is closed under complementation) (23)
3. $X_1, X_2 \ldots \in A \Rightarrow \cup_{i=1}^{\infty} X_i \in A$ (*A* is closed under countable unions) (24)

as for instance defined in (Bingham and Kiesel, 2004; Björk, 2009, p. 461). The definition implies directly (via 1. and 2., respectively 2. and 3.) that the empty set is an element of A

$$\emptyset \in A \tag{25}$$

and that A is closed under countable intersections Hence given a set S, A is a collection of subsets of this set, which includes the whole set itself, the empty set, the complementary sets and is closed under countable unions. It will be utilized later, e.g., for (risk) events.

The pair (S, A) is called a measurable space or Borel space (Bielecki & Rutkowski, 2004, pp. 33-34; Björk, 2009, p. 461; McNeil et al., 2015). An example is the real numbers with all open intervals of the real numbers as A. The measurable space is hence the mathematical foundation to really measure the probability of a certain risk event.

Given now a sample space S and an associated sigma-algebra A, hence a Borel space (S, A).

Then a probability function (or probability distribution or distribution measure) is a function P on a Borel space with domain A – therefore P maps from the basis A, P: $A \rightarrow \mathbb{R}$, $\mathbb{R}$ is the field of real numbers – such that (Björk, 2009, p. 484):

1. $\mathrm{P}(X) \geq 0, \forall X \in A$ (i.e., P is positive definite) (26)

2. $\mathrm{P}(S) = 1$ (i.e., the measure of the whole sample space is one) (27)

3. If $X_1, X_2 \ldots \in A$ are pairwise disjoint, then $\mathrm{P}(\cup_{i=1}^{\infty} X_i) = \sum_{i=1}^{\infty} \mathrm{P}(X_i)$ (28)

Similar standard descriptions are found throughout the literature, e.g., in (McNeil et al., 2015). Number three says that the probability of pairwise disjoint (i.e., nonintersecting) events can be simply added up. These properties make sense in the practical world. The probability of an event is always nonnegative as an event can (will) happen with a certain chance or probability (then $\mathrm{P}(X) > 0$) or it can never happen (then $\mathrm{P}(X) = 0$). But it cannot be less than “not happen (ever, at all)”, hence never have a probability less than zero assigned. Furthermore, a probability can never be more than 100 %, hence one (1), when the event happens for sure (“no matter what”). If one has a whole (complete) sample the probability of the whole sample is naturally one (see the definition of (S, A)). The last property can be heuristically motivated as an (exclusive) “or”-event, connected then with “+”, where the probabilities are hence just summed up.

These three properties are known as axioms of probability or the Kolmogorov axioms. Any function P that is satisfying the axioms of probability is called a probability (measure) function (Bingham & Kiesel, 2004). For instance, a function that maps for five different balls the probability of one-fifth (1/5) for drawing one of them evenly on each ball fulfills all the axioms and is hence a probability function. One can directly derive the following properties from the rather intuitive definitions.

If P is a probability function, X is any set in A, then (where $\emptyset$ is the empty set):

I. $\mathrm{P}(\emptyset) = 0$ (The probability of an empty set is zero) (29)

II. $\mathrm{P}(X) \leq 1$ (A probability is always less or equal than one, i.e., 100 %) (30)

III. $\mathrm{P}(X^c) = 1 - \mathrm{P}(X)$ (31)

(The probability of the complementary set is one minus the probability of the set)

as shown in (Bingham & Kiesel, 2004; Björk, 2009, pp. 485-486; Chen, 2018, p. 3). This can be also seen when recalling that by definition:

$$S = S \cup \emptyset \, , \, S^c = \emptyset \, . \tag{32}$$

$$\mathrm{P}(\emptyset) = 0 \tag{33}$$

is clear, when the third point is established and by writing

$$\mathrm{P}(S^c) = \mathrm{P}(\emptyset) = 1 - \mathrm{P}(S) = 1 - 1 = 0 \tag{34}$$

where

$$\mathrm{P}(S) = 1 \tag{35}$$

comes from 2. further above. The same is true for the second property

$$\mathrm{P}(X) \leq 1 \tag{36}$$

When the third point is established then

$$\mathrm{P}(X) = \mathrm{P}((X^c)^c) = 1 - \mathrm{P}(X^c) \tag{37}$$

and as by 1. further above

$$\mathrm{P}(X^c) \geq 0 \tag{38}$$

it follows

$$\mathrm{P}(X) \leq 1. \tag{39}$$

Therefore, only the third point has to be proven. Yet, point three directly follows from 3. further above as X and the complement X^c do not intersect (are independent) by their very definition as complementary sets, further

$$S = X \cup X^c \tag{40}$$

by definition and by

$$P(S) = 1 \tag{41}$$

from 2. above. As a result,

$$1 = P(S) = P(X \cup X^c) = P(X^c) + P(X) \tag{42}$$

thus

$$P(X^c) = 1 - P(X) \tag{43}$$

The last point means that the probability of a complementary ("opposite") event is one minus the probability of the event. However, one needs to be precise when regarding complementary events. As an example, the complementary event of "The color of the ball I have drawn is black" is not "The color of the ball I have drawn is white" but logically just the negation of the first event, hence "The color of the ball I have drawn is not black" so "The color of the ball I have drawn is any color except black".

If P denotes again a probability function, furthermore X_1 and X_2 are any sets in A, then one can from the definition of a probability measure above also directly deduce (an alternative way to the own one below is shown in Chen, 2018, p. 3):

- $\mathrm{P}(X_1 \cap {X_2}^c) = \mathrm{P}(X_1) - \mathrm{P}(X_1 \cap X_2)$ (44)
- $\mathrm{P}(X_1 \cup X_2) = \mathrm{P}(X_1) + \mathrm{P}(X_2) - \mathrm{P}(X_1 \cap X_2)$ (45)
- If $X_1 \subset X_2$, then $\mathrm{P}(X_1) \leq \mathrm{P}(X_2)$ (46)

The first equation can be rewritten as:

$$\mathrm{P}(X_1 \cap {X_2}^c) + \mathrm{P}(X_1 \cap X_2) = \mathrm{P}(X_1) \tag{47}$$

As

$$(X_1 \cap {X_2}^c) \cup (X_1 \cap X_2) = X_1 \tag{48}$$

by definition of the complementary set of X_2, and as ($X_1 \cap X_2$) and ($X_1 \cap {X_2}^C$) are disjoint by definition of the complementary set, together with 3. further above

$$\mathrm{P}(X_1 \cap {X_2}^c) + \mathrm{P}(X_1 \cap X_2) = \mathrm{P}(X_1) \tag{49}$$

regrouping shows the first point. The third point can be proved as follows. As $X_1 \subset X_2$ one can write

$$Y \cup X_1 = X_2 \tag{50}$$

for some set Y, being disjoint from X by construction. Then with 3. further above one gets

$$\mathrm{P}(X_1 \cup Y) = \mathrm{P}(X_1) + \mathrm{P}(Y) = \mathrm{P}(X_2) \tag{51}$$

and as by 1. further above

$$\mathrm{P}(Y) \geq 0 \tag{52}$$

it yields

$$\mathrm{P}(X_1) \leq \mathrm{P}(X_2). \tag{53}$$

From

$$X_1 \cup X_2 = (X_1 \cap {X_2}^c) \cup ({X_1}^c \cap X_2) \cup (X_1 \cap X_2), \tag{54}$$

which are disjoint by definition follows with 3. further above directly

$$\begin{aligned} \mathrm{P}(X_1 \cup X_2) &= \mathrm{P}(X_1 \cap {X_2}^c) + \mathrm{P}({X_1}^c \cap X_2) + \mathrm{P}(X_1 \cap X_2) \\ &= \mathrm{P}(X_1) - \mathrm{P}(X_1 \cap X_2) + \mathrm{P}(X_2) - \mathrm{P}(X_1 \cap X_2) \\ &+ \mathrm{P}(X_1 \cap X_2) \end{aligned} \tag{55}$$

With the help of point 2. and hence by regrouping and summing up one gets:

$$\mathrm{P}(X_1 \cup X_2) = \mathrm{P}(X_1) + \mathrm{P}(X_2) - \mathrm{P}(X_1 \cap X_2) \tag{56}$$

One can also derive the following result in table 1 from measurement theory and counting, showing the “number of possible arrangements of size r from n objects” (Chen, 2018, p. 6; Witte & Witte, 2010).

Table 1 Arrangements of objects with and without replacement in an ordered vs. unordered fashion. Thereby *r* and *n* are denoting natural numbers and *r* ≤ *n*.

	Without Replacement	**With replacement**
Ordered	$\frac{n!}{(n-r)!}$	n^r
Unordered	$\binom{n}{r}$	$\binom{n+r-1}{r}$

Source: Own illustration in line with (Chen, 2018, p. 6).

From the definition above, it is further defined (Bingham & Kiesel, 2004; Björk, 2009, p. 484):

A probability space is a triple (Ω, *A*, P) where Ω is a set of samples (sample space, which is the set of all possible outcomes of a given experiment), *A* is a σ-algebra (a set of events, where each event is a set containing zero or more outcomes as defined above) and P is a probability measure function (mapping a probability to each event, as also defined before).

For risk management purposes and theory, probability spaces are fundamental objects as for certain risk events the corresponding probabilities and exposures are mapped onto them utilizing probability spaces (Bingham & Kiesel, 2004).

In the annex the probability-based concepts of a filtration, random variables as variables that assign values to events of a domain space with a certain probability, i.e., "randomly", (in)dependence of random variables, random processes as a collection of random variables indexed by a set like the natural or real numbers, adopted processes and martingales – colloquially a special stochastic process with a conditional expectation of the process at point *t* with knowledge of time *t-1* being equal to the process and random variable at *t-1*, hence zero expected increment – are formally defined. For the purpose of the thesis an intuitive understanding and the descriptions given in the text are sufficient. In finance the existence of the mentioned special (martingale) measure in a probability space, denoted as a risk-neutral probability measure, with expectation zero is equivalent to no-arbitrage opportunities as already seen before in the first fundamental theorem of asset pricing. To return to the central definition of a risk measure one can formally define:

A risk measure is a mapping

$$\rho\colon M\ (\Omega, \mathbb{R}) \ \rightarrow \ \mathbb{R}\,, X \ \mapsto \rho(X) \tag{57}$$

where M (Ω, $\mathbb{R}$) ⊂ $M_{\mathscr{B}}$ (Ω, $\mathbb{R}$) is a ρ-dependent sub-vector-space. Hence, a real number (measure) is assigned to the random variables in the sub-space by the mapping (Björk, 2009; McNeil et al., 2015). This concept gives the possibility to measure and quantify risks now.

Many risk measures include a parameter α ∈]0,1[with which, accompanied by the specific measure, the desired "security level" is defined (Hull, 2015). This important parameter is hence referred to as confidence level (Hull, 2015). A mathematical concretization of the "security level" is then defined by three elements, a certain risk measure, a confidence level like 0.95 hence 95 % and a certain time horizon like a year, to which the (gains or) losses then correspond (Bouteillé & Coogan-Pushner, 2021, p. 239; Hull, 2015, pp. 294-298; McNeil et al., 2015).

The literature mainly differentiates between so-called moment-based risk measures (MBR), which are grounded in the statistical moments like the mean, variance or higher moments then Value-at-Risk measures, alternative tail or conditional Value-at-Risk measures, which are also sometimes called expected shortfall measures, and finally the spectral measures (Adam et al., 2008; Albrecht, 2004; Bouteillé & Coogan-Pushner, 2021, pp. 78-80; Hull, 2015, p. 298; McNeil et al., 2015; Raskin, 2006). All of which will be defined in more detail in this chapter.

In statistics, a cumulative distribution function (or a cumulative probability distribution, often referred to as cdf or cpd) of a real-valued random variable X is the function

$$\mathrm{F}_X\,(x) \;=\; \mathrm{P}\,(X \;\leq\; x) \tag{58}$$

where P ($X \leq x$) is the probability that the random variable X takes on a value less than or equal to x (Bingham & Kiesel, 2004; Björk, 2009, pp. 484-485; McNeil et al., 2015). In the discrete case, this is just the summation up to that part. If there exists – it is often not the case – a function f_X such that

$$\mathrm{F}_X\,(x) = \int_{-\infty}^{x} f_X(t)\;dt \tag{59}$$

then f_X is called the (probability) density function of F_X (Björk, 2009). The integration (or summation in the discrete case) of the density up to x hence leads to the cumulative probability distribution. It follows directly from the definition that every cumulative distribution function F_X is right-continuous and non-decreasing, which makes it a so-called càdlàg[118] function (Bielecki & Rutkowski, 2004; Björk, 2009; Bingham & Kiesel, 2004).

In statistics, especially in descriptive statistics, apart from the cdf of a random variable X one is interested in parameters like the mean, median, or variance of a distribution, generally in statistical moments, hence a general n-th moment is defined where n can be any natural number.

[118] continue à droite, limite à gauche

If F is a cumulative probability distribution function of a probability distribution P then the n-th moment of the probability distribution is given by (where just the n-th power of x is taken):

$$\mu_n = \mathrm{E}[x^n] = \int_{-\infty}^{\infty} x^n d\mathrm{F}(x) \tag{60}$$

E[X], hence the first moment, is also the called expectation value of the random variable X. μ is called the mean (Björk, 2009, p. 485; McNeil et al., 2015). For a cumulative probability function with a density function f as defined above, the equation then simplifies to

$$\mu_n = \mathrm{E}[x^n] = \int_{-\infty}^{\infty} x^n f(x) dx \tag{61}$$

Substituting X with X - E[X] one can derive for the second moment

$$\sigma(X) = \sqrt{\mathrm{E}[(X - E(X))^2)]} = \sqrt{\mathrm{var}(X)} \tag{62}$$

which is called standard deviation, in many situations in finance referred to as volatility, and where var(X) := E[(X − E(X))^2] is called the variance (Björk, 2009, p. 485; Hull, 2015, pp. 240-248; McNeil et al., 2015). This formula gives the opportunity to explicitly calculate the mean (expectation value) and volatility of a random variable or realized random process and hence to quantify risk in a certain way. To summarize, moments-based risk measures are the standard deviation (or vola) indirectly, the variance (second moment), sometimes skewness and kurtosis (with moment n equals three and four), or higher moment measures (Hull, 2015).

For different random variables X and Y, one calls then

$$\mathrm{E}[(X - \mathrm{E}(X))(Y - \mathrm{E}(Y))] = \mathrm{cov}(X, Y) \tag{63}$$

the covariance of X and Y, which generalizes the variance concept (Hull, 2015, pp. 272-274; McNeil et al., 2015).

While volatility – a kind of deviation from some expected value as the mean – is an often-used concept for measuring risk, it should never be the sole one. Other concepts like the maximum drawdown within a period and further measures based on VaR-like capital ratios, performance and risk ratios, RWA and leverage ratios – or even factors like net in-/outflows, active share, or the Kelly factor in case of investing portfolios should be regarded (Van Deventer et al., 2013; Kelly Jr., 1956). The Kelly factor is stemming from game theory and the idea behind it is searching for the optimal proportion of the investment money to invest in each step (Kelly Jr., 1956). Active share describes what proportion of a portfolio is managed in a real active way as opposed to which part is mainly tracking indices (benchmarks) and is hence considered passive investing (Cremers, 2017). The concept of net in-/outflows is self-describing and measures which amount of capital is flowing in and out of assets (or normally a whole portfolio respectively fund) considering different time frames, investor groups, and relations. The last three figures are mainly used within portfolio and fund management, whereas volatility, maximum drawdown and VaR-like measures are generally applied in the financial industry (Albrecht, 2004; PMI, 2017).

The concept of applying multiple (in some cases partly complementary) measures was already described in the risk management process before, where a strong focus was

1. on a set of various indicators (volatility, maximum drawdown, risk/return ratios) and metrics collected in a dashboard,[119] as well as
2. on (dis-)aggregable VaRs/TVaRs throughout the institutes' departments and processes

Furthermore, especially the occurrence of extremely rare unforeseen events, in the financial industry known as "black-swans", unhappy foreseen circumstances hence events labeled as "white-swans" or "grey-rhinos" or changing correlations in stress situations are essential (Hull, 2015; Taleb, 2008). Hence, the VaR measure and related concepts will be broadly discussed in the following parts, stress tests are again mentioned later in the thesis.

One property of the standard deviation is that positive deviations have the same impact as negative deviations. Especially for risk management purposes, however, one often prefers to concentrate on the negative side, the "loss-side" (Hull, 2015). One solution is to just consider these losses which exceed the expectation value, and one subsequently derives

$$\sigma^{+} = \sqrt{\mathrm{E}[\max(0, X - \mathrm{E}(X))^{2}]} \tag{64}$$

[119] The internal capital adequacy assessment process ICAAP with its core of calculating and comparing the risk-bearing capacity with the overall Value-at-Risk, hence potential loss capital needed, can be similarly executed with liquidity (which is just "short-term available/required capital") and its liquidity risks. Hence, keeping the interconnectedness between capital and liquidity in mind the combination is named an ICLAAP process (capital and liquidity). In some contexts, the extensions of the risk metrics are including performance parameters (among risk, capital, liquidity, and return) and structural or regulatory parameters which are then sometimes denoted as IMAAP (or ICLMAAP) with "M" for management.

(Albrecht, 2004).

The "danger" of crossing a certain threshold is generally measured by the Shortfall measures (McNeil et al., 2015; Sortino & Price, 1994; Sortino & Satchell, 2001).

For losses one can then derive the upper partial moments for a threshold a (McNeil et al., 2015):

- $UPM(h, a)(X) = \sqrt{\mathrm{E}[\max(0, X - a)^h]}\, , h > 0$ (65)
- $UPM(h, a)(X) = \mathrm{P}(X \geq a), h = 0$ (66)

Another risk measure sometimes used in the financial industry is the maximum drawdown (MaxDD), which is the already mentioned maximum loss having occurred during a pre-defined period, like a 250-days MaxDD.

Nevertheless, most of these measures are not really suitable for finance, and apart from the standard deviation (and skew, kurtosis) the moments-based measures are not intuitive and imaginable (Albrecht, 2004, pp. 11, 15, 17; Artzner et al., 1999; BIS, 2009). The maximum drawdown[120] lacks certain mathematical properties and is also extreme as it solely concentrates on the very maximum, and the standard deviation is less useful for unsymmetric risks and gradual phases of price declines (Hull, 2015).

To eliminate these properties the next risk measure is defined, which is mathematically feasible and of the highest practical relevance, the Value-at-Risk (VaR) measure, which first appeared in the banking industry with the introduction of Till Guldiman in the 1980s (Afzal & Nawaz, 2011, p. 7475; Bouteillé & Coogan-Pushner, 2021, p. 65; Hull, 2015, pp. 294-298; Holton, 2002; Jorion, 2007). Main ideas however date back much longer, for example to Leavens et al. (Leavens, 1945).

To have the ability to measure also "correlated assets" within the VaR concept correlation needs to be defined more precisely. The Pearson's correlation (coefficient) between X and Y is defined as:

[120] Chekhlov et al. generalized the concept of maximum drawdown (MaxDD or MDD) and introduced the – path-dependent – conditional drawdown (CDD), containing a whole family of drawdown risk measures (Chekhlov et al., 2005; Möller, 2021, p. 3).
The maximum drawdown (MDD) and the average drawdown (ADD), an arithmetic mean of occurred drawdowns, are special cases thereof. A special kind of that family, the so-called conditional expected drawdown (CED), has been introduced in (Goldberg & Mahmoud, 2017; Möller, 2021, p. 3). However, they are only useful in few practical (and theoretical) applications and hence not further considered here (Afzal & Nawaz, 2011).

$$\rho_{X,Y} = \frac{cov\,(X,Y)}{\sigma(x)\,\sigma(y)} \tag{67}$$

with volatility function σ(x) sometimes briefly denoted σ_x (Hull, 2015, p. 270; McNeil et al., 2015). In some contexts like linear regression, $R^2 := \rho_{X,Y}{}^2$ is called the determination coefficient. There is a direct mathematical relationship to bilinear, symmetric, positive semi-definite inner products and regression analysis, one can derive similar principles and definitions for multi-dimensional random variables; furthermore also more general goodness-of-fit measures similar to R^2 (Hull, 2015; McNeil et al., 2015).

The Value-at-Risk $VaR_{\alpha,T}(X)$ is defined as:

$$VaR_{\alpha,T}\,(X) := \inf\{x \in \mathbb{R}: \mathrm{F}_X(x) \geq \alpha\}, \qquad T \in \mathbb{R} \tag{68}$$

where F_X denotes the cumulative distribution function of X, T the time horizon and inf the standard infimum function, i.e., the minimum function in the case of a closed set (Hull, 2015, p. 294; Holton, 2002; McNeil et al., 2015). Considering an illustration of the Value-at-Risk makes sense due to its practical value and its far-reaching applications:

If $VaR_{0.05,\,365}(X) = 100$, then in just 5% of all the cases the loss within a year (365 days) exceeds 100 Euro. Equivalently formulated: The loss in 95% of all cases within one year does not exceed 100 Euro. The Value-at-Risk measure is the most important risk measure in finance and for that reason, it will be also used to quantify portfolio losses in the following chapters (Hull, 2015; pp. 294-295; McNeill et al., 2015).

If one considers stressed values in a (hypothetical) stress scenario or a stressed distribution then the corresponding Value-at-Risk in that regard is labeled Stressed Value-at-Risk – SVaR (Alexander & Baptista, 2017; Dupacová & Polívka, 2005; EBA, 2012).[121] A risk measure closely related to VaR is the tail Value-at-Risk measure (*TailVaR*, *TVaR*). The tail Value-at-Risk or conditional Value-at-Risk (*CVaR*) or expected shortfall (*ES*) is defined as, see (Holton, 2002; Hull, 2015, p. 298):

$$ES_\alpha = TailVaR_\alpha(X) = \mathrm{E}(X|X > VaR_\alpha(X)) \tag{69}$$

It defines the expected loss, under the condition that the worst (1 - α) × 100 % of the cases occur, hence that the α × 100 % threshold is already crossed. Looking at the definition, one can see that the expected shortfall measure is also suitable for rare events and heavy-tailed distribution, where a large mass of the distribution lies in its outer tails (Hull, 2015, pp. 298-300). It is therefore used for stress testing and sometimes the

[121] See the source (Dupacová & Polívka, 2005) for stressing the VaR and CVaR measures.

calculation of extremal values in extreme value theory (de Haan & Ferreira, 2006; Farmer et al., 2022). Especially after the great financial crisis ES-based methods were introduced in the Basel III regulatory package, e.g., the FRTB-concept[122] in the area of market risk measurement. The stressed extension exists for the CVaR as well, denoted as SCVaR (Alexander & Baptista, 2017; Dupacová & Polívka, 2005).

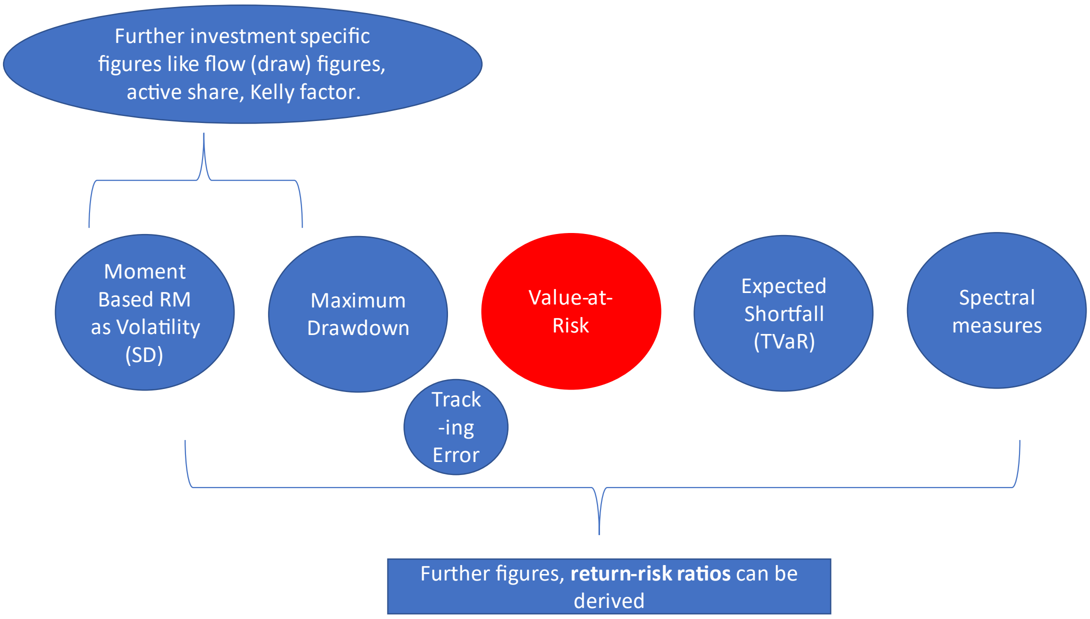

Figure 19 Risk measures with the central VaR.

Source: Own illustration.

An interesting extension of Value-at-Risk and expected shortfall is risk measurement by an expectile[123] τ, which overcomes the shortcomings of Value-at-Risk (not coherent) and of expected shortfall (not/hardly elicitable for effective backtesting) and can be regarded as a generalization of the mean in the same way quantiles generalize the median (Chen, 2018, pp. 11-18). Expectiles are hence a symbiosis of *VaR* and *ES* and can be calculated incorporating them both as

$$ES_{\alpha} = VaR_{\alpha}(1 + \frac{\tau}{\alpha\,(1 - 2\tau)}) \tag{70}$$

[122] Fundamental review of the trading book, as mentioned before.
[123] A synthetic word, combined from the words expectation and percentile.

That is shown in (Chen, 2018, pp. 11-18).

Similarly, other “weighted combinations” of *VaR* and *ES* were investigated (Happersberger et al., 2019). While expectiles possess great theoretical (and potentially regulatory) appeal, they are however hardly used in the financial industry at all and hence not dealt with in more detail for the purpose of this thesis (Chen, 2018, p. 12).

If the Value-at-Risk (or expected shortfall, sometimes also denoted as tail conditional expectation – TCE[124]) refers to time series of the past and historical values it is regularly denoted as HS-VaR (respectively HS-TailVaR := HS-TVaR) for the historical or historical simulated Value-at-Risk (respectively historical tail Value-at-Risk) as in (Bohdalová & Greguš, 2016; Hull, 2015, pp. 318-330).

There exist however some assets like bond portfolios with inconvenient maturities or bonds named multi-callables, which basically yield options that can be exercised at multiple points in time, or also private equity assets, newly listed companies, merger companies, or private placements, for which historical price information is not directly available (Andersen et al., 2005). Hence, “pseudo” historical prices must be constructed using either standard quantitative pricing models, factor models, or proxies as comparisons (Andersen et al., 2005; pp. 2-3; Bohdalová & Greguš, 2016; Hull, 2015). The assets which lack historical prices might in this case be matched to “similar” or “comparable” assets by their characteristics as capitalization, industry sector, business structure, or duration. The historical pseudo asset prices and returns can then be constructed with the help of these properties of the substitute assets.

However, the lack of conditionality, hence time-dependency in the HS-VaR approaches might be a reason for concern first (Bohdalová & Greguš, 2016; Hull, 2015, pp. 318-334; Pritzker, 2001). To solve the problem, several methods are available for remedying this deficiency, and they are introduced in line with the paper “Financial Risk measurement for financial risk management” (Andersen et al., 2011). The two concepts used within this field are the directly implementable RiskMetrics (RM) and the generalized autoregressive conditional heteroskedasticity (GARCH) models (Constantinides et al., 2012; Hansen & Lunde, 2005; Hull, 2015, pp. 253-265; J. P. Morgan & Reuters, 1996; Nelson, 1990).

In the RM framework, VaR is defined as, see (J. P. Morgan & Reuters, 1996),

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[124] Or sometimes also CTE: Conditional tail expectation.

$$RM - VaR^{p}_{T+1|T} \equiv \sigma_{T+1}\, \Phi^{-1}_{p} \tag{71}$$

where $\Phi^{-1}{}_{p}$ denotes the p-quantile of the standard normal distribution (SND) and also where

$$\sigma_t{}^2 = \lambda\, \sigma_{t-1}{}^2 + (1-\lambda)\, r_{t-1}{}^2 \tag{72}$$

with σ_t denoting the conditional volatility, more precisely one could also write $\sigma_{t|t-1}$, λ is further a parameter between zero and one, r the (asset) returns and hence recursively (for $j > t\text{-}1$ the r_{t-1-j} are set to zero):

$$\sigma_t^2 = \sum_{j=0}^{\infty} ß_j\, r_{t-1-j}^2 \tag{73}$$

where $ß_j$ denotes $ß_j = \lambda^j\,(1-\lambda)$ (J. P. Morgan & Reuters, 1996, p. 82). Therefore, one also refers to “exponentially weighted moving averages” for this method. Various distributions and quantiles could be used instead of the standard normal distribution (SND), but the assumption of conditional normality is the one most commonly used (J. P. Morgan & Reuters, 1996).

Furthermore, the smoothing or decay parameter λ can be conveniently calibrated to best fit the specific historical returns. However, it is typically fixed at the preset value of 0.94 for daily returns (McNeil et al., 2015; J. P. Morgan & Reuters, 1996, pp. 236, 240).

The GARCH framework mentioned also incorporates a VaR and herein VaR is defined as (Bollerslev, 1986; Bollerslev et al., 1992):

$$GARCH - VaR^{p}_{T+1|T} \equiv \sigma_{T+1}\, \Phi^{-1}_{p} \tag{74}$$

where

$$r_t = \mu_t + \sigma_t z_t \tag{75}$$

z_t is independently identically distributed, hence denoted as $z_t \sim$ i.i.d., the expectation value is zero and the variance is one, i.e.,

$$\mathrm{E}(z_t) = 0 \tag{76}$$

$$\mathrm{Var}(z_t) = 1 \tag{77}$$

GARCH is the generalized version of ARCH, the autoregressive conditional heteroskedasticity model introduced by Engle in 1982 (Bollerslev, 1986; Engle & Bollerslev, 1986; Box et al., 1994; Engle, 1982; Engle, 1983). As the name indicates homoscedasticity is not necessary, and conditional heteroskedasticity is considered which allows to model (randomly) changing volatility processes and volatility clusters as, e.g., during a financial crisis (Engle, 1982; Bollerslev, 1986). For simplicity, henceforth a zero conditional mean

$$\mu_t \equiv 0 \tag{78}$$

is assumed. This is without loss of generality (W.L.O.G.) as conditional mean dynamics is then evidently implemented by considering demeaned returns $r_t - \mu_t$ in place of r_t (Bollerslev, 1986).

Further, the GARCH(1,1)[125] process is described as

$$\sigma_t^2 = \omega + \alpha\, r_{t-1}^2 + \beta\, \sigma_{t-1}^2 \tag{79}$$

hence recursively (again for $j > t$ the r_{t-j} are set to zero):

$$\sigma_t^2 = \frac{\omega}{1-\beta} + \alpha \sum_{j=1}^{\infty} \beta^{j-1}\, r_{t-j}^2 \tag{80}$$

GARCH parameters and also the GARCH volatility are estimated using rigorous statistical methods which utilize probabilistic inference (Bollerslev, 1986; Hull, 2015, pp. 255-260). Therefore, (ω, α, β) are estimated by – in most cases where no analytical calculation possibility exists numerically – maximizing the corresponding log-likelihood function based on the assumption that z_t is i.i.d. in N[0,1] as shown in (Bollerslev et al., 1992). The log-likelihood function selects the parameters as the ones implying the maximum likelihood of outcomes respectively the logarithms of it (Hull, 2015, pp. 255-260). The solution is stable in the defined sense that if the conditional return distribution is a nonnormal one, the quasi-maximum likelihood estimator in general still produces consistent, useful – and asymptotically normal – results, which are valuable for practical purposes of risk measurement (Bollerslev et al., 1992; Hull, 2015).

Another possibility is extending GARCH with a HAR-RV type model to GARCH-X[126] or to T-ARCH (Andersen & Bollerslev, 2005; Bollerslev et al., 2016; Hansen & Lunde, 2005, p. 876). In that way, structural breaks or jumps of the price process and volatilities can be included in the process.

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[125] GARCH(p,q) is the general form, here $p = q = 1$ hence the process goes one step back (from t to t-1) for returns and for the volatility. Generally, it is utilizing p return and q volatility predecessors. For α + ß < 1 the important case of a covariance stationary (ergodic) process is derived. The GARCH(1,1) process suffices for most price processes in practice as shown by (Hansen & Lunde, 2005). Models which include the impact of asymmetric news were, e.g., proposed by (Nelson, 1990) or (Engle & Ng, 1993).

[126] GARCH-X (or GARCHX) is including covariates in the model, T-ARCH is including a threshold T for jumps.

In a straightforward manner, the same extensions as of the VaR with RM and GARCH methods are then directly applicable to the ES measure as well, as it is defined upon the VaR measure.

However, the expected shortfall (ES) measure also has an additional mathematical property. It is a coherent risk measure (Artzner et al., 1999; Hull, 2015, p. 299; Embrechts & Wang, 2015). That means it contents the property of sub-additivity and hence usual summations under the condition of the triangular inequality hold, which makes it a convenient measure for, e.g., comparing cross-calculations (Hull, 2015, p. 299). For further details on coherent risk measures, convexity, sub-additivity and spectral measures which are not (core) parts of this thesis, the reader may consult the Annex.

A risk metric is an attribute (or property) of a risk being measured. Ultimately, however, after measuring the amount of risk which can occur, an institute needs enough capital consisting of its own funds and possibly certain eligible liabilities to cover those losses and be able to continue its business as seen. This holds at least for the named going-concern business continuity perspective, on the contrary, the turning point on which losses result in bankruptcy or resolution of a bank is then labeled the liquidation or gone-concern perspective (EBA, 2017). The available amount of capital was defined as risk cover potential or risk-bearing capacity and is of utmost importance for institutions (EBA, 2017).

The credit risk side of transactions was often overlooked in that context in the past which, along with other factors, led to tremendous crises in times (Martin et al., 2014). The miscalculation or in most cases pure non calculation of counterparty credit risk in the field of financial derivatives and hence missing risk covering capital accelerated – alongside wrong assumptions of normality conditions, wrong rated CDOs and the misuse of Gaussian copulas underestimating tail risks – the great recession of 2009 (Hellwig, 2008; Hull, 2021, p. 152-166; Martin et al., 2014; Wigmore, 2008; Witzany, 2017; Zopounidis, 2021). This led to the development of a whole new CVA framework and the so-called regulatory “big bang” within the Basel accords (Martin et al., 2014; Witzany, 2017, pp. 8-10, 217-220). Furthermore, multi-curve approaches taking the (own) funding and counterparty credit spread risks, defined earlier, into account are used when discounting and valuating financial products (Martin et al., 2014). In the times before, often a single curve was used for all discounting purposes and for both sides of a financial contract. The counterparty credit risk and credit spread for short-timed deals were simply (falsely) neglected. An example where multi-curve approaches are applied is the pricing of the so-called “legs” (sides) of a credit default swaps CDS (Martin et al., 2014). It is an instrument that aims to protect the buyer of such a CDS in case a default of the referenced company occurs (Bielecki & Rutkowski, 2004; Martin et al., 2014). Implied correlations of CDS and the link to credit spreads are for instance examined in (Witzany, 2017, p. 186).

Rendleman Jr. also contributed to research on risk premiums associated with risky debt and quantified the impact debt financing even has on equity values, in the case of included taxes and without taxes (Bhandari, 1988; Lee et al., 2010, chapter 2.4/2.5; Rendleman Jr., 1978). As a consequence also in the field of modern corporate funding this risk component meanwhile comprises a necessary part – other than in original CAPM-based considerations as illustrated later.

Within the class of credit risks and losses, one has two differentiate between expected losses and unexpected losses (Bouteillé & Coogan-Pushner, 2021; Hull, 2015; Martin et al. 2014).

Expected loss (EL) is not the risky part as it is “expected” and is directly calculated as the expected value or probability (Bouteillé & Coogan-Pushner, 2021, p. 65). An institute accounts for that value, holds risk reserves and pre-calculates the costs of it for the margin or price of credit (Standard Cost of Credit). The unexpected loss (UL) is the part excessive of that expected loss, the loss “not expected” for an observer which “rarely” appears (Bouteillé & Coogan-Pushner, 2021). It is the risky part of losses, and the calculation is more complex. This part is not accounted for at first in the credit pricing and hence has to be covered by (additional) regulatory capital (Hull, 2015; Rutkowski & Tarca, 2015; Witzany, 2017).

Using the VaR framework the unexpected loss over a time horizon *T*, with confidence level α and a random variable X is defined as in (Hull 2015, p. 294-298; McNeil et al., 2015):

$$UL = VaR_{\alpha,T}(X) - EL \tag{81}$$

where

$$EL = \mathrm{E}[X] \tag{82}$$

Often the UL is referred to as Credit Value-at-Risk (*CreditVaR* or *CVaR*, not to be confused with the Conditional VaR, often also denoted *CVaR*, as in (Hull, 2015, pp. 500-513)). Sometimes, however, one defines

$$VaR := CreditVaR = CVaR \tag{83}$$

hence without subtracting the expected loss.

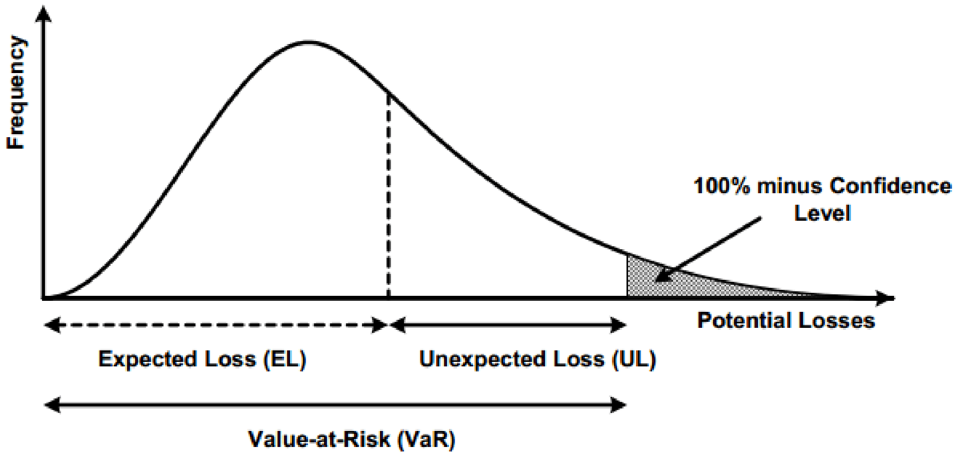

Figure 20 Frequency – VaR diagram.

Source: Graphic from (Bouteillé & Coogan-Pushner, 2021, pp. 78-79) and financetrainingcourse.com ®.

As seen before, once having calculated the Value-at-Risk, by multiplication with 12.5 one can generally derive the risk-weighted assets, *RWA* (Hull, 2015, p. 380). If the expected loss (*EL*) is already provisioned for (as it should) and *VaR* as in the first definition (including *EL*) then

$$RWA = 12.5\,(VaR - EL). \tag{84}$$

Otherwise by definition

$$RWA = 12.5\,VaR \tag{85}$$

when *VaR* equals *CVaR*. The formulas are then evidently, as in (European Commission, 2019):

$$RWA = EAD \cdot RW \tag{86}$$

where *RW* denotes the risk weight (per unit) and *EAD* the exposure (size) at the (potential) default time – in terms of on-balance, non-derivative exposure often just the book value or book value and (at maximum) undrawn credit line. Hence, for a fixed exposure (potentially stressed by the undrawn credit line) the risk weight (per unit) determines the risk-weighted assets. Further, one then gets

$$RW = K \cdot 12.5 \tag{87}$$

where

$$K \cdot EAD = VaR \tag{88}$$

above. *K* is the *VaR* per exposure unit. In the IRB formula (also labeled Gordy or Vasicek[127] formula) of the CRR III article 153[128] one has a form of

$$K = (UDR\,(PD) - PD) \cdot LGD \cdot M \tag{89}$$

with *UDR*: Unexpected default rate, *PD* again the probability of default (BIS, 2005; European Commission, 2019, §153; Cech, 2004, p. 7).

One could also regard *UDR (PD)* as a type of "stressed PD" (Witzany, 2017). *LGD* denotes again the loss once a default occurs ("loss given default") and *M* is only a regulatory maturity factor, often equal to one, e.g., for retail receivables (BIS, 2005; European Commission, 2019).

In the IRB formula under some assumptions, like normally distributed returns of an asset shown later, the formula above reduces to the following expression – without the at that point less important maturity factor (European Commission, 2019, §153; Cech, 2004, p. 7):

$$K = \left(SND \left(\frac{1}{\sqrt{1-R}} G(PD) + \frac{\sqrt{R}}{\sqrt{1-R}} G(0.999) \right) - PD \right) LGD \tag{90}$$

where *SND* is the standard normal distribution and *G* the inverse standard normal distribution (*G* is defined as equal to SND^{-1}). One can directly see that

$$UDR\,(PD) = SND \left(\frac{1}{\sqrt{1-R}} G(PD) + \frac{\sqrt{R}}{\sqrt{1-R}} G(0.999) \right) \tag{91}$$

In that case

$$K = UDR\,(PD) \cdot LGD - PD \cdot LGD \tag{92}$$

and

$$RWA = (EAD \cdot UDR\,(PD) \cdot LGD - PD \cdot LGD \cdot EAD) \cdot 12.5 \tag{93}$$

and as the expected loss is

$$EL = PD \cdot LGD \cdot EAD \tag{94}$$

---

[127] Named after Michael B. Gordy respectively Oldřich A. Vašíček. The American spelling "Vasicek" is used when referring to the associated formula or model – as common in the literature.

[128] See again (European Commission, 2019, §153).

one derives

$$RWA = 12.5\,(EAD \cdot LGD \cdot UDR\,(PD) - EL) \quad (95)$$

One has to keep in mind when regarding the Value-at-Risk, the probability of defaults is considered in stressed circumstances as the formula shows - not just the (expected) “average” probability of default but a distribution of defaults containing the 99.9 % quantile – as well as the exposure at default through regarding the extreme default case with the undrawn credit line. However, as the VaR is the 99.9% quantile distribution of defaults frequencies and also the loss severances, the loss given default parameter (LGD) should also be stressed. Regulators hence require a so-called downturn-LGD (calculated within three possible types) including recession scenarios as in (EBA, 2019; Witzany, 2017, pp. 114, 155). If that suffices is debated and watched by regulators, however, a real, larger problem is a potential correlation between creditors and between the parameters LGD and PD (Witzany, 2011; Witzany, 2017, pp. 114, 154-155).

In that regard, the IRB formula should be viewed in a critical way (Witzany, 2017, pp. 154-155).[129] First, correlations among creditors are considered – in the IRB formula reflected through a certain correlation (R) for whole exposure classes and sometimes PDs to the overall business cycle, as seen – later in the thesis also the LGD-PD nexus, which is not regarded in the IRB formula.

As calculating the VaR for a single asset is quite straightforward, the VaR for a whole diverse portfolio of assets yet has to take into account the various mentioned asset correlations (or even migration and default correlations), which is a much more complex task (Hull, 2015).

[129] This might lead to a surprising effect. Banks have, as stated in the thesis when considering the UTP criteria, some degree of freedom in choosing a definition (events) of default. In case this definition is “too soft”, hence counting counterparties as defaulted “relatively early”, as a consequence many borrowers will be later defined as “cured” after potential restructuring and after some time (under living forbearance) are considered as non-defaulted with a recovery rate of one (Witzany, 2017, p. 155). Therefore, a very “soft default definition causes the empirically observed PD to be higher, while the empirical LGD is lower“, or even zero for many borrowers (Witzany, 2017, p. 155). As a result, that induces lower capital requirements when applying the IRB formula compared to a standard definition of default as shown in (Witzany, 2017, p. 155). The effect is hence caused by insufficient modeling of the LGD.

In this situation (credit) portfolio models are playing out their strengths and are used to solve the problem with underlying correlations and thus higher credit losses (Duffie & Singleton, 2003; Hull, 2015; J. P. Morgan, 2007; Witzany, 2017).

2.3 Asset Portfolio Models

Having introduced the mathematical and terminological foundations for investment portfolios, risk and risk management in this chapter, it is briefly concluded with effective portfolios according to Markowitz's theory and the CAPM and APT models. The historical development in the area of portfolio theory was in the following order (Bodie et al., 2011; Fuller et al., 1987).

The Markowitz model, also known as modern portfolio theory (MPT), was introduced first in (Markowitz, 1952). It was followed by the less known single-index model (SIM), a special case of an asset pricing theory model by Sharpe (Sharpe, 1963). There are several comparisons between the two models, generally, the single-index model is regarded as an extension performance-wise (Vargese & Anoop, 2018).

In a scientific breakthrough, the capital asset pricing model (CAPM) was then introduced and is still considered a standard model in the field used, e.g., for calculating weighted average costs of capital or premiums and optimal capital structures of companies (Hull, 2015, p. 27; Lintner, 1965; Sharpe, 1964). An extension, the intertemporal capital asset pricing model was introduced by Merton (Merton, 1973). Later, the CAPM was succeeded by a less restrictive and more flexible model based on a multi-factor linear combination of risk factors, the arbitrage pricing model or theory (APM/APT) by Ross (Huberman & Wang, 2005; Hull, 2015, p. 32; Ross, 1976a; Ross, 1976b). Model enhancements were further introduced by Chen et al., or Shanken and Weinstein (Chen et al., 1986; pp. 383–403; Shanken & Weinstein, 2006). Finally, APT was extended by behavioral components (B-APT), e.g., influenced by methods of Thaler (Thaler, 1993). One idea of that approach is to include a consumers' or investors' confidence factor (Kahneman & Tversky, 1979; Emrul, 2010). The latest developments to create optimal portfolios, also including rebalancing considerations and time-dependent solutions, use machine learning techniques as reinforced learning methods (seen later), which apply stochastic dynamic programming as the Bellman optimality – the theory behind it and examples are shown in (Dixon, 2020, pp. 25-28; Dori et al., 2018).

Further applications for (special) stock portfolios utilizing G-learning and for wealth management can be found in Dixon's book as well (Dixon, 2020, pp. 380, 401).

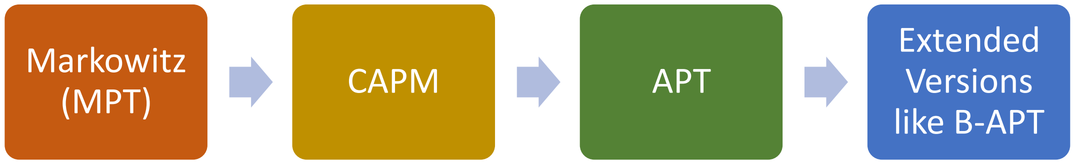

Figure 21 Asset portfolio models.

Source: Own illustration.

The Markowitz model or MPT first achieved the goal to answer two main questions of portfolios and were thus shaping the financial world substantially (Mangram, 2013).

First, it offered a positive proof and quantification of diversification on risk and return in a portfolio, while introducing and explaining the role of correlations (Hull, 2015; Mangram, 2013; Markowitz 1952).

Second, it showed a method for selecting an "optimal (efficient) portfolio". The model is built up as follows (Elton & Gruber, 1995; Markowitz, 1952; Markowitz, 1959). It needs the assumptions of an investor who is using only publicly available information like prices/quotes, dividends, cash flows and who acts rational and hence opportunistic – as usual in standard economic theory (Holton, 2003; Hull, 2015). The investor is additionally utility-maximizing and risk-averse (Markowitz, 1959, pp. 77-79, 81, 83).

Furthermore, the model assumes a complete capital market as defined before, which means a market that is arbitrage-free, friction-less, and especially all claims in it can be replicated (Markowitz, 1952; Markowitz, 1959 pp. 82-85). It requires that not all assets are perfectly correlated as this is the trivial case, the mixture of assets is then irrelevant at all.

The model aims to find and select an optimal "combination" of assets (Markowitz, 1952). This implies a portfolio bearing an optimal risk/return profile, which means a portfolio that has the highest returns while having the same risk or the same return with lower risk – among and compared to all possible combinations of other assets and portfolios (Hull, 2015; Markowitz, 1959, pp. 79, 82).

For the model one can define that a portfolio A "dominates" a portfolio B, when for the affiliated returns

$$r(A) \geq r(B) \tag{96}$$

and for the volatilities

$$\sigma(A) < \sigma(B) \tag{97}$$

or equivalently:

$$r(A) > r(B) \tag{98}$$

and

$$\sigma(A) = \sigma(B) \tag{99}$$

(Gollier, 1997; Markowitz, 1952).

A is called efficient when it dominates all other portfolios *B* for this set of assets, with *r(i)* denoting the expected return of asset *i* (and *r(A)* the return of the whole portfolio *A*), i.e., the fraction *(p(t) - p(t-1)) / p(t-1)*, with *t* in time [0;*T*] when *p(t)* denotes the asset portfolio price at time *t* (Markowitz, 1952; Markowitz, 1959). σ(*i*) is the volatility, the standard deviation of the (time-dependent) returns of asset *i*, hence the "risk" for an asset *i* (Markowitz, 1959).

Therefore, the portfolio return might be regarded as the proportionally weighted sum of the different assets' returns and the portfolio volatility is a weighted sum of the correlations ρ(*i, j*) of the different assets (Markowitz, 1959). Hence, for all combinable asset pairs (*i, j*), *i*=1,…, *n* and *j*=1,…, *n* for a portfolio with *n* assets the volatility is

$$\sigma = \sum_i \sum_j w(i)\, w(j)\, \sigma(i,j) \tag{100}$$

where σ(*i, j*) is the covariance, i.e.,

$$\sigma(i,j) = \sigma(i)\, \sigma(j)\, \rho(i,j) \tag{101}$$

and *w*(*i*) respectively *w*(*j*) denotes the weights of the assets in the portfolio (Elton & Gruber, 1995; Markowitz, 1959).

An optimal portfolio depends on the risk preference of an investor. Therefore, for an optimal portfolio, it holds that the slope of the indifference curve equals the slope of the efficiency line, as defined below, with the maximum return when the risk or indifference curve is given or the minimal risk when the return is given (Markowitz, 1959). Consequently, to solve the two questions above one has to use the following scheme developed by Markowitz (Markowitz, 1952; Markowitz, 1959):

$$Minimize\ w^T \Sigma\ w \ \text{ s.t. } \ R^T\ w\ =\ \mu, \tag{102}$$

where Σ is the covariance matrix, μ is given as the expected return, or equivalently

$$Minimize\ w^T \Sigma\ w - m^* R^T w\ , \qquad m^* = \frac{1}{m} \tag{103}$$

In the last line $m*$ is denoting the "risk tolerance", m is the slope of the indifference curve, as stated above. Both equivalent optimization problems, also denoted programming problems, are standard economic problem settings and are usually solved by Lagrange multipliers (Markowitz, 1956). As a result, the weight vector w for an optimal portfolio is obtained.

As mentioned before, the Markowitz idea can be also applied to bonds and loans with nonnormal returns as opposed to equity (e.g., shares) with supposed normal returns, and then loan portfolios are optimized in the same way – one only substitutes the volatility σ with the Value-at-Risk *VaR* in the formula and respective concept above, as also described in (Witzany, 2017, pp. 111, 119).

A special case and finding of MPT is, that the capital allocation line (CAL) for a combined portfolio (P) of one risky (C) and one riskless asset, $R*$ denoting the risk-free rate, is the following (Markowitz, 1952; Merton et al., 1973):

$$\mathrm{E}\big(R(C)\big) =\ R^* +\ \sigma(C)\,\frac{\mathrm{E}\big(R(P)\big) -\ R^*}{\sigma(P)} \tag{104}$$

Generally, the CAL looks like as in the following figure, and similarly for two non-risk-free assets or "mutual fund portfolios" (Karatzas et al., 1986).

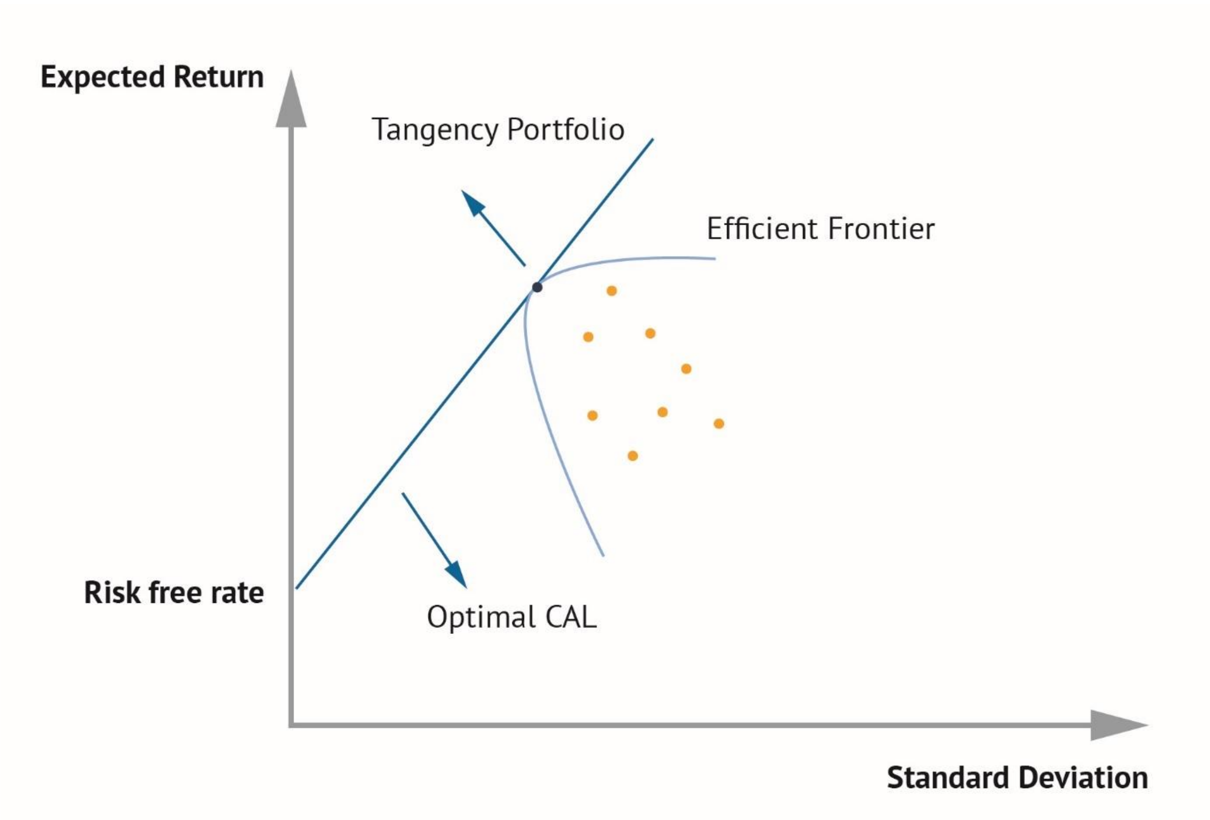

Figure 22 Capital allocation line (CAL) and a tangency portfolio intersecting the efficient frontier.

Source: Own illustration.

An extension of the MPT or Markowitz model is the CAPM (Hull, 2015, p. 27). The aim and main idea of CAPM is to consider all (or at least in case of weaker model assumptions "very many") market participants hence "the whole market", i.e., extending MPT, which considers just one investor in that sense (Hull, 2015). The assumptions for the investors are again that they just act upon publicly available information (Lintner, 1965). The agents and investors are furthermore classified as rational and opportunistic again, they are acting utility-maximizing, and they behave in a risk-averse manner (Lintner, 1965; Sharpe, 1964; Tobin, 1958).

The assumptions for the model are further that everyone in the market owns the risky assets in identical ratios to one another and hence these ratios are pre-defined by the so-called efficient or tangency portfolio (Sharpe, 1964). This is one of the most criticized assumptions of the CAPM, for further details see e. g. (Dumas & Allaz, 1996). When a market equilibrium is reached in the CAPM, the existence of such an equilibrium under certain conditions is proven in standard literature by, e.g., Arrow-Debreu building on works of Wald or by Mantel, the risky assets' prices and hence returns have adapted in such a manner that the ratios in the tangency portfolio equal the ratios in which the supply of risky assets to the market takes place (Arrow & Debreu, 1954; HKT, 2021, p. 3; Mantel, 1976; Sharpe, 1964). Hence, the model is involving also a macroeconomic factor besides the micro-agents.

The market view on risk is consequently involving two risk components, which were already briefly discussed in the thesis, unsystematic risk as an individual, idiosyncratic risk specific to an asset, which is diversifiable (away), and systematic risk, e.g., "market risk" as a whole, which cannot be diversified (Lintner, 1965). Systematic risks yet can be managed by a strategy known as a "market neutral" portfolio, a portfolio that holds long as well as short positions in assets for that purpose (Fama & French, 2004; Satchell, 2015).

The core idea of CAPM is that the fair price paid for an extra asset *x*, *p*(*x*), must equal the incremental effect on expected return and risk, when the additional risky asset, *x*, was added to the market portfolio (Sharpe, 1964).

Hence, with the additional risk – the in-brackets expression in the denominator of the following formula – and additional expected return – denoted after *w*(*x*) in the nominator – and weighting function *w* one gets the following result (Sharpe, 1964):

$$\frac{w(x)\big(\mathrm{E}\big(R(x)\big) - R^*\big)}{2\, w(m)\, w(x)\, \rho(x,m)\, \sigma(x)\, \sigma(m)} = \frac{w(x)\big(\mathrm{E}\big(R(m)\big) - R^*\big)}{2\, w(m)\, w(x)\, \sigma(m)\, \sigma(m\,)} \tag{105}$$

where R^* is again the riskless return, $R(x)$ the return of *x*, $R(m)$ the former market return. Thereby, one is rationally assuming $\mathrm{E}(R(x)) > R^*$, i.e., that the expected return should be more than the risk-free rate (Sharpe, 1964). On the right-hand side of the expression, the former market ratio is extended and multiplied by *w*(*x*) in the nominator and denominator. The equation further simplifies, by multiplication times 2 *w*(*m*) and using the covariance definition, to

$$\mathrm{E}(R(x)) = R^* + \frac{\mathrm{E}(R(m) - R^*)\, \sigma(x,m)}{\sigma(m,m)} \tag{106}$$

The ratio of the covariance between *x* and *m* in the nominator and the variance of *m* in the denominator is called β as before (Lintner, 1965). In practice β and the surplus return α are calculated via regression on real, available market data (Fama & French, 2004). A special finding in that context is the security characteristic line SCL:

$$R(x) - R^* = \alpha + \beta(R(m) - R^*) \tag{107}$$

as in (Fama & French, 2004; Hull, 2015). This is basically the "extended version" and analogion of the capital allocation line (CAL).

"Post-modern portfolio theory" (PMPT) even extends CAPM by, e.g., other measures of risk or by allowing nonnormal, asymmetric ("skewed"), or even "fat-tailed" distributions for risk factors (Chaudhry & Johnson, 2008; Rom & Ferguson, 1993). Another approach exploits the methods of the Black-Litterman portfolio theory, which requires the formulation of "how assumptions about expected returns differ from the markets" and the degree of confidence in the alternative assumptions plus expected utility (Chen et al., 2018; Black & Litterman, 1992; Noguer i Alonso & Srivastava, 2020, p. 2).

As the last model the state-of-the-art portfolio model APT is introduced, the arbitrage pricing theory, however concentrating on the classical version excluding behavioral proprietary extensions. The aim is to insert multi-factors to represent market influence (Hull, 2015, p. 32; Ross, 1976a). The reason is the empirical evidence that many factors in reality impact asset returns (Fama & French, 1993; Fama & French, 2004; Ross, 1976a; Ross, 1976b). Furthermore, market equilibrium in that model is not necessary, but only freedom of arbitrage. The assumptions for agents are again that they behave in a rational and opportunistic manner plus arbitrageurs are in the market to guarantee the mentioned no-arbitrage-condition, often named NA-condition (Ross, 1976a). Further risk preferences are also not necessary, e.g., a certain risk-averse, Bernoulli, or (μ,σ)-principal behavior is not a required pre-condition (Ross, 1976a).

The model assumptions include that there is a basket of assets $x(i)$, $i = 1, \ldots, n$ existing. The return $R(i)$ of an asset is dependent on many micro and macro factors $F(j)$, $j = 1, \ldots, n$ and sensitivities to these factors $\beta(i, j)$, while $\epsilon(i)$ is the idiosyncratic risk (Ross, 1976a). The return is therefore written as

$$\begin{aligned} R(i) = \mathrm{E}(r(i)) + \beta(1,j)\,F(1) + \beta(2,j)\,F(2) + \cdots + \beta(n,j)\,F(n) \\ + \epsilon(i) \end{aligned} \tag{108}$$

(Ross, 1976a; Ross, 1976b). That equation needs to be solved for the betas. This objective is achieved via a standard multi-(linear)-regression on empirical data for the Fs, though calibration on that data might be in some cases relatively complex (Ross, 1976a). Ross proved that the expected return (under a NA condition) in a complete capital market is then

$$\mathrm{E}(R(i)) = R^* + \sum_j (\mathrm{E}(R(F(j))) - R^*)\, \beta(i,j) \tag{109}$$

while freedom of arbitrage is reached here by ensuring that assets with the same risks expect the same returns (Fama & French, 2004; Huberman & Wang, 2005; Ross, 1976a, pp. 341-360). Therefore, unsystematic risk is already assumed to be diversified away, $\epsilon(i)$ is vanishing. This is approximately the case for very large and nonconcentrated portfolios. Special findings are the following ones related to the APT concept. Research by Ross showed that five macro-factors (denoted "5f") explain an asset return quite precise (Gibbons et al., 1989; Ross, 1976b):

These are the long-term inflation (expectations), the short-term inflation, real interest rates, an index of industrial production or on output data and the probabilities of default (PD) for the assets.

There are also other factors that can influence asset prices, but they are generally less significant or in the realm of "policy", i.e., institutional economics or monetary policy – which affects asset prices in a longer-term than previously expected (Bianchi et al., 2022). Possible extensions – like including consumers' confidence in a six-factor model (6f), albeit more complex – were mentioned before. Also in other related economic fields, five or six factors are commonly used to model prices or investment decisions, often utilizing socio-demographic factors or attitudes and behaviors as e. g. factors influencing the prosumer's investment decision on the use of "renewables" like solar power (Rausch & Suchanek, 2021).

Most APT models mainly (except for the PD) or even solely use macro-factors hence a macroeconomic model is derived when considering all agents and all assets. The criteria and variables mentioned above are also commonly used – among various further variables – for many modern macro-economic models like dynamic stochastic general equilibrium models (DSGE) as the Smet-Wouters model from the ECB, which is used for forecasting and measuring policy response functions, it is normally estimated by Bayesian estimation (Smets & Wouters, 2003; Smets & Wouters, 2007; Uhlig, 2007).[130] Yet the last feature in the APT model above, the probability of default, sticks out and can be quite individual. For investors and lenders, the probability of default or reciprocally stated the (credit) quality of a company is of utmost importance.

[130] For a well-explained model introduction see (Uhlig, 2007).

Hence, the aim is to develop a grade or more general a whole scoring – in the retail case – or rating system, which is judging the creditworthiness of a company on a certain measurable scale and also directly links and calibrates it to its probability of default (Brusov et al., 2021; Witzany, 2017). This “rating process” is furthermore of general importance for the wider scope of credit risk and as input for industrial credit portfolio models as CreditMetrics® or CPV® as well as the support vector regression applied in the thesis, which all utilize ratings to quantify the portfolio risk. After having introduced the Markowitz, CAPM, and APT portfolio models building on risk and return measures, the models’ assumptions, applied methods, corresponding quantification formulas, main findings and ratings as “risk model input” are hence now discussed in detail (Chen et al., 2018; de Laurentis et al., 2010; Fama & French, 2004; Izzi et al., 2011; Witzany, 2017).

CHAPTER 3
RATINGS AND RISK MODELS

3.1 Rating Systems, Their Characteristics and Validation

Through regulatory efforts like the Basel II and III accords the use of ratings became increasingly widespread and standardized (Izzi et al., 2011; Quattrociocchi, 2016; S & P, 2016, pp. 6-9). The credit risk or creditworthiness of a bank's debtors has to be judged by the institute with the help of external or internal, bank-internally calculated, ratings (Daldrup, 2006, pp. 11-15; Kumar et al., 2012). After the experience of the great recession, which was also spurred by inaccurate, far too positive ratings for CLOs[131] and other subprime mortgage packages, the new Basel III finalization – commonly referred to simply as "Basel IV" within the banking industry – requires due diligence when using external ratings from rating agencies in the standard approach (SA/ST) for credit risk (Abdo, 2020; BCBS, 2022; Financial Crisis Inquiry Commission, 2011; Hull, 2015, pp. 152-164; Podkul, 2019a). That necessity is legally implemented in the EU within the CRR II and CRR III legislations. Further lessons of preceding crises concerning ratings and credit risk in general, for instance the Herstatt insolvency in 1974 or the savings banks crisis in the 1980s, were already incorporated in Basel II and the CRR I (BCBS, 2000b; de Laurentis et al., 2010, p. 15; Riedl, 2002).

Generally in Europe, most debtors of banks are micro, small, and medium-sized companies (SMEs) or retail clients[132], which are not listed on capital markets and as non-public companies mostly do not bear an external rating (Abdo, 2020; Kumar et al., 2012). Hence, according to the CRR and even more when considering the fact that banks with the Basel III finalization need to make their own due diligence also in the case of using external credit ratings within the SA, banks are (partly) incentivized to make use of the internal methods (Daldrup, 2006; Oyama & Yoneyama, 2005; Witzany, 2017, pp. 11-16). These rating methods are the foundational internal ratings-based approach (F-IRBA), where institutes have to determine an own internal creditor rating and a corresponding

[131] As mentioned, CLO: Collateralized loan obligation, special form: CMO Collateralized mortgage obligation, where the loans are mortgages. Both forms are bundling and packaging many, often subprime level loans and through diversification and the securitization of the loans (often via special purpose vehicles), their rating gets (often disputable) better.
[132] Retail clients here refers to SMEs plus private clients as used in the credit risk approach by the BCBS.

PD for every debtor in that rating class by means of their own internally developed rating system, or the A-IRBA (advanced IRBA) as applicable for the retail segment (European Commission, 2019; BCBS, 2000a; BCBS, 2022; Oyama & Yoneyama, 2005; Witzany, 2017, pp. 15, 108, 112-116). The use of IRB approaches has to be permitted and is controlled by regulators (European Commission, 2019, §143). In the case of the A-IRBA, banks not only determine the PD but also another classification or rating, then often called transaction rating and probability, the so-called LGD mentioned before (European Commission, 2019; Witzany, 2017, pp. 112-116).

The LGD is the loss given default, hence the loss once default occurs, which equals one minus R when R specifies the recovery rate (Bouteillé & Coogan-Pushner, 2021; Daldrup, 2006; Kumar et al., 2012):

$$LGD = 1 - R \tag{110}$$

The loss given default is commonly expressed as the percentage of the exposure at default that one does not retrieve back after a default, sometimes also denoted as LGD% (Daldrup, 2006). LGDs for models are frequently obtained from empirical workout-LGDs, i.e., the LGDs a bank experienced in its real, comparable cases after a defaulted exposure was worked out by its NPL[133] and workout units, with collateral sold, estate foreclosed, assets sold, etc. and after costs for lawyers and administration (Schuermann, 2002; Witzany, 2017, p. 98). LGDs, including their time-variability, can be yet also treated within a statistical framework and model, commonly utilizing Beta or mixed Beta distributions (Schuermann, 2002; Witzany, 2017, p. 98). These distributions are able to describe many empirical LGD distributions rather precisely (Witzany, 2017, pp. 96-100, 129-131). When employing the advanced IRBA the institutes also need to determine the exposure at default (EAD or EaD), taking especially into account the "CCF", as the third parameter (de Laurentis et al., 2010, pp. 15, 21-22, 396; European Commission, 2019). That is the credit conversion factor, which describes and models the part, or more precisely the percentage of a credit line or other off-balance-sheet exposure, still to be drawn in the future until the default event occurs and is taken from the UCL, the undrawn credit line (Bouteillé & Coogan-Pushner, 2021; Daldrup, 2006, pp. 92, 93; de Laurentis et al., 2010, pp. 21-22, 396; European Commission, 2019; Witzany, 2009c; Witzany, 2017, pp. 112-116). It can be written as:

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[133] Non-performing loans

$$
\begin{aligned}
&Exposure\ at\ Default\ (EAD) \\
&= Drawn + (Limit/Line - Drawn) \cdot LEQ \\
&= Drawn + Undrawn \cdot LEQ
\end{aligned}
\tag{111}
$$

LEQ here denotes the loan equivalent factor or credit conversion factor CCF[134] (de Laurentis et al., 2010, p. 21).

The worst-case here is that a company draws its entire free (open) line briefly before it defaults. Empirical studies found that the credit conversion factor is indeed much higher at/after a default of a company and showed that there is further a positive correlation between the economic cycle and the undrawn line (limit), as well as a positive one between the riskiness, i.e., worse rating of a company and the drawn line or CCF (Zhao & Yang, 2019, p. 1). The EAD is therefore a time-dependent function.

Time is generally a crucial parameter – alongside PD, LGD, and EAD (with CCF). Therefore, maturity and fictional maturities (e.g., for daily withdrawable sight deposits) are modeled for some exposure classes in the A-IRB as well (Daldrup, 2006; European Commission, 2019; Witzany, 2017, pp. 112-116). Generally speaking, more time means more time for default or worsening creditworthiness (rating) and hence more credit risk (Bouteillé & Coogan-Pushner, 2021; Hull, 2015). Different approaches considering that time and (rare) models of a time-dependent EaD were developed, e.g., by Moral, Jacobs, and also Witzany (Jacobs, 2008; Moral, 2006; Witzany, 2009c). Generally, a so-called fixed time horizon approach of normally a year and used for a PD weighted concept, a cohort approach that also normally endures a year, is standard and used otherwise, e.g., for mean weighted concepts, or a variable time approach beginning from one month, month-wise up to one year, mainly used for seasonality concepts, are differentiated therein (Witzany, 2017, pp. 100, 102). Especially Moral's and partly Jacob's ideas also influenced the latest CRR III regulation with its CCF approach (Witzany, 2017, pp. 100, 102).

A reminder that a highly leveraged counterparty reveals enormous risk and might drive up the EAD substantially was the collapse of the Archegos fund in 2021 – a method to include such leveraged cases in an intensity model was recently published by Dickinson

[134] Banks may calculate the EAD of the unfunded part of the loan utilizing LEQ or CCF. Within the CRR of the European Union and Basel, the terms LEQ and CCF are used interchangeably. The American OCC yet defines the CCF "as the balance at default to balance 12 months prior to default" (Zhao & Yang, 2019, p. 1). As before, the thesis sticks to the first, interchangeably used definition.

(Dickinson, 2022). Apart from PD and LGD modeling in the A-IRB approach, the often less regarded EAD should therefore form a crucial model part as well (Jacob, 2008; Witzany, 2017).

An advantage of the IRB is that institutes can calculate their own internal PD and in the case of the A-IRBA also the LGD, EAD (with CCF) and partly the maturity factor M – and the resulting capital requirements are generally (slightly) lower than the ones required by the standardized approach (BCBS, 2006b; Witzany, 2017, pp. 109, 154).

A further argument for using the IRB approach is that banks in the European Union can more easily decide to just use the IRB for certain specific exposure classes or even portfolios, known as partial use (PU) within the CRR III and can "return it" in exchange for a switch to the standardized approach faster, whereas in the past they had to use it throughout all the segments or portfolios then (European Commission, 2019, §149-150). On the other hand, also the standardized approach (SA) for credit risk is much more risk-sensitive than in the past (Hull, 2015; Witzany, 2017). The use of the IRB will be floored at 72.5 % of RWAs calculated by the standard method covering all risk categories (phased in from 50% from 01.01.2022 until 01.01.2027)[135][136] – which hence has to be calculated in either way – and the use of the A-IRBA now is limited to certain credit exposure segments as will be shown in the next paragraph (ECB, 2019; European Commission, 2019; Hull, 2015; Witzany, 2017, pp. 112-116).

Therefore, it remains to be seen whether the IRB approach is used more frequently than before,[137] less frequently as maybe expected by supervisors[138] or – after considering costs-and-benefits – rather similar than nowadays (Behn et al., 2022b; Bundesbank, 2022b; Witzany, 2017).

The concept of the PD case, which is the primary parameter and the only one not covered in detail yet is treated in the following part of that chapter. It represents the sole parameter needed for the A-IRBA as well as for the F-IRBA[139], and the definition and

[135] 01.01.22: 50 %, 01.01.23: 55 %, 01.01.24: 60 %, 01.01.25: 65 % ,01.01.26: 70 %, 01.01.27: 72.5 %
[136] In 04/2020 in the wake of the COVID-19 pandemic the finalization of Basel III and the transition rules and dates were scheduled for one year later.
[137] See (Woodall & Bhollah, 2019) and for German banks the Bundesbank list on (Bundesbank, 2022b).
[138] The regulators are cautious because of their assumption that the institutions use these models mainly to lower their RWA and capital burden and that there are limits to model-based regulation as shown in (Behn et al., 2022b).
[139] With the finalization of Basel III and the adjusted SCRA and IRB approaches the A-IRBA is mainly left to the exposure class of small companies, it is even required for retail portfolios - whereas for banks and larger companies with more than five hundred millions in revenue just the F-IRBA, hence just the PD estimation, is allowed by regulators (Witzany, 2017, p. 15).

generating of a rating are crucially linked with it (S & P, 2016; Witzany, 2017). A rating is a standardized and (mostly) objective valuation of an object or person by a grade or score, which follows at least an ordinal but, in most cases, a metric scale (Bielecki & Rutkowski 2004; Daldrup, 2006, p. 3; de Laurentis et al., 2010, p. 33; Izzi et al., 2011). However, most ratings include a qualitative component, regarded as a difference compared to automated "scorings", and rating agencies who publish them then declare ratings, mainly for legal reasons, as an "opinion about credit risk" (de Laurentis et al., 2010, pp. 67-78, 320-322; S & P 2016, p. 4). Scoring is mainly used for retail exposure, where an overall assessment, including a large amount of qualitative data, is considered too expensive (Bouteillé & Coogan-Pushner, 2021, pp. 136-140; de Laurentis et al., 2010, pp. 116, 452). At the point of loan application and before loan origination for natural persons, the income or wage verification of at least the last three years, shown by pay stubs and tax files/statements (e.g., the form 1040 of the IRS[140] in the U.S.) as well as possible collateral like a stock portfolio, personal jewelry, an owned car or house are examined by the bank – often combined with the score from a consumer credit reporting agency as FICO®[141] in the US or Schufa®[142] in Germany (Bouteillé & Coogan-Pushner, 2021, pp. 136, 138-139; de Laurentis et al, 2010). Later in the time of the credit-relationship lifecycle with a client, the focus in scoring shifts more from past-looking creditworthiness statements to dynamic, behavioral-based payment patterns of the borrower (Bouteillé & Coogan-Pushner, 2021; de Laurentis et al., 2010). Increasingly, data based on internet shopping patterns and behavioral data from certain apps, labeled as super-apps, as the Chinese WeChat® is used for scoring purposes (Roa et al., 2021).

As the main focus of the thesis lies within the realm of a middle-sized or larger company's creditworthiness the following part deals with ratings (and not scorings, though many aspects are common).

If a rating occurs in the context of a company's willingness and ability to pay all its obligations completely and on time, hence its creditworthiness, then it is called a credit rating[143] (S & P, 2016, p. 4). Since the scope of the thesis is on credit, the terms rating and credit rating are henceforth used interchangeably. A rating can be generally done in a

[140] Internal Revenue Service, U.S. tax authority. Form under: https://www.irs.gov/pub/irs-pdf/f1040.pdf?msclkid=cfd937aebb0011ec920d4a09ce6141da (Retrieved Mai 16, 2022).
[141] Fair Isaac Cooperation® (Website: Retrieved Mai 15, 2022, from https://www.fico.com/en)
[142] Schutzgemeinschaft für Allgemeine Kreditsicherung® (SCHUFA® e. V., Website: Retrieved Mai 14, 2022, from https://www.meineschufa.de/de/)
[143] Sometimes also denoted credit-rating.

public or in a private fashion (de Laurentis et al., 2010). There are also two further ways of categorizing ratings. The category then depends on the rating object. If the rating aim is the creditworthiness of a whole company, of a state (or supranationals, sub-sovereigns, and agencies - SSA) or a municipality one speaks of an issuer rating or issuer credit rating (Izzi et al., 2011; S & P, 2016, pp. 15-18). If the rating refers to a certain special investment instrument or issuance with its own properties, collaterals, covenants, tranches, seniorities, and specifications or a project without recourse rights and hence solely dependent on the corresponding forecasted project cash flows, one talks of an issue rating (S & P, 2016, pp. 15-18). In the case of object or project finance a corresponding rating is thus known as an object, transaction, or project rating (Izzi et al., 2011; Joseph, 2013; S & P, 2016, pp. 15-18).

If a whole branch or country is rated, instead of a company, a sole financial object or a natural person, one refers to that circumstance as branch rating or sovereign rating (Bouteillé & Coogan-Pushner, 2021, pp. 121, 145, 163; de Laurentis et al., 2010, p. 42; S & P, 2016). In case the entity level is a sub-national level, one can further break down the categories into municipal ratings and regional or communal ratings (European Commission, 2019; SEC, 2012).

These ratings often act as so-called ceilings or caps for the creditor rating (Daldrup, 2006; IMF, 2005; Quattrociocchi, 2016). The term ceiling (cap) in this regard means that the rating of a subsidiary or company within a country should not be better than the one of the parent entity or state itself (Daldrup, 2006; IMF, 2005). This is due to the fact that generally profits (and losses) are upstreamed within a group or holding and often patron declarations or unlimited guarantees for subsidiaries exist (de Laurentis et al., 2010; Quattrociocchi, 2016). The parent entity thus serves as a "parachute". In the case of a company within a country, the company is dependent on the political system and laws, its resources, infrastructure, technological environment and human capital of that country, and its ability to raise taxes to pay for infrastructure and state expenses in general (de Laurentis et al., 2010). So, it is exceedingly rare that companies perform better than their overall environment and available infrastructure – hence they are normally capped by the state or country ceiling. Only in seldom cases and, e.g., for multinational, globalized companies who have worldwide sources of income and can even use arbitrage between the various tax and business environments the company is able to achieve higher ratings than its country of origin or the country where its headquarter currently resides (Moody's, 2022). Similarly, under normal circumstances, also for instance regional or municipal ratings are capped by the state or national ratings thus the government level above, where the states can be viewed as support factors or sometimes even explicit guarantors of debt – though in its detail depending on the infrastructure, laws and constitutions, inter-state transfer schemes, and possibly additional factors (Naciri, 2015; Quattrociocchi, 2016; SEC, 2012).

On the other side the term "rating floor" means that a specific rating cannot be worse than the general floor rating. This is referred to as "the weakest link in the chain" and an example is the worst rated state in a union of states – the overall union rating, for reasons of arithmetical averaging of indicators, cannot be worse than the one of the worst rated member state (Quattrociocchi, 2016). One calls a rating a "stand-alone rating" (SAR), when it comes without any support/burden factors of affiliated companies or governments and before an eventual expert override [144] or currency conversion [145] (Daldrup, 2006; Izzi et al., 2011; Moody's 2022; S & P, 2016, p. 16; Quattrociocchi, 2016). This stand-alone rating is sometimes also referred to as baseline credit assessment (BCA), for example by the rating agency Moody's® (Moody's, 2022). It is the core of a company's ratings. The rating methodology of the most relevant agencies as S & P Global Ratings®[146], Moody's Investors Services®, or Fitch Ratings® developed substantially since their inception and the use of ratings grew enormously since the 1990s (Cantor et al., 1999; Nickell, 1998; S & P, 2012). The origins of judging companies' creditworthiness already date back until the beginning of the nineteenth century, with Dun & Bradstreet pioneering, and accumulated in the second part of that century, when railway infrastructure financing was a preeminent national task and the issuing of bonds became common (de Laurentis et al., 2010, pp. 14-15). Moody's was founded in 1900 (1909 under a new name) and Standard & Poor's in 1941, merging from Poor's Publishing Co. which traces back until the 1860s and Standard Statistics Co. from 1906, subsequently S & P was acquired by McGraw-Hill in 1966 (de Laurentis et al., 2010, p. 15; S & P, 2022b; Sinclair, 2005). Internal ratings were not introduced by banks until the second half of the 1980s after the loans and savings crisis, during which more than 2800 savings banks collapsed, when the FDIC and OCC subsequently required a classification of bank loans

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[144] Sometimes it is also called overwriting.
[145] Receiving first a foreign currency rating (FCR) and then after a rating transfer a local currency rating (LCR).
[146] In the course of the thesis sometimes just briefly denoted as S & P ®, Moody's ® or Fitch®.

into at least six grades (de Laurentis et al., 2010, p. 15). For external as well as internal rating systems the BCA forms the basis of a rating.

After the BCA and the adding of affiliate support as well as government support – and in case a bank is the rated issuer, also after the LGF[147] analysis of the risk in the event of failure of the bank has taken place – one derives at the issuer credit rating, the ICR (Moody's, 2022; Quattrociocchi, 2016; Witzany, 2017). In the very rare cases mentioned above the affiliate (group) and government factors might be a burden instead of support and one then speaks of government burden (Izzi et al., 2011).

The term rating is often used for the result and actual rating grade as well as for the rating process in general (Bouteillé & Coogan-Pushner, 2021).

Having defined credit ratings and rating objects, considered entities for which ratings are used as well as support/burden-factors transforming the individual BCA (SAR) to a final ICR the two main rating paradigms are regarded.

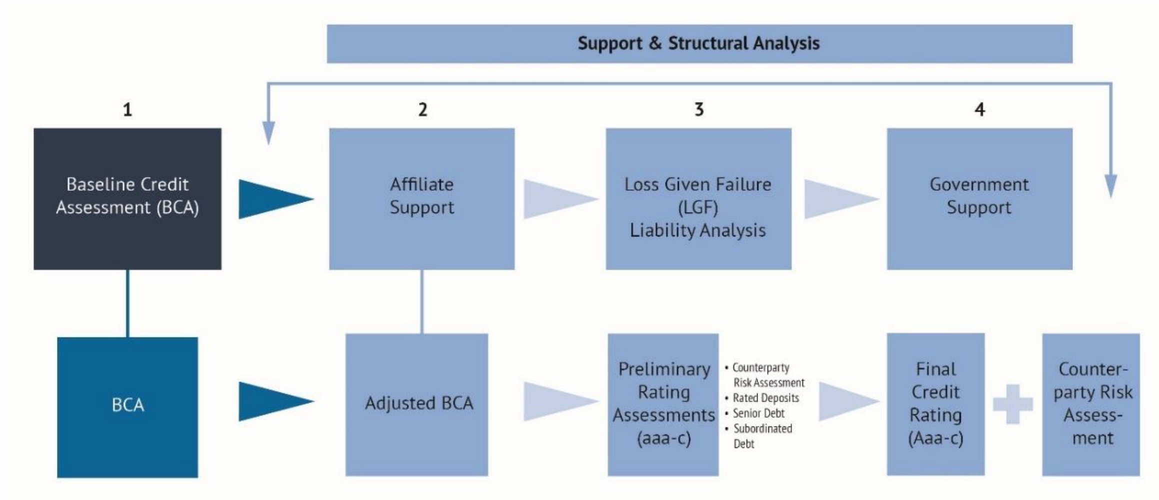

Figure 23 Rating with support and structural analysis, from BCA to ICR in case of a rated bank.

Source: Own illustration, according to (Moody's, 2022) and NORD/LB® Fixed Income Research.

[147] Loss given failure

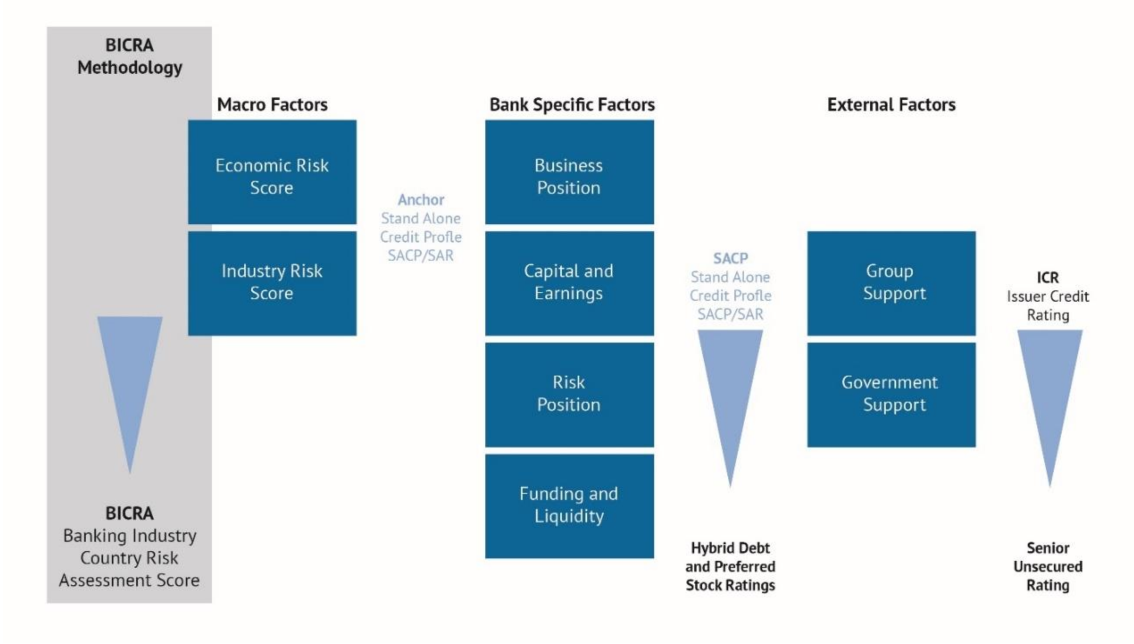

Figure 24 BICRA rating process methodology of S & P ®.

Source: Own modified illustration, from (S & P, 2022): "How we rate financial institutions".

In the banking industry it is possible to either use point-in-time (PIT) ratings, which are done at a specific, concrete point in time and for a certain short-term horizon, e.g., a year backward and through-the-cycle ratings (TTC), which are taken over a certain period of time including a whole economic cycle, especially also a recession or downturn time (Daldrup, 2006, p. 13; Oyama & Yoneyama, 2005, p. 11; Trueck & Rachev, 2008; Witzany, 2017, pp. 63-64). The first paradigm is more exact at the specific time and more sensitive, hence useful for real credit risk and bank controlling in the internal control systems (ICS) as well as for fair value pricing (Izzi et al., 2011; Witzany, 2017). On the negative side, however, it is pro-cyclic, hence better ratings are assigned during an upturn or boom and worse ones in a downturn phase of the economy (de Laurentis et al., 2010; Gordy & Howells, 2006; Izzi et al., 2011; Oyama & Yoneyama, 2005, pp. 11-12; Repullo et al., 2009; Witzany, 2017, p. 64). Therefore, the PIT approach is amplifying the cyclical effects and the VaR and loan provisions needed instead of smoothing the economic cycle effects and hence ratings are less stable compared to TTC ratings, which change less and are generally more long-term oriented and therefore often preferred for regulatory settings (Gordy & Howells, 2006; Repullo et al., 2009; Witzany, 2017, p. 63-64, 66, 113). However, this pro-cyclicality is partially countered within the CRR III framework and with the introduction of the buffers CCyB and CCB as shown before (Witzany, 2017, pp.

155-156). Hence, it depends on the use case and application, when deciding what approach to prefer. Regulators principally allow both to be utilized for the IRB approaches in the CRR III, but downturn effects have to be considered, which is a prerequisite tending toward the TTC method; however, the period is on the other hand a short-term perspective, which might better suit the PIT approach (Izzi et al., 2011; Oyama & Yoneyama, 2005, p. 12; Witzany, 2017, pp. 112-113). Principally, the downturn effect is also assignable to PIT ratings by means of a MOC[148], by a history containing, e.g., the GFC or by a capped procedure (de Laurentis et al., 2010; European Commission, 2019). As it is moreover the approach mainly used for internal controlling and economic capital determination purposes and better confirming the vision of regulators to converge the ideas of a regulatory, normative perspective and an economic perspective in RBC, PIT ratings might be preferred (de Laurentis et al., 2010, p. 406; Izzi et al., 2011). Furthermore, the PIT methodology has to be applied for stress testing purposes in the EBA stress test and the LSI stress tests for small banks (EBA, 2019). Generally, an institute is well advised in either case to precisely describe which approach it is using and for what reasons – to be on the safe side a PIT and TTC calculation might be executed (Daldrup, 2006; Witzany, 2017).

A rating, especially if it shall be accepted and accredited by regulators, in both cases of rating philosophies has to fulfill some prerequisites (Daldrup, 2006, pp. 15-24; Witzany, 2017, pp. 18-20, 110). A rating is usually given by an associated (whole) rating system, which assigns it during the rating process and especially includes a rating function for that purpose (de Laurentis et al., 2010). Formally, a rating system "comprises all of the methods, processes, controls, and data collection and IT systems that support the assessment of credit risk, the assignment of internal risk ratings, and the quantification of default and loss estimates" (Basel Committee, 2004, p. 394; de Laurentis et al., 2010, p. 14).

Such a rating system ought to be constructed in a way that the following axioms hold truth (Brusov et al., 2021; European Commission, 2019; Izzi et al., 2011; Krahnen & Weber, 2001; Moody's, 2022; Naciri, 2015):

The rating system is (fairly) objective, measurable and verifiable, transparent, information efficient, and understandable (Daldrup, 2006, pp. 15-25; de Laurentis et al.,

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[148] Margin of conservatism

2010, p. 35). It divides the rated objects into certain classes or grades, as mentioned. Furthermore, also the best rating class has a certain probability of default assigned to it which is bigger than zero, as described in (Moody's, 2022). Otherwise, the investment would be risk-free, which is only the case for a risk-free rate or bonds issued by an "ideal government". Real governments and their bonds yet bear some risks, as one could observe during the euro-crisis in the case of the so-called "PIIGS", Portugal, Ireland, Italy, Greece, and Spain, whose bond and CDS spreads "soared" in the wake of the crisis (Bouteillé & Coogan-Pushner, 2021, p. 53; Musabegović et al., 2010). The CRR generally sets a minimum input floor at least for certain exposure classes' PDs (European Commission, 2019).

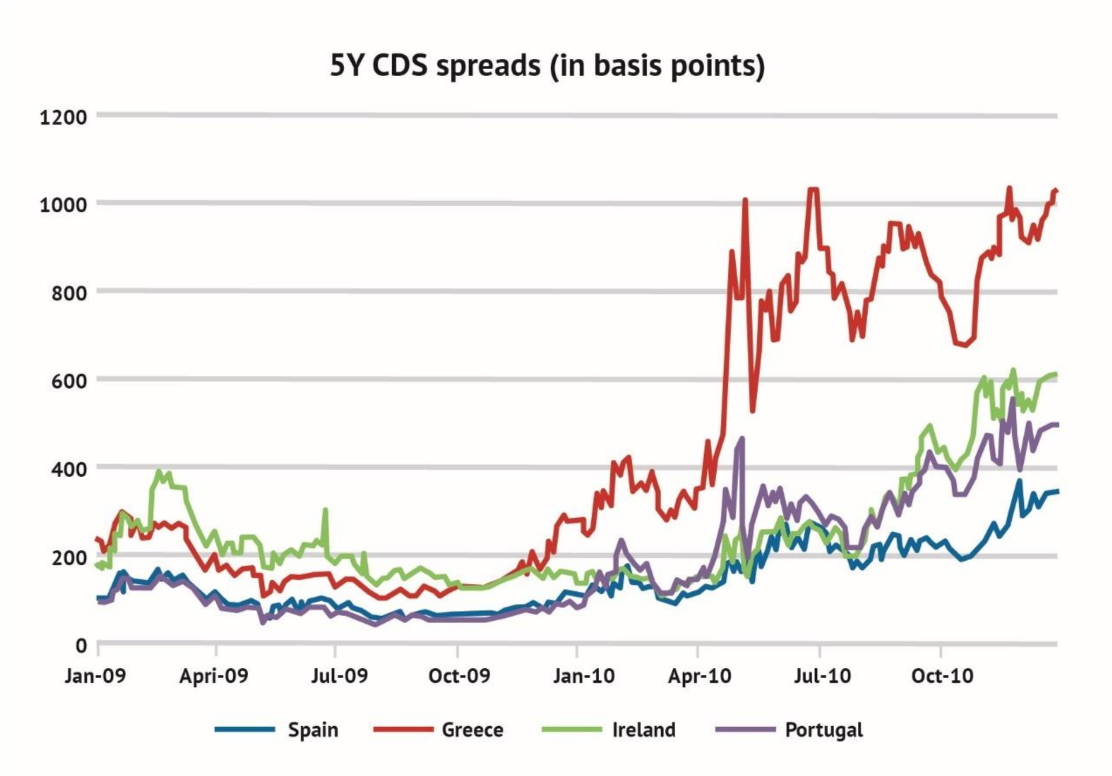

Figure 25 CDS Spreads of Spain, Greece, Ireland, and Portugal during the euro-crisis of 2010.

Source: Graph according to (Witzany, 2017, p. 167).

The system, as a further condition, has to be monotonous – as intuitively clear (Daldrup, 2006, p. 18; de Laurentis et al., 2010, pp. 176, 229). This means if the probability of default of a certain obligor ($PD1$) is lower than the one of another obligor ($PD2$), then the rating has to be better as well or at least equal if the PD is "nearly" the same (Daldrup, 2006, pp. 18-19). The later case implies that companies are in the same "bucket" or class when constructing the discrete rating function. Mathematically, that reads as

$$PD1 < PD2 \Rightarrow Rating\,1 > Rating\,2 \tag{112}$$

and vice versa, and if

$$PD1 = PD2 \Rightarrow Rating\,1 = Rating\,2 \tag{113}$$

(Daldrup, 2006, p. 19). The rating function is also transitive when comparing three or more creditors and absorbing (Daldrup, 2006; de Laurentis et al., 2010). Transitivity means when

$$PD1 < PD2\,;\, PD2 < PD3 \Rightarrow Rating\,1 > Rating\,3 \tag{114}$$

(Daldrup, 2006, p. 19). Absorbing means that if a company has already defaulted it cannot be cured for a certain time, the rating is the default rating, e.g., with the symbol or "rating code" D and stays in default (Izzi et al., 2011; Scandizzo 2016). This is a rather conservative assumption in practice and can be released under certain circumstances. For the reason of completeness, one should further keep in mind that a PD can be either calculated as counting the number of defaults per period divided through the number of all debtors, which is the standard way also used within the thesis and known as count-weighted or by taking the volumes of exposures denoted as volume-weighted and rarely used in practice (Scandizzo, 2016; Witzany, 2017).

A rating function must contain enough rating classes to differentiate between obligors and make use of "the width of the scale and segments universe", but at the same time not too many to have a representative number of obligors in each class and to stay comparable (Brusov et al., 2021; Daldrup, 2006, p. 19; de Laurentis et al., 2010, pp. 93-95). Further properties one can often observe in the financial sector are that the rating classes are not necessarily equidistant and that the ratings change over time and are not completely constant or "stable" (de Laurentis et al., 2010, pp. 17, 34, 345-346; S & P, 2016, p. 19; Scandizzo, 2016; Trueck & Rachev, 2008). A "relatively good" stability is yet important (de Laurentis et al., 2010, pp. 233, 294, 329, 347-351).

A rating system ought to further have the greatest possible flexibility such that it can (theoretically) be used to rate all past, present, and future customers without having to make fundamental changes to the system itself (Daldrup, 2006, pp. 17, 24; de Laurentis et al., 2010, p. 92-94; S & P, 2016). The rating system should therefore be flexible enough to be able to rate at least all foreseeable types of companies and types of transactions in that area it was developed for. It hence has to be granular, of adequate complexity, specificity, and complete using all possible available data at that point in time (Daldrup, 2006, p. 23-24; de Laurentis et al., 2010; Izzi et al., 2011; Scandizzo 2016).

As ratings are important for pricing and reaching out loans or bonds as well as for internal controlling purposes and a proper information flow, the assembling of ratings is a task, which has to be implemented with much rigor and on a solid data base (Brusov et al., 2021; Witzany, 2017). This data base can be an own one of a bank or credit agency or partly externally enriched, pooled – which is done, e.g., by mutual and savings banks, that often use centrally collected data and rating service providers or units within their associations (de Laurentis et al., 2010; Witzany, 2017).

In detail, each rating agency (and most banks) makes use of its own specific method to calculate its corporate ratings (Moody’s, 2022; S & P, 2016, p. 10).

These methods take into account quantitative aspects, which are mainly financial data and statistical data of an entity, as well as qualitative characteristics, which are more subjective “soft facts” like business strategy, competitive situation e. g. by applying the Porter’s five® or ESPEL® methods and considering the management qualities of a company – but also by examining the surrounding political and environmental stability of a country or company (Bouteillé & Coogan-Pushner, 2021, pp. 81-104; Daldrup, 2006, p. 7; Izzi et al., 2011; Moody’s, 2022; Oyama & Yoneyama, 2005, pp. 13-15; Porter, 1980; Porter, 1985; S & P, 2016; Scandizzo 2016).[149] Quantitative data is collected through the companies’ statements published on their websites[150], data from preferably independent analysts and financial data providers like Bloomberg® or Refinitiv® and research via official databases like the ones of national statistics bureaus or the SEC’s EDGAR.[151]

The information used for the quantitative part is mainly past or present-oriented (Daldrup, 2006; Witzany, 2017). The named qualitative aspects are forward and future-oriented (S & P, 2016, p. 5). Additionally, contextual criteria are considered as changes in an industry as a whole or specific events and current crises at the time of the rating assembling process (Eigermann, 2002; Oyama & Yoneyama, 2005, p. 13). The collection

[149] See Moody’s ® scheme in (Moody’s, 2022b).
[150] which ought to be critically double-checked however
[151] The EDGAR system is explained here: (SEC, 2021).

of all these criteria or risk factors is called a "long-list" (Moody's, 2016; Witzany, 2017, pp. 44, 51).

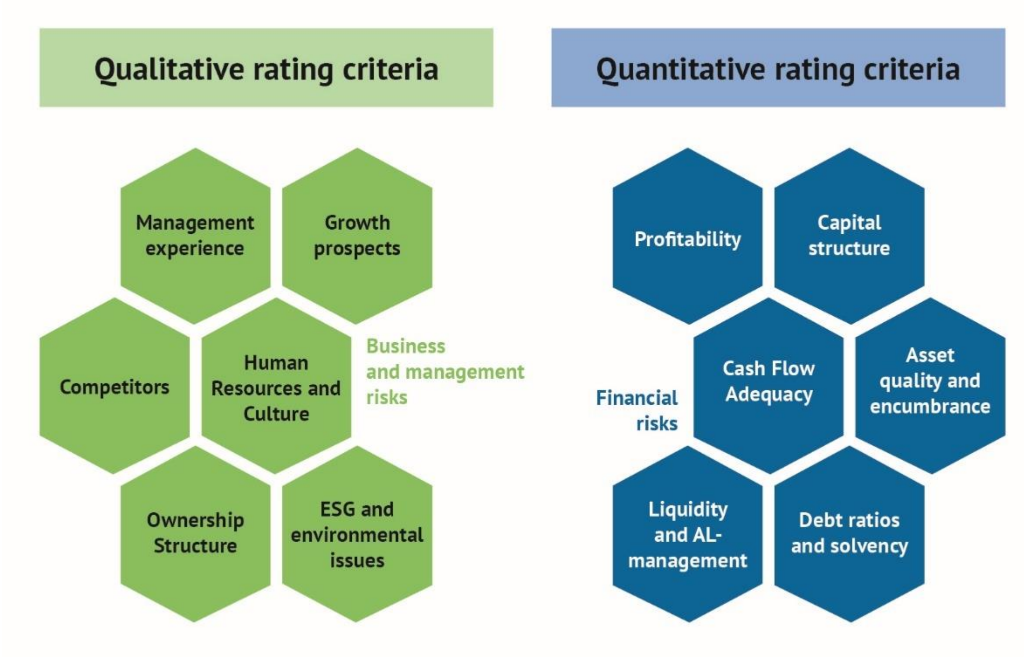

Figure 26 Quantitative and qualitative criteria for a rating.

Sources: Own collection and illustration.

While many banks in the past used creditworthiness estimates, close personal bank-client relationships and often subjective criteria as main factors for credit decisions, modern ratings use complex rating systems, which are quantitatively grounded, backtested, ideally rather objective, and additionally have a professional collateral valuation and management system integrated (Brusov et al., 2021; Chen, 2018; de Laurentis et al., 2010, pp. 93-235; Witzany, 2017). The methods in use changed accordingly, from more relationship-banking-based ones and methods like the four C's (character, capital and capacity, coverage, collateral), to more sensitive and more structured approaches like CAMELS (capital adequacy, asset quality, management, earnings, liquidity, sensitivity) by J.P. Morgan® or LAPS (liquidity, activity, profitability, structure) as shown in (de Laurentis et al., 2010, p. 39). Contrary to popular beliefs about the important role of relationship banking in the theory of banks as financial intermediaries, the increased use of automated scoring systems for SMEs has empirically neither led to fewer SME companies being financed nor higher costs – even rather independently of a bank's size or organizational structure (Berger et al., 2002; de

Laurentis et al., 2010, pp. 408-409, 424-425, 426-427; DeYoung et al., 2003; Frame et al., 2001). Therefore, apart from the already mentioned advantages like higher objectivity, verifiability, homogeneity, etc., automated scoring or more quantitative rating approaches seem to be the preferred method of choice (de Laurentis et al., 2010, pp. 408-409). The remaining criteria and factors finally selected are then given scores or grades on a certain scale, the rating system weights them according to their impact and aggregates them to an overall grade or rating, normally the mentioned baseline credit assessment (Daldrup, 2006; Izzi et al., 2011; Moody's 2022; Trueck & Rachev, 2008).

One of the first approaches to give a company a certain score influenced by macro-economic factors traces back to Altman and is hence denoted as Altman's Z-score (Altman, 1968). It can be regarded as a multi-factor approach assigning a score to the weighed summation (Witzany, 2017, p. 37).

It has further enhancements (Altman, 1989a; Altman, 1989b). Especially also the idea of "unexpected losses" was included in a 1993-version (denoted Z" model) and an extension to emerging market bonds took place (Altman, 1993; Altman et al., 1995). As will be seen in this chapter, nowadays probit or logit (logistic regression, with logarithm on odds)-based models or even AI-based approaches are used to estimate PDs and ratings – which are basically just a code for a PD range in a certain class of obligors (Daldrup, 2006; de Laurentis et al., 2010, pp. 85-91; Witzany, 2017).

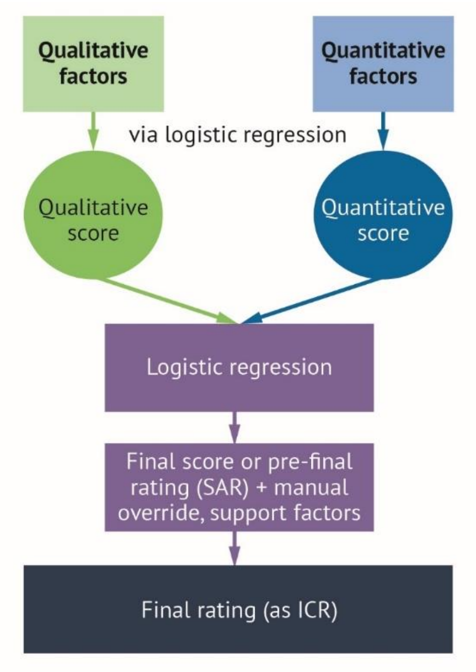

Figure 27 Qualitative and Quantitative factors are weighted, scored, and aggregated via (standard) logistic regression.

Source: Own illustration, based on (Witzany, 2017, p. 63).

By a more general level the model design process, which is beside the data an important part when initiating a model, ought to be regarded as follows:

Figure 28 Model design process.

Source: Own illustration

The process is hence getting more granular from the model and its type, followed by its concrete architecture to a specific definition of default and PD as well as LGD characteristics (Daldrup, 2006). After the model design process and the concrete creation of a rating function and system, regarded later in this chapter, the rating system needs to be initially validated (de Laurentis et al., 2010).

The rating system even has to be (re-)validated regularly, at least annually as well as on "special occasions" (Daldrup, 2006; European Commission, 2019; Scandizzo, 2016). That process might be accompanied and extended by a set of predefined early warning indicators and an internal escalation system (Witzany, 2017). Early warning indicators are important because they may substantially reduce the costs of workouts as the indicators help "limit the exposure at default and increase the safeguard of collaterals and guarantees" (de Laurentis et al., 2010, p. 396).

Qualitative validation is then especially applied to the data quality, the use cases and applications as well as the model design, often including change policies (de Laurentis et al., 2010, pp. 334-345; Scandizzo, 2006). During the validation process the system also has to be backtested or benchmarked quantitatively (Aussenegg et al., 2011; Daldrup, 2006, p. 19; de Laurentis et al., 2010, pp. 324f., 346-357; Oyama & Yoneyama, 2005; Scandizzo, 2016). Quantitative backtesting contains the application of statistical tests, whereas benchmarking means ultimately comparing the rating model with other industrial models, methods or external ratings (Daldrup, 2006, pp. 93-101; de Laurentis et al., 2010, pp. 237-256, 346-357; Oyama & Yoneyama, 2005, pp. 17-24). The backtests include accuracy or discriminatory power tests as the application of CAP and AUC or ROC and AUROC – and the corresponding ratios AR, Gini, or AUROC – to differentiate between defaults and non-defaults or rating classes (de Laurentis et al., 2010, pp. 224, 286; Engelmann et al., 2003; Oyama & Yoneyama, 2005, pp. 17-24; Scandizzo, 2016; Witzany, 2017, pp. 20-30). CAP stands for cumulative accuracy profile, AUC for area under the curve – in connection with the CAP curve (Engelmann et al., 2003; Oyama & Yoneyama, 2005, p. 45; Witzany, 2017, p. 21). AR is the accuracy ratio, sometimes also denoted as Gini (Oyama & Yoneyama, 2005, p. 19; Witzany, 2017, p. 21). ROC denotes receiver operating characteristic and AUROC[152] the area under ROC (de Laurentis et al., 2010, pp. 224, 286f; Engelmann et al., 2003; Oyama & Yoneyama, 2005, p. 46). Generally, the idea is that one relates statistical measures and odds as, e.g., "good units", in the case of the thesis correctly predicted defaulted companies, against other figures like "number of all actual good companies", "number of all companies" as in the case of CAP or to

[152] Sometimes also confusingly abbreviated as AUC.

“wrongly predicted good companies”, “wrongly predicted bad companies” the so-called “bad odds” as in case of the ROC (Scandizzo, 2016, p. 62-67, 87; Witzany, 2017). Then they might be related to ideal models/orders (Witzany, 2017, p. 22).

More precisely for the AUC method, the CAP curve is constructed by plotting the percentage of classified defaulted companies after each step against the percentage of all companies, in ascending order, and looking at the area under this curve – cf. figure 28 (Witzany, 2017, p. 22).

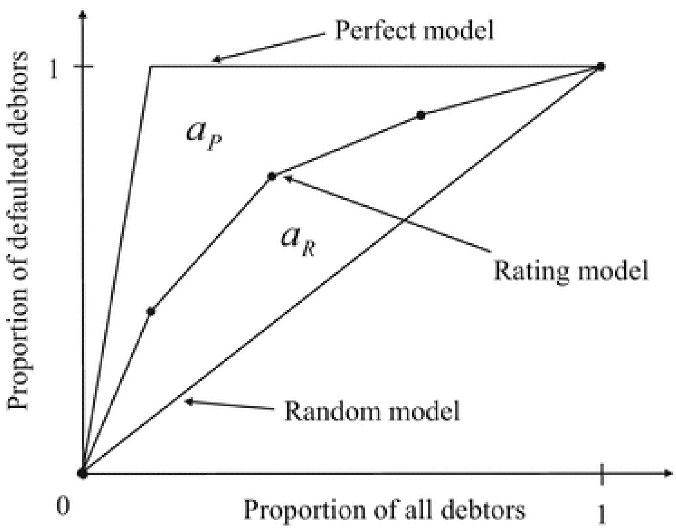

Figure 29 CAP and AUC illustration.

Source: According to (Witzany, 2017, p. 22).

From this area, ½ is subtracted, as this is the area of a random linear model – like a dice and proportional to *m/n*, where *n* is the sample size and *m* are the defaults – with slope 1 (Oyama & Yoneyama, 2005; Scandizzo, 2016, Witzany, 2017, p. 22). The resulting area is denoted as a_R (“area of the rating model”). As next step, a perfect model is considered which means a model for which in the first step 1/*m* of all defaulted companies are correctly detected (and 1/*n* of all companies looked at), where *m* denotes the total number of defaulted companies, *n* the total number of all companies (Oyama & Yoneyama, 2005; Scandizzo, 2016). Then in the second step hence 2/*m* are correctly classified (and 2/*n* looked at), … straight up to *m/m* = 1 in the *m*-th step, when *m/n* debtors were looked at. This yields a straight line from $0 \leq t \leq m/n$ with slope *n/m* and intersect zero followed by a constant one for $t \geq m/n$ (Oyama & Yoneyama, 2005; Scandizzo, 2016; Witzany, 2017, p. 17).

Again, if considering the area under that curve of the perfect model the random model, ½, is subtracted resulting in the area a_P. In the next step, the accuracy rate (AR) is just calculated by the division of a_R and a_P.

$$Accuracy\ Rate\ (AR) = \frac{a_R}{a_P} \tag{115}$$

(Oyama & Yoneyama, 2005; Scandizzo, 2016). It lies between zero and one, and the closer it is to one the better the employed model performs (Witzany, 2017, p. 22).

In the case of *AUROC*, one plots the correct default classifications (already a ratio), hence the true-positive-rate (*TPR*), mentioned before and sometimes also labeled "hit rate", against the false-positive-rate (*FPR*) of wrong classifications of default, also known as "false-alarm-rate" (de Laurentius et al., 2010, pp. 224, 286; Oyama & Yoneyama, 2005; Scandizzo, 2016). The resulting curve is then the receiver operating characteristic (*ROC*) and the area underneath the *AUROC* (Oyama & Yoneyama, 2005; Scandizzo, 2016):

$$ROC = \frac{TPR}{FPR} \tag{116}$$

Then it is also again compared (normally without the subtraction of the area of a random model – ½) to the optimal model with area one and hence the area under the *ROC* curve divided by one, which is still *AUROC* (de Laurentis et al., 2010, pp. 285-286; Witzany, 2017, p. 23). The receiver operating characteristic approach stems from the usual definitions of sensitivity (or recall) and specificity in the confusion matrix of sampling statistics (Witzany, 2017, p. 28). In that "language" it can be expressed as

$$ROC\ = \frac{sensitivity}{1 - specificity} \tag{117}$$

(Witte & Witte, 2010).

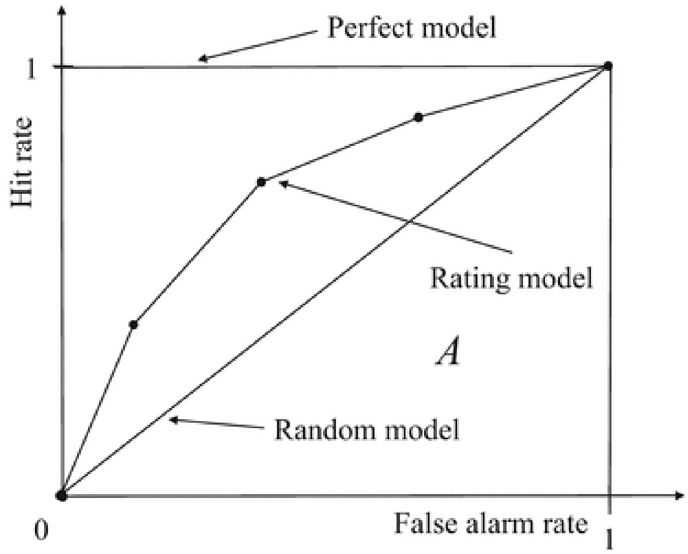

Figure 30 ROC and AUROC illustration.

Source: According to (Witzany, 2017, p. 23).

It is also possible to give probabilistic definitions of the parameters AR and $AUROC$ first, thereby directly receiving the relationship

$$AUROC = ½ (AR - 1) \quad (118)$$

and then show the equivalence to the geometric definition given before as proven in a straightforward way in (Witzany, 2017, pp. 24-25).

The definitions of AR and $AUROC$ given above are numerically approximated in reality, as there is only a finite sample of rated obligors where defaults were observed (validation sample) and not the complete (whole) population sample (Witzany, 2017, p. 25). The accuracy rate or the area under receiver operating characteristic measured on the training sample (historic sample where the rating system is derived from) are usually nearer to one than the ones calculated on different samples, hence a more conservative out-of-sample validation is preferred, as opposed to validation on parts of the training sample (Witzany, 2017, p. 25-27). Rating providers such as Standard & Poor's normally achieve high accuracy ratios (Ginis) of roughly 90 %[153] and even 86 % during the COVID-19 crisis in 2020 (S & P, 2022). Given the estimated accuracy ratio or Gini coefficient, Witzany's advice is that "one should also look at the confidence interval for

[153] See for 2020 (S & P, 2021) and for 2021 (S & P, 2022).

the real coefficient AR on the confidence level α, using an estimator given", as by Whitney-U or asymptotic normal ones (de Laurentis et al., 2010; Engelmann et al., 2003; Greene, 2003; Witzany, 2017, p. 26). This might hence also help in selecting discriminatory relevant and statistically significant factors for a rating system as will be shown later when the univariate analysis is regarded.

Furthermore, as in other statistical contexts, one can compare two rating systems with accuracy ratios *AR1* and *AR2* – which might be correlated and have interlapping confidence intervals – by utilizing a *t*-statistic test (Witzany, 2017, p. 27). Other discriminatory measures as the weight of evidence (WoE), the use of ordinary analysis of variance (ANOVA) or the information value (IV) exist but are considered less important and are rarely used in the realm of discriminatory analysis in practice in the financial industry (Scandizzo, 2016; Witzany, 2017, pp. 48 – 51).

As the next step in the whole validation process, the correlations between the chosen risk factors are measured and tested, e.g., via Spearman's rank correlation or Kendall's tau (de Laurentis et al., 2010, pp. 233-235, 376; Oyama & Yoneyama, 2015, pp. 17-24). That is apparently important to see which risk factors impact each other and in which "strength" and "direction". Highly correlated ones (e.g., more than 70 %) might be omitted (Oyama & Yoneyama, 2015). Furthermore, a principal component or factor analysis is often utilized in regard to correlation structures (de Laurentis et al., 2010, pp. 95, 98-113).

Then as another key step, the calibration is tested, i.e., the ability of the model to assign accurate PDs to each rating in the longer term (de Laurentis et al., 2010; Oyama & Yoneyama, 2015). That is executed via statistical tests. Utilized are standard Binomial, Spiegelhalter, or Hosmer-Lemeshow tests or nonparametric tests like a Wilcoxon rank sum test in the independent or Wilcoxon rank sign test in the dependent case (Daldrup, 2006, pp. 93-101; de Laurentis et al., 2010; Scandizzo, 2016, pp. 70-73). Often, also comparison mappings from internal (and its master scale) to external rating systems are performed (Bouteillé & Coogan-Pushner, 2021). In practice, the Binomial test or Hosmer-Lemeshow test, which ought to be preferred, are most frequently used (Witzany, 2017). Backtesting calibration is one of the most crucial components of backtesting besides discriminatory (accuracy) tests (Scandizzo, 2016, p. 69). For a rating or PD backtest the before-mentioned "traffic-light"-approach can be again utilized when deciding about the denial or acceptance of a model (Scandizzo, 2016, p. 73; Tasche, 2003).

Besides backtesting it is also often convenient to compare different (nested) models, i.e., benchmarking them by comparing their log-likelihood ratios, the Akaike information criterion (AIC), or the Schwarz information criterion (SIC), also known as the Bayesian information criterion (BIC), and testing if they are for instance extensions of each other with additional variables (de Laurentis et al., 2010; Oyama & Yoneyama, 2015, p. 19; Scandizzo, 2016). General comparing concepts like goodness-of-fit tests are also available (Izzi et al., 2011).

The last validation area concerns the stability of a rating system, which is tested for instance via causal relationships, transition matrices, tools like stability indices or the Kullback-Leibler divergence and indicates the consistency of the system over time (Scandizzo, 2016). A rigorous execution of the quantitative validation is a must to avoid pitfalls and impreciseness (Aussenegg et al., 2011).

The challenged models and their results have to be finally reported and documented exactly. Model risks themselves have to be kept in mind as well, quantified in “quantified heatmaps”, scorecards, or similar concepts, and the results considered with a certain a model risk buffer, known in risk management as margin of conservatism – MOC (Breinich-Schilly, 2021; Hull, 2015, pp. 587-605; Reinwald, 2022b).

Three dimensions of quantitative validation – Implication for PD model validation

	What is measured?	Typical Tests	Implication for validating a model
Discriminatory Power	Ability to discriminate between defaults from non-defaults (and between rating classes)	- Range of accepted statistical tests e.g. (ROC, AR, Gini, K-S), based on sample of goods and bads - When external indicators available, test with e.g. ext. agency rating, int. expert ratings - Rank correlation or matching matrices	Validation/comparison of ranking ability (e.g. versus agency or internal expert ratings)
Accuracy (Calibration)	Ability to assign accurate long-term PDs to each rating (band)	- Comparison of Actual vs. Predicted PD – is performance within expected error bounds? - Comparison to PDs implied by external agency ratings - Cross comparison across portfolios to asses accuracy of relative rating profile	Comparison across portfolios should be reviewed to get an expert assessment that absolute PDs are reasonable across and within segments
Stability	Stable, causal relationships between factors and credit quality over time	- Statistical tests would rely on the ability to monitor model performance over time, and ensure model continues to show discriminatory power in line with expectation, and that model factors continue to be significant and predictive	Need to demonstrate consistency of rating assignment

Figure 31 Three dimensions of quantitative ratings.

Source: Own illustration, in line with (Farooq, 2016; Witzany, 2017).

3.2 External and Internal Rating Processes

In contrast to a more classical creditworthiness check of companies by their house banks to decide about loans, which is generally a detailed due diligence of the balance sheet, financial statement and P & L, a rating should also include some future-oriented or at least current information (Izzi et al., 2011; S & P, 2016, pp. 5, 11). The following description deals with the most frequent case of rating a company – nearly all parts however can be transferred for an application to other entities like states or individuals as well. The latter case is often referred to as scoring as mentioned when completely using quantitative data, normally in a fully automated way and often in retail segments (Bouteillé & Coogan-Pushner, 2021, p. 59). Firstly, the entire process which is followed by a rating agency is illustrated – most parts again can be transferred to the use of internal ratings as in (Daldrup, 2006; de Laurentis et al., 2010, pp. 93-235; Oyama & Yoneyama, 2015). Apart from the agency-committee-company interaction – the bank's internal data and history is used instead – and the missing presentation to the rated company and non-publishing of the rating, the process stays the same.

As already explained, there are external and internal credit ratings – external ones are drafted by a regulated and registered credited rating agency abbreviated often as CRA (Daldrup, 2006, pp. 13-15; European Commission, 2013). These private companies are making their profit by rating other companies, states and other municipalities, supra-nationals and state-like actors, complete branches, and single financial instruments (Naciri, 2015; Witzany, 2017, p. 35). There are only a few accredited and registered rating agencies in the Western hemisphere, and mainly relevant are the American companies Standard & Poor's Global Ratings® (S & P Global Ratings®), Moody's Investors Service®, and Fitch Ratings®[154] (Naciri, 2015).[155][156] In the EU they are called external credit assessment institution (ECAI, sometimes external credit rating agency ECRA) and

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[154] Fitch was partly French-owned by Fimalac® until Hearst® took over the remaining 20 % stake in 2018. See, e.g., the press statement on (Hearst, 2018).

[155] The three of them own a combined market share of more than 95 % in the US and similarly globally (SEC, 2022, p. 23).

[156] After the great recession and the perceived overarching power and American oligopoly of rating agencies there was an effort in the EU to set up a foundation-based rating agency, but the market was not very comfortable with this idea, and the effort largely failed. The plan of a European CRA is described in (Spiegel, 2012). A further important rating agency in North America is the Canadian Dominion Bond Rating Services (DBRS).

are supervised by the ESMA – the European Securities and Markets Authority (ESMA, 2014; ESMA, 2017; ESMA, 2021; Witzany, 2017, p. 110).[157]

In the US they are regulated by the Securities and Exchange Commission's Office of Credit Ratings (OCR) and called nationally recognized statistical ratings organizations NRSROs (SEC, 2022b; SEC, 2022c). There are other agencies that are just partly accredited for certain types of debt – the so-called exposure classes – in the US and the EU, e.g., for accreditives and foreign payments or special loans, as Euler Hermes, Scope, or Creditreform (Bouteillé & Coogan-Pushner, 2021, pp. 55-62).[158] Rating agencies normally use symbols or rating codes to express their rating judgment, famous are, e.g., the triple-A, "AAA", by S & P ® or symbols like baa1 in the Moody's ® code (Bouteillé & Coogan-Pushner, 2021, p. 57; Moody's 2022; S & P, 2016, p. 13). The aim is to inform investors about the debtors and to provide for an open and transparent market and trustworthy credit pricing (Daldrup, 2006).

Rating agencies possess a substantial amount of power as the use of external ratings is included in legal requirements as the CRR III (as regarded) and more than 34 trillion $ in securities on the financial markets are judged by these agencies (Witzany, 2017, p. 35).

Yet, as most ratings are paid for by the companies themselves and unpaid, free ratings known as unsolicited ratings without any order are less lucrative for the agencies, some studies suggest that the ratings ordered by a company tend to be slightly better than unsolicited ones, as it may serve as an incentive for re-ratings by the same agency and conflicts of interest may arise (Bouteillé & Coogan-Pushner, 2021, pp. 55-62; Daldrup, 2006; de Laurentis et al., 2010, p. 40; Lucas, 2008; Podkul, 2019a; Podkul, 2019b; S & P 2016, p. 12; Witzany, 2017, p. 35). Furthermore, as most of the companies are located in the US and are used to the U.S. capital market based open financial system, some studies suggest a weak bias toward that system and against the European or Japanese bank-loan centered system of financing and funding companies (Bartels, 2019; Behr & Güttler, 2004, pp. 109-112; Ichiue, 2006; Oyama & Yoneyama, 2005). Having rated a company, the process often involves "follow-up ratings" and an "outlook" which can be negative, positive or neutral, and the company stays on the "watchlist" for an "upgrade/downgrade" of further steps (S & P, 2016, pp. 11, 18). These rating steps are called "notches" (S & P,

[157] Different methodologies are shown by the ESMA on (ESMA, 2017).
[158] See again (ESMA, 2022).

2016). The processes are often similarly executed by banks when they use internal ratings. The largest rating agencies use the following scale and rating symbols:

Moody's		S & P		Fitch		Verbal description
Long Term	Short Term	Long Term	Short Term	Long Term	Short Term	
Aaa		AAA		AAA		Prime
Aa1		AA+		AA+		
Aa2		AA		AA		
Aa3		AA-	A-1 +	AA-	F-1 +	High Grade
A1		A+		A+		
A2	P1	A	A-1	A	F-1	
A3		A-		A-		Upper Medium Grade
Baa1	P2	BBB+	A-2	BBB+	F-2	
Baa2		BBB		BBB		
Baa3	P3	BBB-	A-3	BBB-	F-3	Lower Medium Grade
Ba1		BB+		BB+		
Ba2		BB		BB		
Ba3		BB-		BB-		Non Investment Grade Speculative
B1		B+		B+		
B2		B		B		
B3		B-	B	B-	B	Highly Speculative
Caa1		CCC+		CCC		Substantial risk
Caa2		CCC				
Caa3		CCC-		CC		Extremely speculative
		CC				
Ca		C	C	C	C	In default with little prospect for recovery
		SD		RD		
C	Not prime	D	D	D	D	In default

Figure 32 Rating codes.

Own illustration, according to http://www.swiss-rating.agency/rating-scales-codes/

Importantly the “investment grade” section, which indicates rather safe debt, is including the BBB- (by S & P methodology) rating class, while everything below that class is non-investment grade and often referred to as “junk” (Bouteillé & Coogan-Pushner, 2021, pp. 55-57; Everling & Bargende, 2005; Moody’s, 2022; Wieben, 2004). Some institutions, e.g., pension funds and insurance companies, are obliged by law to invest in investment-grade bonds solely (Behr & Güttler, 2004; Bouteillé & Coogan-Pushner, 2021, pp. 145-151).

Figure 33 shows a differentiation of rating codes in terms of the time horizon of a company considered, or the maturity of financial instruments issued by the company, and that motivates the distinction of short-term ratings (P-1 to P-3 at Moody’s ® and slightly more granular A-1+ to A-3 at S & P ®), generally debt due in less than 12-13 months, and long-term ratings (Moody’s, 2022c). Rarely medium-term ratings for a range of 3-5 years are published (Izzi et al., 2011). It is also important to look at what kind of debt is rated. Is the company itself rated or normally equivalently its Senior Unsecured bonds or loans, or also other debt like subordinated/junior debt or covered bonds or collateralized loans (Moody's, 2022). If the loan or bond is subordinated and hence lower in the insolvency regime, the ranking of debtors, or classified as non-preferred senior unsecured, which is a "middle-stage" between preferred senior debt and subordinated debt, the rating is generally worse (Bouteillé & Coogan-Pushner, 2021; Moody's, 2022). In terms of a collateralized loan or covered bond the unsecured exposure (part) is naturally lower or even zero when fully (over)collateralized, and hence the recovery rate and thus the rating, especially for the LGD, is higher.

As presented before, in most cases external ratings are ordered by a client (S & P, 2016, p. 12). The rating is often henceforth published together with its rationale, denoted a public rating (Moody's 2022; S & P, 2016, pp. 11, 21). Unpublished but still used ratings are then private ratings. The (public) rating process can be viewed in diverse ways yet as to illustrate a standardized way the ICRA® rating process (of the Indian ICR-Agency®, a subsidiary of Moody's ®) is directly shown below.

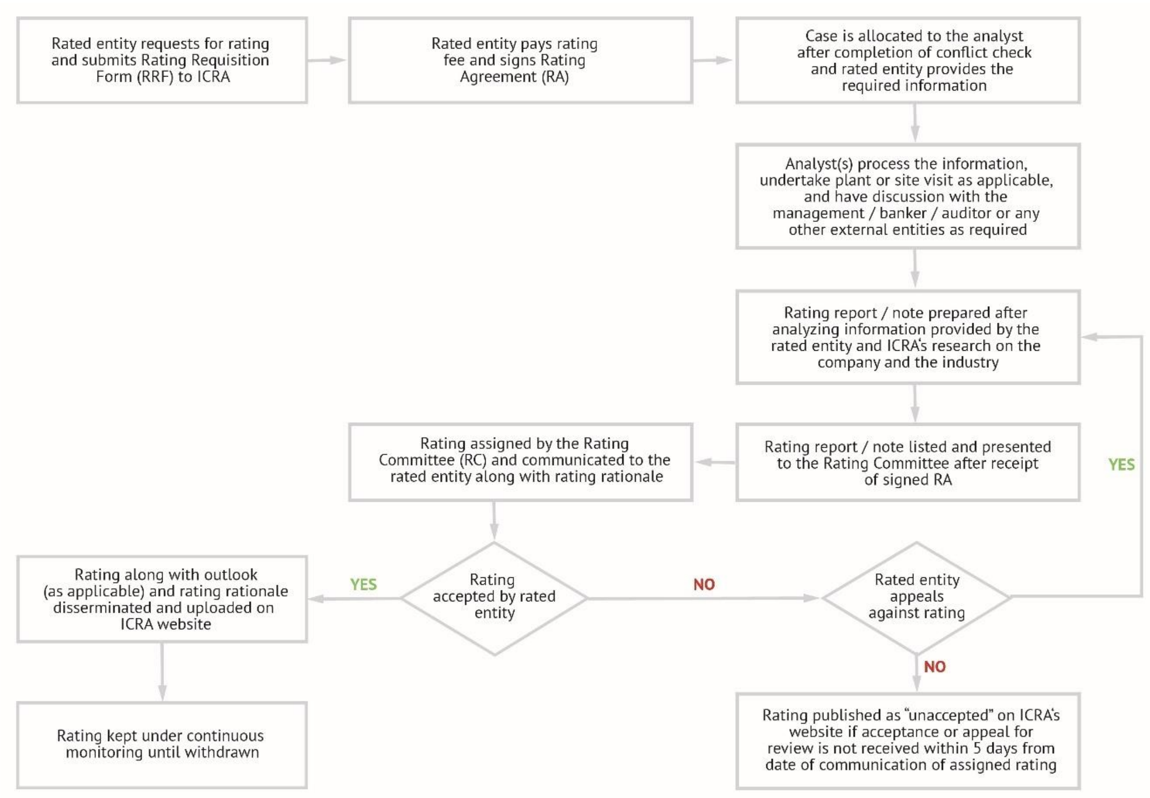

Figure 33 ICRA® rating process.

Source: Own illustration in accordance with www.icra.in (Retrieved Mai 24, 2022), copyright by: ICRA® India.

As the details of the rating processes and methodologies of major agencies are usually kept as business secrets there are some regulatory and transparency requirements in place, which force the ECAIs to publicly disclose their methodologies to some extent and some agencies even publish more information than that to (re-)gain the trust of the public and of investors (BCBS, 2000a; Daldrup, 2006, p. 6; de Laurentis et al., 2010; Moody's, 2022b) If the input parameters and the corresponding weights of a rating are roughly known, it is quite usual that the rating function behind it is emulated and tested ("copied") by others, who then build a "shadow rating" (Engelmann & Rauhmeier, 2006; Everling & Bargende, 2005; Izzi et al., 2011; Wieben, 2004; Witzany, 2018, pp. 36, 66). Even though the methodologies of rating companies are often rather similar, the comparability has its limits when it comes to different definitions in use, different populations taken by the agencies and varying conceptual details (de Laurentis et al., 2010, pp. 53-56).

Generally, an agency begins with assembling a team of analysts and with the pre-analysis including information collection from public sources and in its own databases, also used for a peer group review and comparison, in addition to the information offered by the company itself (Eigermann, 2002, p. 32; Everling & Bargende, 2005, pp. 263-264; Izzi et al., 2011; S & P, 2016, pp. 10-11; Wieben, 2004, p. 92). Following a rather standardized bottom-up approach, the team first estimates the home country and branch risks and ratings, hence the environment a company is operating in (Daldrup, 2006, pp. 6, 30, 33-41; Bouteillé & Coogan-Pushner, 2021, pp. 55-56; de Laurentis et al., 2010, p. 43; S & P, 2016 pp. 10-11). Then it analyzes the company-specific risk, the idiosyncratic one, which can be divided again into individual business risk which is, as seen before, mainly depending on the strategy, management and competitive environment and is hence future-oriented and financial risk, which is mainly objective and based on financial figures, planning scenarios and hence more backward or present-oriented (Caouette et al., 2008; de Laurentis et al., 2010; Everling & Bargende, 2005; Moody's, 2022; Witzany, 2017, pp. 33-34). Adding the peer group analysis the analysts derive a set of checklists, questionnaires and a first and preliminary rating assessment, which is then discussed with the company's management – in most cases at least with the CEO and CFO – in an analysis talk or meeting (Bouteillé & Coogan-Pushner, 2021, pp. 55-63; S & P, 2016, p. 11). Often that meeting might be accompanied by further quantitative information presented by the management and a tour through the company's (production) facilities.

The exchange is subsequently followed by the assembling of a final rating recommendation by the analysts, who are further trying to compile an outlook for the future of the company (Moody's, 2022). The analysts are considering especially the following areas, as shown in (Bouteillé & Coogan-Pushner, 2021, pp. 81-104; Caouette et al., 2008; Daldrup, 2006, pp. 7, 33-40; Eigermann, 2002; Everling & Bargende, 2005; Moody's, 2022; Oyama & Yoneyama; 2005, S & P, 2016, pp. 16, 18; Witzany, 2017, pp. 33-34):

- Financial figures (financial statement and disclosures, balance sheet, cash flow and income statement, P & L/earnings, and other KPIs)
- Financial management
- Production
- External stakeholders and relations (clients/customers, sales, and suppliers)
- Shareholders
- Overall management and strategy
- Business and risk controlling
- Employee capabilities, satisfaction and education, and a company's attractiveness for young talents
- The use of new technology, IT, including cyber security, AI (artificial intelligence), and automatization

Rating agencies like Standard & Poor's also utilize the business and financial risk side division – dating back to Modigliani and Miller – while considering for the latter one mainly profitability ratios from historical and projected operations, coverage ratios as cash flow from operations divided by outstanding principal and interest which has to be paid, leverage ratios, and further the common "quick and current liquidity ratios" (de Laurentis et al., 2010, p. 43; Modigliani & Miller, 1958).

All these factors are regarded in a similar fashion when using internal ratings, as seen before (Daldrup, 2006). They are (again) given a kind of score or assigned value on a certain predefined scale then. The score is weighted by the impact and importance of the respective factor and finally, they are assembled into an overall rating (score). In some circumstances the score also includes supporting factors/affiliates or country caps, and the rating is finally adjusted by an expert overwriting and possibly a rating transfer (Daldrup, 2006; Everling & Bargende, 2005; Izzi et al., 2011; Moody’s, 2022; Oyama & Yoneyama, 2005, pp. 13-15).

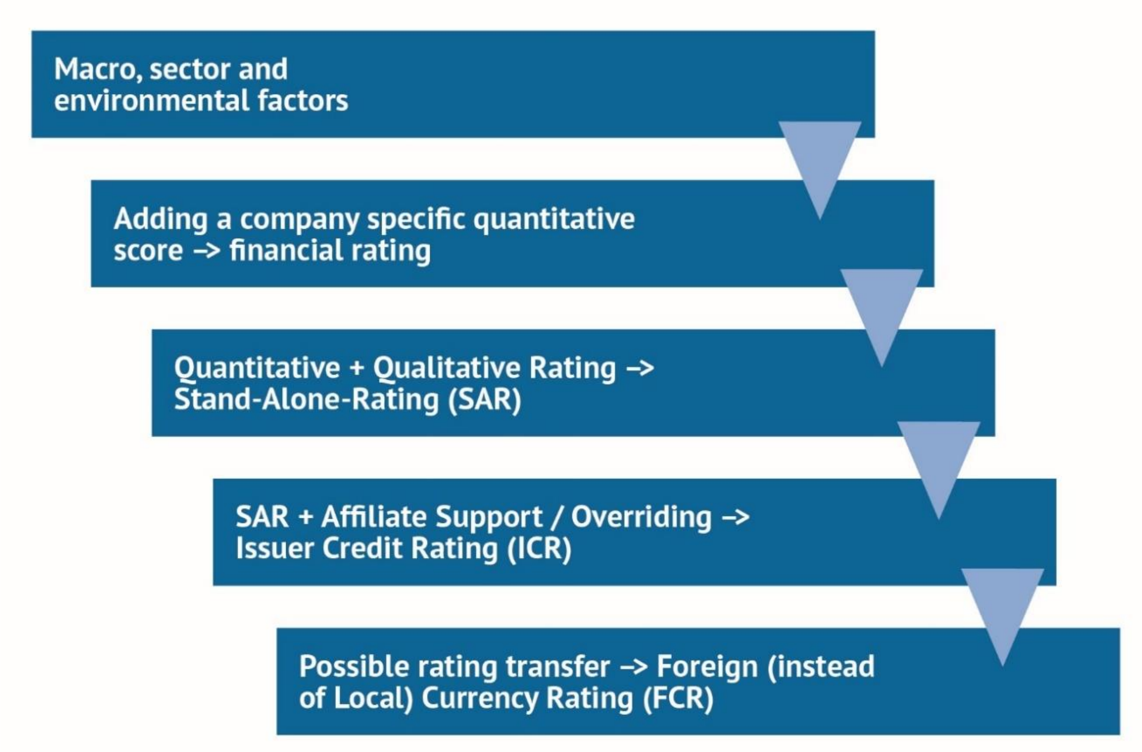

Figure 34 Rating process as a whole, e.g., within a rating agency when rating a financial institute.

Source: Own illustration.

In rating agencies normally a rating committee finally decides about the rating decision, often including five to seven voting members (de Laurentis et al., 2010, p. 44; Moody’s, 2022). The result is transmitted to the company at the end, and it can decide whether to accept it or not (Moody’s, 2022; S & P, 2016, pp. 11, 21).

In the event of a general rejection by the company, the rating process is terminated, and the rating is not published (Moody’s, 2022). Yet in many cases in practice, there are “unaccepted published ratings”, especially for re-ratings (Wieben, 2004). If the company objects to the result of the rating process because new information regarding the company's development may be available, a new analysis and evaluation of the company are carried out, taking into account the updated information, and a new rating assessment is adopted by the Rating Committee (Everling & Bargende, 2005; Wieben, 2004; S & P, 2016).

If the rating assessment is accepted, it is published by the rating agency. At this point, however, the agency's activities are not finished yet, but the company as well as the development of the industry and the market are continuously monitored, such that an up- or downgrade of the rating assessment may occur (Bouteillé & Coogan-Pushner, 2021, pp. 55-63; S & P, 2016; Witzany, 2017). Regarding that context, the companies under review are expected to submit significant changes, monthly and quarterly reports as well as annual financial statements to the rating agency (Izzi et al., 2011; Naciri, 2015). In addition, new meetings are usually held at least annually between the company’s management and the agency’s analysts to discuss the company’s development and current industry trends (Witzany, 2017). If the agency discovers signs of a change in the credit rating of a company under its monitoring, a review process is initiated, and the company is placed on the so-called “watchlist” as described above (S & P, 2016).

The use of (external) ratings as an indicator function is however not always (completely) fulfilled (Witzany, 2017). In many cases, the downgrading of a company is time-lagged and appears when the worsened creditworthiness and quality of a company is already obvious and known throughout the market, often even in a cycle through previous upgrades from other agencies, hence already included in the market prices (quotes) and thus in many cases not even influencing them further (Alsakka & Gwilym, 2010; Kou & Varotto, 2008; Oyama & Yoneyama, 2005; Witzany, 2017). Especially between rating cycles of normally a year that might be the case and one should add further, timely information published by the company (ad-hoc reports and news, business figures), analysts (reports and recommendations like the hold/buy/sell votes), regulators, or markets as important information included in credit spreads and credit default swap premiums of a company, to get a full picture (de Laurentis et al., 2010). Witzany describes credit spreads and their role as early warning indicators, e.g., in the case of the euro-crisis (Witzany, 2017, pp. 93-94, 161, 167).

Furthermore, the rating updates may add further pro-cyclicality to the market and economic cycles especially through downgrades during recessions and might in extreme cases even contribute to credit crunches and further defaults (Auh, 2015; Kiff & Kisser, 2018).

The incentives for companies to seek official ratings from agencies one hand consist of the fundamental possibility of gaining access to the capital markets and on the other hand the opportunity of being able to obtain more favorable funding conditions in the capital markets through a favorite rating (Aktan et al., 2019; Daldrup, 2006; Naciri, 2015). A company's investment decisions and capital structure are significantly influenced by ratings and rating changes (Aktan et al., 2019; Kisgen, 2019).

However on the contrary, in less developed economies many companies might not consider getting a rating as useful as it may not serve as an advantage without developed capital markets or without the attraction of investors, or some companies simply cannot afford a (constant) rating fee.

In analogy to external ratings, internal ratings have the goal of rather objectively assessing the creditworthiness of companies, with internal ratings showing the creditworthiness of bank borrowers and external ratings generally showing the creditworthiness of bond issuers on the capital market (Daldrup, 2006, pp. 13-15; de Laurentis et al., 2010; Oyama & Yoneyama, 2005). In contrast to external ratings, however, they do not increase market transparency due to their internal and non-public use by definition, nor do they serve potential investors as an indicator of the future viability of a company (Daldrup, 2006, pp. 13-15). Rather, they represent the central component in a bank's overall credit risk management (Brusov et al., 2021; de Laurentis et al., 2010). Internal ratings are mainly used in the context of lending as a decision-making aid for the acceptance and prolongation of loan applications and for the approval and review of credit lines, furthermore, to allow differentiation and prevent adverse selection of debtors (Daldrup, 2006, pp. 13-15; Oyama & Yoneyama, 2005).

In the non-American world, banks will predominantly use the approaches based on internal ratings, as only a few larger companies, especially in the European market, have an assigned external rating and thus the necessity arises for banks to implement their own internal rating systems (de Laurentis et al., 2010; EBA, 2021c, pp. 6-7; Elsäßer, 2015).

In addition to these areas of application, which in principle relate to individual borrowers, internal ratings are also a prerequisite for a number of credit risk models for quantifying portfolio risk as illustrated later in the thesis (BCBS, 1999a; Daldrup, 2006; Gordy, 1998). The goal of these credit risk models is to quantify the credit portfolio risk by taking into consideration possible changes in creditworthiness during the lifetime of the loan. Internal ratings tend to be less complex and cheaper compared to external ratings also being proportionate to the volume of a bond emission, many millions in denomination at least, versus an in most cases considerably smaller bank loan (Daldrup, 2006, pp. 13-15). Internal rating systems are considered in the following. If an internal rating system must also have a component for a transaction rating for LGDs, which is volume-driven, in addition to a credit rating for PDs relating to the frequency of defaults and rating migrations-driven-relative frequencies, internal rating systems should adhere to the basic requirement of two-dimensionality[159] (Daldrup, 2006, p. 16; Oyama & Yoneyama, 2005, p. 8; Witzany, 2017, p. 112). These two-dimensional systems tend to be much more precise, granular and transparent, which is required by the Basel accords (BCBS, 2019). They have to further fulfill all the criteria defined at the beginning of the chapter as well, like neutrality, completeness, objectivity, and granularity (Daldrup, 2006, pp. 15-24; de Laurentis et al., 2010). At least seven living rating classes and one default class have to exist, and at least a certain minimum PD for some classes like 0.03 % and a PD for defaulted exposure of one have to be included (Daldrup, 2006, p. 17; European Commission, 2019, §160; Oyama & Yoneyama, 2005; Witzany, 2017, p. 112). The next graph shows and resumes an internal rating process in a bank with the development of a rating function. As already explained the validation process and the available tools concentrate on further steps.

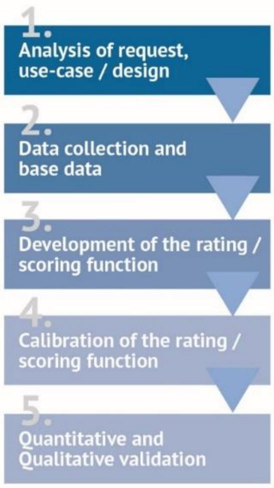

Figure 35 Setting up an internal rating function.

Source: Own illustration.

[159] For non-retail exposures it is even required (Witzany, 2017, p. 112).

Generally, the process follows a typical analysis and design, data development and implementation, and test feedback loop (Daldrup, 2006; Witzany, 2017). The steps in the figure above are the following.

1. Requirement analysis and use cases/design
2. Data collection and cleansing, and generating a data set and sample
3. Development of the rating function
    - 3.1 Collection of quantitative risk factors –long-list
    - 3.2 Univariate analysis: discriminant analysis and correlations
    - 3.3 Multivariate analysis
    - 3.4 Selection and aggregation of the risk factors
    - 3.5 Soft facts, qualitative criteria

3.6 Support and override, and overall risk function

4. Calibration of the rating model, assignment PDs to score or rating values
5. Quantitative and qualitative validation, using discriminatory power and calibration, stability tests in the first, quantitative case and a test of model design, use cases and applications, and data quality in the second, qualitative case

The requirement analysis (RA) and the subsequent set-up of use cases and the rating system's design include further steps. At first, an inventory of the existing risk classification procedures of the bank has to be created (Daldrup, 2006; Witzany, 2017). In the next step, areas of potential application (denoted as use cases) for the rating system have to be identified, the use cases are then written down precisely with the help of the employees intending to apply and use the model later (Oyama & Yoneyama, 2005, pp. 32-34). These are the basics for the requirement analysis which also, besides requirements from the users, include regulatory requirements for rating systems as mentioned above, e.g., at least seven living classes and one default class. That is manifested in the CRR and CRD. The analysis also includes the requirements from the risk control unit for instance in terms of consistency with internal guidances and existing models, the internal audit team and eventually from the management board (Daldrup, 2006; European Commission, 2019; Oyama & Yoneyama, 2005). The requirements are documented in a requirements catalogue, streamlined and transformed into a form that is implementable (Izzi et al., 2011). A methodological design of the scoring or regression procedures and their calibration is following (de Laurentis et al., 2010). For the final calibration the institute's "master scale" is utilized, a single, integrating, binding reference rating scale of an institute, to which every subsystem needs to be transformed (Daldrup, 2006). This is accompanied by a review of already existing internal documentation, like the development documentation, general requirements for risk models and former validation results (Bouteillé & Coogan-Pushner, 2021; Daldrup, 2006; Izzi et al., 2011).

Expert interviews with representatives of the market and back-office are conducted, with model development and validation units integrated into the process as well (Daldrup, 2006). Other stakeholders are questioned regarding their expectations of a new risk classification procedure (RCP). Finally, the determination of the design and methodology, the selection of relevant use cases and overall requirements as well as first possible rating criteria for a risk classification procedure to assess the rating takes place (de Laurentis et al., 2010; Izzi et al., 2011). These actions are well documented as part of the creation of a framework concept.

The second overarching step concerns data collection (Oyama & Yoneyama, 2005, pp. 25-27, 37-38). The identification of high-quality data sources on creditworthiness relevant factors for the rating function is crucial (Oyama & Yoneyama, 2005). As mentioned before external as well as internal sources should be exploited (Oyama & Yoneyama, 2005, pp. 25-26). Therefore, the identification of internal institute data and if applicable data from peer groups have to take place, followed by a comparison of information sources (de Laurentis et al., 2010). A limitation of the relevant entities for the development sample is a further necessity and achieved, e.g., by considering a debtors sector and market cap, rating segment or exposure class[160] (Witzany, 2017). The last sub-step is the final preparation of a development sample (often labeled as development RDS [161]) for further analysis, including the standardization of key figures, the determination of a specific reference period for historical data, and categorization of qualitative information (Izzi et al., 2011; Oyama & Yoneyama, 2005, pp. 25-27).

[160] In (European Commission, 2019, §147) at least seven exposure classes are considered for internal models, namely central governments and central banks, financial institutions, corporates, retail, equity, and securitizations (and other assets). Keep in mind that also the equity class is hence equipped with some default probability, exposure size, and potential loss given default. For the SA more exposure classes are differentiated, as in (European Commission, 2019, §112).

[161] Development record data set (RDS)

The main part of the rating system set-up process is the development of the rating function (de Laurentis et al., 2010, pp. 142-324; Witzany, 2017, pp. 36-63).

At first, as stated before, a collection of quantitative risk factors and key figures for a long-list is executed, containing all factors that might be valuable for the rating (de Laurentis et al., 2010, pp. 209-216, 228-236, 303; Izzi et al., 2011; Oyama & Yoneyama, 2005; Witzany, 2017, pp. 43-44). That process is done in a similar fashion as for external ratings as illustrated before and for instance in (Moody's, 2022). In terms of companies, the fundamental business figures as the revenues and sales, the EBITDA[162], the gross and net profit, the free cash flow, the leverage ratio, the equity and assets, the return on assets and return on equity, the turnover rate, the liquidity ratios and figures, the costs, CAPEX and wages, productivity if available, related growth rates, and other factors might be considered (Caouette et al., 2008; Witzany, 2017, pp. 33-34, 43-44). To decide in the next step, whether these factors fulfill the necessary requirements and help to quantify a certain debtors rating or probability of default a univariate analysis with every single risk factor is carried out (Daldrup, 2006, pp. 43-45; de Laurentis et al., 2010, p. 67; Giri, 2004; Scandizzo, 2016, p. 59; Witzany, 2017, pp. 44, 51). It includes a discriminant analysis and determining the accuracy ratio (AR) or AUROC for the risk factor, the associated *p*-values of the individual risk factors assuming a given distribution like an asymptotically normal distribution (feasible) or Whitney-U distribution as seen before in the context of validation, which can be hence partly viewed as a "repetition (test) of the rating function construction" just out-of-sample (Daldrup, 2006; de Laurentis et al., 2010, pp. 67 - 76; Scandizzo, 2016, pp. 63-65). (Pre)selection of variables is made only if they turn out to be statistically significant usually with respect to a 95% confidence interval, i.e., a 5% level of significance. Some variables might be transformed, e.g., log-linearized or box-cox-transformed to be suitable (Witzany, 2017, p. 52). It is important that not significant factors are excluded at that stage, as well as the ones which are economically counterintuitive (as with opposite sign of the coefficient) and contradict expert judgments as they might not perform well out-of-sample (Witzany, 2017, pp. 43-44). Generally, not more than 20-30 factors are selected at that point in the process (Witzany, 2017, pp. 43-44). Furthermore, the Spearman rank correlation of a risk factor to the entire grading of obligors and to the other risk factors is calculated (de Laurentis et al., 2010; Scandizzo,

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[162] Earnings before interest and taxes, (before) amortization and depreciation

2016, p. 66; Witzany, 2017, pp. 46, 52). A correlation cutoff is typically set at more than 30 (50) -70 % depending often on expert judgment, somewhere at 50 - 70 % is a common bound (Witzany, 2017, p. 44). With the help of a principal component or factor analysis (PCA/FA) redundant and highly correlated risk factors can be reduced or eliminated later, similar to other contexts, e.g., interest rate multi-factor models (de Laurentis et al., 2010, pp. 95-112; Hull, 2015, p. 230). As a result, a segmentation of the data due to statistical properties is carried out.

Once this procedure is completed for all single risk factors, one obtains the "short-list" of suitable factors, and these risk factors are grouped and multivariate discriminant analysis with ultimately all potentially relevant factors is carried out (Daldrup, 2006, pp. 44-45; Scandizzo, 2016, p. 66). That can be achieved by a stepwise either forward or backward process and comparing the log-likelihoods or, e.g., AIC of the nested models against each other (Scandizzo, 2016; Witzany, 2017, p. 44). Therefore, a (linear) combination and logit regression analysis of the remaining risk factors to different components and with different weighting factors, if necessary, is executed to decide on the final factor set (max. 10), hence the final selection and aggregation of the risk factors (de Laurentis et al., 2010, pp. 85-91; Witzany, 2017, pp. 46, 52-53). The boundary condition is that risk factors are used as sparsely as possible while ensuring the broadest possible coverage and differentiation of risks across the entire rating scale (Daldrup, 2006; Scandizzo, 2016). After the selection, scaling according to the master scale, weighting, and aggregation of the risk factors the quantitative rating or scoring function is the result. A "reject bias" in the sample, stemming from the fact that banks can only observe defaults on exposures that were approved before – though mainly of theoretical interest – can be further avoided by using reweighting and augmentation techniques as described in (Anderson et al., 2009; Crook & Banasik, 2004; Witzany, 2017, pp. 61-62). The quantitative approach however is also generally applied successfully (Witzany, 2017, p. 62).

Having comprehensively regarded the quantitative factors further qualitative criteria and soft facts, which might influence a credit assessment, are taken into consideration (Brusov et al., 2021; Daldrup, 2006; Witzany, 2017, pp. 62-63). An aggregation of the qualitative rating module in particular by including soft facts such as competition and market environment, commonly referred to as SWOT[163] analysis or Porters Five analysis as seen, an investigation of the legal and political environment as well as of management quality follows (de Laurentis et al., 2010, p. 39; Porter, 1980; Porter, 1985; Wieben, 2004; Witzany, 2017). Additionally, the available (human) resources of a company, market sentiment toward it, public trust in the firm and its brands, and other criteria might be useful in many cases (Daldrup, 2006). Whenever achievable, the factors are weighted – possibly by expert judgments – and included in the entire model to form the own creditworthiness, hence the stand-alone-rating (SAR), otherwise they serve at least as a correction function or “add-on” (de Laurentis et al., 2010; Witzany, 2017, p. 63). An examination of a support or burden component, e.g., with regard to obligations from equity investments, liability associations, government support factors, etc., and of overriding due to specific events or experts assessments again lead to the overall rating function, as in the external case (the ICR).

Then a last calibration of the rating model is carried out (again similar to the backtesting ideas mentioned before) with a definition of PD limits for the individual obligors’ rating levels a final comparison and mapping of the rating levels to the internal master scale of the bank (de Laurentis et al., 2010; Witzany, 2017). A possible determination of “margins of conservatism” (MOCs) follows, due to calibration uncertainty (Daldrup, 2006, pp. 71-81; de Laurentis et al., 2010; Witzany, 2017, p. 64).

Finally, an initial quantitative and qualitative validation as described above and performed on a validation sample or via cross-validation takes place and is documented in a first initial validation result report (Daldrup, 2010; de Laurentis et al., 2010, pp. 324-357; Scandizzo, 2016). As an amendment to the third step, one should stress the following. The development of the rating (scoring) function can be implemented with the help of expert-based or rule-based models, heuristic-based models or shadow rating approaches, with statistical-based models, or with causal-based models (Daldrup, 2006, pp. 41-70; de Laurentis et al., 2010, p. 36; Witzany, 2017, pp. 36-39). The latter two are sometimes labeled under the common term mathematical-statistical models, and the last one sometimes also structural approach (de Laurentis et al., 2010, pp. 36-44, 57-62). Apart from special situations or project/object ratings, where often cash flow or simulation models are in use or limited information situations (like few defaults and sparse data),

[163] Strength and weaknesses, opportunities, and threats

where shadow ratings are utilized, modern models normally use statistical models like (linear or logistic) regression models – which were hence also described in the thesis (de Laurentis et al., 2010, pp. 114-118, 360-367; Witzany, 2017, pp. 36, 38-39). Alternatives to the mainly used probit or logit functions, often connecting multiple linear regression for example with a link function of Wilson-type, are AI-based techniques as K-nearest-neighbors or artificial neural networks (Daldrup, 2006, pp. 66-70; de Laurentis et al., 2010; Witzany, 2017, pp. 36-39, 73, 78, 81, 82, 85).

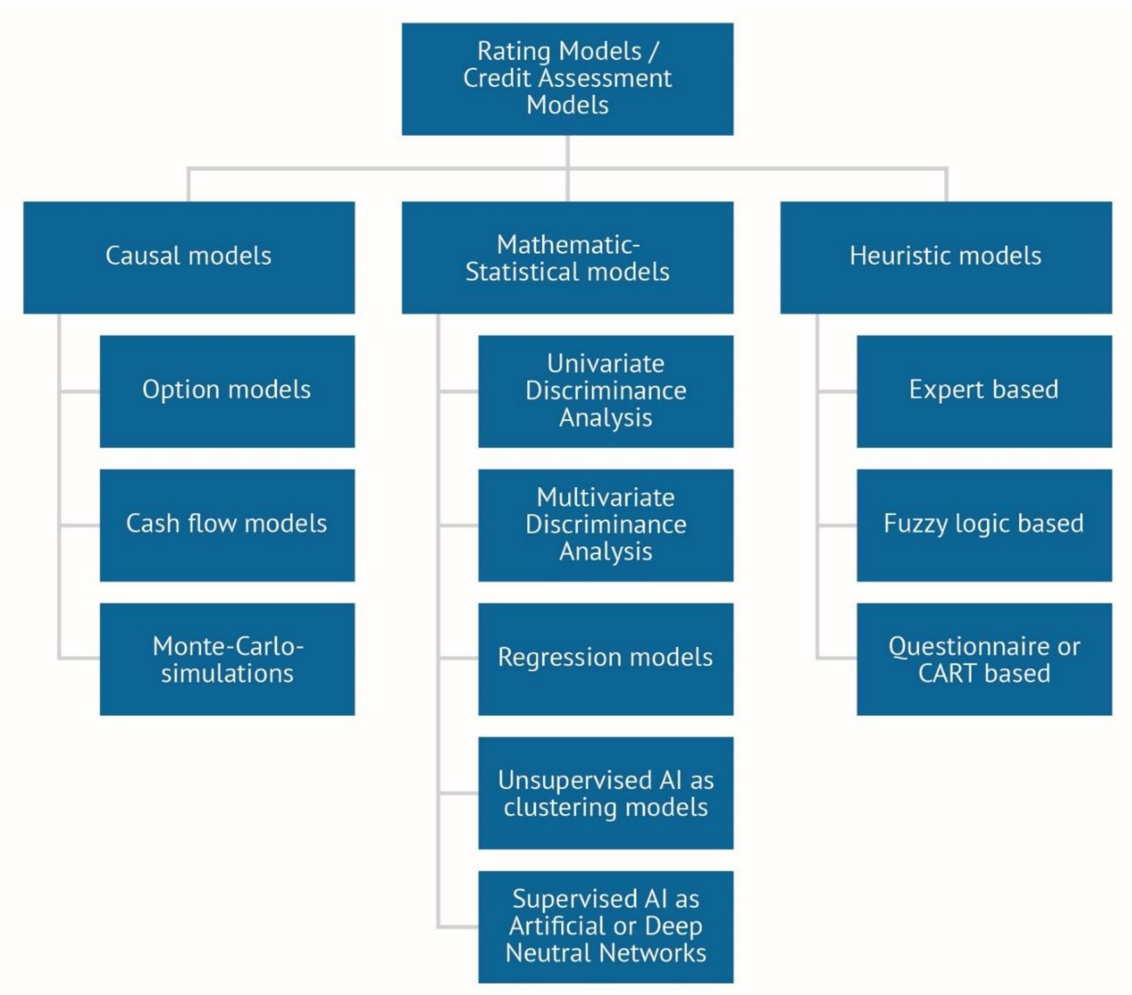

Figure 36 Rating models – causal, mathematic-statistical, and heuristic models.

Source: Own illustration.

Rating models are a special sort of internal models for ratings respectively implicit finally PDs, and these ratings are also (re-)used as input for further internal models, building on them to measure credit risk. The thesis aims to quantify the credit risk inherent in certain types of portfolios.

To put it in context, the models are recalled which are generally used within the financial sector to assign a price and hence, by considering the fluctuation of the price movement and corresponding parameters, a risk to various assets. This is the quantitative foundation of managing the risk associated with assets in portfolios. In Chapter 4 the state-of-the-art models to quantify credit risk are presented then.

3.3 The Applied Models for Different Risk Types in Finance

Risk types typically appearing in banks were discussed at the beginning of Chapter 2 followed by (optimal) portfolio pricing models. Having introduced specifically credit rating models at the beginning of Chapter 3, other models in use to identify, control, and especially quantify various banking risks are hence described in the following paragraphs. As the Value-at-Risk measure is the most common risk measure the models generally calculate a Value-at-Risk (Hull, 2015, pp. 294-295; McNeil et al., 2015; Rutkowski & Tarca, 2015).

A model by its nature should be as precise and as general as possible. Whereas for risk analysis several "simple" methods exist as just sensitivity analysis of certain risk factors, factor analysis or scenario analysis the concentration is on "complete" quantitative models here. These hence aim to be rather generalizable and many of them normally[164] rely on multi-factor frameworks – though some are also combined with individual cash flow modeling techniques like discounted cash flow models, DCF (de Laurentis et al., 2010, pp. 114-118). A comprehensive overview regarding valuation (especially of firms) and DCF can be found in (McKinsey and Company, 2021).

While some models have analytical solutions others also apply numerical approaches or simulations, like Monte Carlo simulations (Hull, 2015; Reinwald, 2022b). If no individual data is available sometimes proxy data and factors can be used as well as will be seen.

Whereas risk models for all kinds of typical financial sector risks as credit risk, market risk, operational risk, interest rate risk, and liquidity risk are presented to have a

[164] Except, e.g., liquidity models as will be shown later.

comprehensive overview and reference, the ones which are not in the main scope of the thesis are just briefly touched on.

In the area of credit risk – again the main risk area for most banks in terms of materiality and impact and the focus risk type of this thesis – the industry applies the following models. The model which is the foundation of internal credit risk models and of the IRB formula in the CRR framework is the so-called Gordy model also known as the asymptotic single risk factor model, briefly ASRF model (Gordy, 1998b; Gordy, 2002; Hull, 2015, p. 384). It was given its name by one of its inventors, director Gordy of the Federal Reserve Bank of the United States. The main underlying idea of one common economic single risk factor usually considered "the economic cycle" also traces back to Merton and Vašíček (Hull, 2015, p. 283). It should be yet noted that, while the economic cycle influences stock prices a lot, (aggregated) stock prices and indices themselves are on the other hand not necessarily a good predictor for recessions (Gómez-Cran, 2022).

The Gordy model is directly feasible and straightforward to implement, and is mainly used for homogeneous, unrated, large retail portfolios and often as a benchmarking model for validation purposes, besides the required IRB formula calculation in Pillar 1 risk management as mentioned (Claessens et al., 2005; Gordy, 1998). For internal risk measurement purposes and more sophisticated applications or complex portfolios one usually makes use of a more comprehensive model like CreditRisk+®[165], CreditMetrics®, or CreditPortfolioView®.[166] The first one is an industrial implementation of a hazard rate model or actuarial model, the second one a structural model application and the third one an implementation of an econometric model.

CreditMetrics® as later described in depth in this thesis relies on modeling the asset or firm value process – generally derived from the stock price as a proxy for listed companies – that is assumed to be governed by a normally distributed dynamic (Gordy, 2002; J. P. Morgan, 1997, pp. 24-26; RiskMetrics Group, 2007, p. 37). It simulates the movements of correlated assets, discounted by corresponding forward rate measures and the joint probability of rating migrations and defaults (Gordy, 1998; Gordy, 1998b; J. P.

[165] Rarely also (wrongly) denoted as Credit Risk+® in the literature.
[166] CreditRisk+® is a commercial product by CreditSuisse®, CPV® by McKinsey®, and CreditMetrics® is a commercial product by J. P. Morgan®. There are further versions of credit risk models existing, e.g., another structural model called KMV® as will be shown later. Further implementations are, e.g., RiskCalc® or RiskFrontier® or PortfolioManager® all by Moody's® or CreditPortfolioManager® or CreditManager® as a further commercial implementation of CreditMetrics® by J. P. Morgan®. All of them can be traced back to the three model types hazard rate models, structural models, or econometric models, and refer to the proponents most in use and most developed that will be regarded later.

Morgan, 1997; RiskMetrics Group, 2007, pp. 36-38, 60-61, 81-101). To recall, a single company defaults at latest when its assets are worth less than its debt and its equity becomes negative. J. P. Morgan® with Gupton et al. implemented and offered the model since the late 1990s (J. P. Morgan, 1997, p. 1).

CreditRisk+® was implemented around the same time by Wilde et al. from Credit Suisse First Boston® (Credit Suisse First Boston, 1997; Gundlach & Lehrbass, 2004; Gupton et al., 2001). Its foundation stems from actuarial mathematics and the insurance industry (Credit Suisse First Boston, 1997; Gordy, 1998b). In contrast to the endogenous asset-value model CreditMetrics® CreditRisk+® models defaults as exogenous random events, their frequency and appearance counted by a typical Poisson process with intensity parameter λ and the common joint risk and ultimately under circumstances prompting default factor modeled by a Gamma function (Diaz & Gemmill, 2002; Gordy, 1998b; Hickman & Koyluoglu, 1998). It has a closed-form solution and is mainly used for retail portfolios, which constituents are often unrated and SME companies, whereas CreditMetrics® is used mainly for larger, listed, standard "blue chip" companies (Diaz & Gemmill, 2002; Engel, 2008; Gordy, 1998b; Kaltofen et al., 2006; Rösch, 2005). In most comparisons, as will be proven later in this thesis, CreditMetrics® outperforms CreditRisk+®. More banks got it deployed, though both are used very often in practice (Hull, 2015, p. 613; Resti, 2000).

CreditPortfolioView® is more similar to CreditMetrics® but relies on a macroeconomic multi-factor linear combination of risk factors as the economic cycle, unemployment, inflation, and interest rates and their final transformation, e.g., via a Wilson type logistic link function to PDs (Frey & McNeil, 2001; Wilson, 1998). It is mainly used by savings and loans institutes like the German so-called Sparkassen or state banks and further local banks. Hence, structural models utilizing the asset and debt characteristics and dynamics of a company, as well as hazard rate models that employ actuarial-statistical methods and finally econometric models using multi-factor approaches exist to deal with credit portfolio risk. Default risk as well as migration risk are within the modelling objectives of those. Furthermore, there are also several credit spread models in use, e.g., the Das or the Nielsen and Ronn models. They are generally labeled two-factor models which model the risk-free (short-term) interest rate and the dynamics of a short-term credit spread (Bielecki & Rutkowski, 2004, p. 264; Das 1995; Downing & Covitz, 2007; Nielsen & Ronn, 1997). They are mainly used for specific –

often market risk induced – questions and are not the method of choice when considering portfolio risk in most banks (Fons, 1987; Fons, 1994).

All of the three other model types above and their corresponding methodologies are principally able to adequately cover the generally nonnormal, one-sided, or skewed and frequently heavy-tailed distribution function of credit losses (Bouteillé & Coogan-Pushner, 2021, p. 237; Gordy, 1998b; Hickman & Koyluoglu, 1998).

In the area of market risk, the institutes also mainly employ three general approaches and corresponding models. Market risk is often – as daily quotes and prices are commonly available through public exchanges, open platforms, and large financial data providers, most instruments are liquid and the market contains many arbitrageurs – relatively comfortable to measure (Hull, 2015; Szylar, 2013). Models are convenient to implement, in many cases a (log) normal or student-*t* distribution assumption of risks and returns is sufficient (Hull, 2015).

Modern historical simulation, the variance-covariance model (VCM), that is also called delta-normal model, or the Monte Carlo simulation (MC) are the preferred methods of choice (Hull, 2015, pp. 184-341; Milanova, 2010; Szylar, 2013).

The first one makes use of historical prices, connected risks and their paths and then applies and extrapolates them to current real portfolios (Hull, 2015, pp. 318-331; Milanova, 2010). Hence, this method is distribution-independent and flexible (Bohdalová & Greguš, 2016). However, it may lack, depending on the chosen historical time frame, rare but severe events and crises, like the great financial crisis when just considering the last ten years. In some particular cases, it might also lack representativeness overall (Hull, 2015; Pritzker, 2001). Furthermore, additional information about current and possibly future prices cannot be implemented and exploited as again in (Hull, 2015).

Another approach, the variance-covariance model, makes use of the underlying covariance (hence correlation) and variance (hence volatility) structure of the assets and risk factors and implements associated sensitivities (Milanova, 2010). Thus, information can be swiftly implemented, and hypothetical scenarios created and calculated. Especially for (approximative) aggregation of different market risks many banks employ that approach (Hull, 2015). However, the restrictive assumption of normally distributed risk factors is made. The model has analytical solutions (Hull, 2015; McNeil et al., 2015).

A further powerful method, the Monte Carlo simulation – named after the largest borough of Monaco popularly linked with its casino gambling and “randomness” –

creates random scenarios, which are pseudo-random numbers technically (Metropolis, 1987; Metropolis & Ulam, 1949). It generally uses normal distribution assumptions, may implement current information, and is not offering analytical solutions but various simulated paths which are then be probabilistically weighted and summed up (Hull, 2015; Milanova, 2010). It does not have to rely on an SND[167] as by distribution transformations like Levy-Rosenblatt, Box-Cox, Yeo-Johnson, or uniform-to-normal other distributions are applicable as well (Box & Cox, 1964; Draper, 1952; Van Albada & Robinson, 2007). For complex and high-dimensional settings Monte Carlo simulation is the method of choice (Hull, 2015; Milanova, 2010).

For market risk, the typical basis areas and assets are stocks, bonds, interest rates, foreign exchange rates (FX), currencies, and commodities as regarded before and commonly extended by real estate (European Commission, 2019; Hull, 2015; Milanova, 2010). Upon them, different combinations, derivatives like forwards, swaps, and options and collections like baskets or funds might be set up in a second step. First, for the basis market risks, apart from the three named methods, tailored approaches exist.

Especially for FX, a certain model denoted as the Garman-Kohlhagen model, a Black-Scholes derived approach, is used by banks (Garman & Kohlhagen, 1983). The Garman-Kohlhagen model utilizes Brownian motions for describing exchange rate dynamics (Garman & Kohlhagen, 1983). For currencies, Lustig described a suitable standard multi-factor approach and for cryptocurrencies, a novel multi-factor model was proposed by Liu et al. (Lustig et al., 2011; Liu et al., 2022). Regression on explanatory factors to model an asset price and its risk is an idea also employed in the realm of commodities, stocks, or even bonds.

Regarding commodities and their pricing as well as risk valuation, the famous Fama-French-5 model, a model by Szymanowska, or the BCM model are the proponents most frequently used in practice (Schaeffer, 2008; Fama & French, 2014; Szymanowska et al., 2014; Blocher et al., 2016). The BCM model, named after its authors, also outperforms the so-called “FH approach” and the one by Gorton, Rouwenhorst and his colleagues (Fung & Hsieh, 2001; Rouwenhorst & Tang, 2012; Rouwenhorst et al., 2012; Blocher et al., 2016).

[167] Standard normal distribution

Considering stocks or shares the main model applied is the Carhart four-factor model which is an extension of the Fama-French-3 model adding a so-called momentum factor to the before-mentioned factors value[168], market return and size (Fama & French, 1993; Carhart, 1997; Ehsani & Linnainmaa, 2022). When modeling the price movement of a stock or derivatives on a stock also a geometric Brownian motion is often applied to describe the dynamics (Björk, 2009).

Bonds can be directly valued via multi-factor approaches as well, however, also calculated using the coupons and discounted cash flow (DCF) methods, and their underlying interest rates, if variable, should be then quantified with special models for interest rates (Bai et al., 2019; Brigo & Mercurio, 2006). A comparable way can be implemented when dealing with IRDs – interest rate derivatives. For interest rates hence, e.g., for the IRRBB approach (interest rate risk in the banking book) or similarly for common interest rate risk in the trading book (CIRR) the following models are used then (Brigo & Mercurio, 2006; Hull, 2015).

To simulate short (-term) rates, one-factor short rate models like the famous Vasicek model which allows for negative rates and is based on Ornstein-Uhlenbeck processes and the mean-reversion principle – which is saying that rates tend to return to their long-term mean from either side – are used (Brigo & Mercurio, 2006; Vašíček, 1984). The Black-76 or the exponential Vasicek model as an extension, which is forbidding negative interest rates through the use of logarithms in the unshifted version, or the Hull-White model, forbidding negative interest rates through the application of square root processes, are the standard methods of choice (Bielecki & Rutkowski, 2004; Black et. al, 1990; Black & Karasinski, 1991; Brigo & Mercurio, 2006). All of them build on and enhance the idea of mean-reversion. Other examples are the Hoo-Lee model, the Black-Derman-Toy model, or the Cox-Ingersoll-Ross model (CIR) or even multi-factor extensions of short rate models hence possibly also including a credit risk component like the two-factor Hull-White model, the three-factor Chen model, or the Das model for credit spreads (Das, 1995; Ho & Lee, 1986; Hull, 2015; Hull & White, 1990; Hull & White, 2015). Empirical studies show that a second factor is necessary for a real-world

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[168] Value in terms of book-to-market factor or conversely stated price-to-book value (and later also price earnings ratio).

calibration of short rate models that possess high predictive performance (Das, 1995; Hull & White, 2015).

For forward (term) rates, a very flexible model which is able to model instantaneous forward rates and to capture the whole dynamic (tenor) of the interest structure curve is the Heath-Jarrow-Morton framework (Heath et al., 1992; Schönbucher, 2003). It includes many short rate models as special or limit cases. A solution for forward rates with adjustment to real rates – hence straightforward to calibrate – is another one. The industry standard which allows modelling of the whole interest structure curve and that can be directly calibrated with real market data is the BGM or LMM – LIBOR market model. A state-of-the-art version that includes the possibility of modeling sticky volatilities is called SABR LMM (Brace et al., 1997; Miltersen et al., 1997; Henry-Labordere, 2005). The before-mentioned remaining market risk, real estate risk, is approached in a different fashion, again including multiple combined impact factors.

Regarding real estate risk, in most cases a step-wise, hierarchical model describing the macro-economic environment like GDP, CPI, or unemployment rate which amounts to a multiple regressive factor model, a real estate (regional) market simulator containing regional vacancy rates, household income, etc., and a (discounted) cash flow model describing the individual idiosyncratic risk factors and may include factors like the condition of a certain building or the accumulated reserves can be assembled to a Monte Carlo cash flow simulation (Webb et al., 1988; Ziobrowski & Ziobrowski, 1997). Therefore, the multi-factor approach is combined with Monte Carlo simulation.

For infrastructure investments, classified as alternative investment, similar approaches can be used to measure market risk – often multi-factor approaches with factors like illiquidity, investment thresholds, tangibility, leverage, diversification, and possibly given correlations[169] to traditional asset classes are in place as described in papers by EDHEC among others (EDHEC, 2015; Hürlimann, 2018; RICS, 2018). Sometimes also proxy-time series where, e.g., the maximum drawdown equals another risk measure as for example a calibrated 99 % VaR profile and where the series is (linearly) regressed onto base risk factors with their given time series and in the optimization case possibly even with given volatilities and correlations are used (EDHEC,

[169] In the case of given correlations or other restrictions, instead of linear regression optimization, methods like (conjugate) gradients have to be exploited.

2015). Renewable infrastructure is a special kind of infrastructure and usually well-tailored models are employed.

With renewables, generally the renewable energy sort is modeled and the interest rate risk as well, e.g., with a S/CIR approach for discounting purposes. While energy (market) exchange prices, political and regulatory risks are systematic, the specific location and circumstances determining the speed of the wind, the solar intensity, etc. are idiosyncratic (Hürlimann, 2018, p. 192). Especially in Europe, thanks to the renewable energy laws such as the "Erneuerbare Energien Gesetz" EEG in Germany, the year-on-year energy yields on the sources like solar or wind are well predictable, rather stable, and with an advanced project status like Brownfield or so-called turn-keys, well-insured technology like advanced solar cells with a high-efficiency rate, the remaining risk factor is just the available amount of the energy source (Bundesgesetzblatt, 2014; Kabel & Bassim, 2019; Kovacevic et al., 2013; Kunjumuhammed et al., 2020; Logan et al., 1994). In other cases, without guaranteed feed-in tariffs or 20 years fixations as common within the EEG, it is widespread to agree on minimum purchasing amounts, rather predictable (power) exchange prices, or hedged fixed rates (Hürlimann, 2018). The remaining energy factor can be further broken down to the intensity of the sun and sunshine days in the case of solar energy and wind speeds in terms of wind power – similar water power fluid intensity (RICS, 2018). For these factors – which are basically location factors and hence highly idiosyncratic – exact time series, historical (weather) data, and written expertise, dealing with the underlying physics-determined distributions and hence allowing for precise historical simulations, exist (Kunjumuhammed et al., 2020).

Another kind of market-related risk yet often categorized as a risk in its own kind is liquidity risk. It is closely connected with market sentiment and especially important during times of crises as illustrated before (Hull, 2015). In liquidity risk mainly deposit-drawing models by inter alia Kobayashi, Sakiyama, or Takemura are used and often again combined with Monte Carlo simulations for the liquidity maturity statement (Sakiyama & Kobayashi, 2018; Takemura et al., 2012, pp. 116-127). Regarding liquidity risk, historical events as the great financial crisis and the subsequent collapse of the interbank market, a so-called "credit crunch", sudden regulatory or central bank action triggered shifts – cf. the "whatever it takes speech"-speech from Mario Draghi in (Alcaraz et al., 2019) – as well as bank runs like in Argentina in the 2000s or in Russia 1998 should be considered for averse scenarios and downturn situation (Blustein, 2005; Bouteillé &

Coogan-Pushner, 2021; Brunnermeier, 2008; Feldstein, 2002). Realism and preciseness concerning model and scenario assumptions are the decisive factor in the area of liquidity risk (Hull, 2015; Takemura et al., 2012). In line with the CRR regulation, the availability of high-quality liquid assets of regulatory level I-III, the maturity structure, sell-ability, in-encumbrance, and central bank collateral-ability of assets have to be considered and the link of liquidity and solvency as essential nexus ought to be investigated (Martin et al., 2014; Takemura et al., 2012).

The different kind of models mentioned are precisely adapted to their corresponding sub-category of market risk and to calculate its economic impact. An alternative is to utilize individual, project-specific or asset-specific information on cash flows and fee structures – if available. For this purpose, assumptions on cash inflows and outflows have to be made, e.g., extrapolation of historical cash flows, new scenarios and the cash flows have to be discounted employing DCF and simulated via the Monte Carlo method. This modeling can be very precise, but a lot of project information is needed, and the calculation quickly becomes relatively extensive (Hürlimann, 2018; RICS, 2018).

In a "combined risks"- project like debt financing of a wind park where credit risk, market risk, and special project-specific operational risks are involved, the overall risk can be often measured directly by disassembling the involved components as sole underlying market risks, credit risks, or operational project risks and quantify them by using known models, e.g., historical simulation or for the MPR[170] and CreditRisk+® for ADR[171] (Hull, 2015; Kunjumuhammed et al., 2020). Afterward, for aggregation usually the risk measure is the Value-at-Risk VaR over 1 year in the 99.9 % quantile, often conservatively summed up of the single risk components like ADR and MPR without making use of diversification effects. Alternatively, for very volatile projects or expected heavy-tailed return distributions expected shortfall (ES) is applied as risk measure in the same way.

Often, a top-down or cascaded approach is also possible, as in the case of project rating according to the IRB approach, in which first country risks and ratings, as well as macroeconomic influencing factors, then industry sector ratings, additionally regional factors and finally individual, idiosyncratic company or even in the general nonrecourse

[170] Market price risks
[171] Address (default) risks, a synonym for credit risk

case project-specific factors, are considered (Hull, 2015). This is crucial for the (credit) spread and thus the discount curve.

If there is no sufficient data available, it is often possible to use deal and transaction data of similar investments or as the worst case to trust the pricing of third-party target fund managers and their appraisal data (EDHEC, 2020; Lazzari & Bentley, 2017). In case of doubt, this is yet an unreliable and often not verifiable method and proxy data should be preferred (Lazzari & Bentley, 2017). After describing the financial risks and related models, operational risks are regarded for reasons of completeness and classification.

This third main risk area that is often briefly denoted as opRisk covers a broad range of risks (Hull, 2015, p. 538-553). Overall risks stemming from human errors or misconduct, system or IT failures (or manipulation, cyber risks), and environmental factors should be considered and researched (Hull, 2015, pp. 545-553; McNeil et al., 2015; Moscadelli, 2005). Operational risks are important to be controlled for many business lines and directly influence various divisions, e.g., enterprise resource planning (ERP). A comprehensive overall enterprise risk management (ERM), including operational risk, is therefore crucial for the entire institute (Hull, 2015, p. 630).

Increasingly important besides cyber risks and fraud is money laundering with a strong recent focus of supervisors on anti-money laundering and anti-terrorist financing processes coined AML/ATF (Behm et al., 2013; EBA, 2022d; Hull, 2015, p. 616; OCC, 2019). As sustainability, green finance, and the fight against global warming become ever more important international issues – accompanied by the described climate stress tests of the ECB, the European Commission's green taxonomy, the sustainable development goals (SDGs) of the United Nations and ESG ratings in the CRR – banks regard this area as major management and top implementation task and a potential operational risk (EBA, 2022c; ECB, 2022; Oliver Wyman, 2022). Internal operational risk models for Pillar 1 were used mainly by large banks in the past (Woodall & Bhollah, 2019b). The approach is known as advanced measurement approach (AMA) in the Basel terminology and existed besides the standard approaches which were labeled basic indicator approach (BIA) for the simplified case and standardized approach, and basically mainly used the gross profits multiplied with a certain classified indicator to account for opRisk losses (European Commission, 2013; Witzany, 2017, p. 14).

In many cases, Gamma or Pareto-Type distributions and their convolutions describe operational risk events very well (Bousquet & Bernardara, 2021; Embrechts, 2005). Generalized pareto distribution models (GPD) are the main class for opRisk and are often levered as a solver for peak-over-threshold problems (POTs) and in the area of extreme value theory known as EVT (Bousquet & Bernardara, 2021; de Haan & Ferreira, 2006; Embrechts, 2005; Embrechts & Neslehová, 2006; Hull, 2015, pp. 330-334).[172]

Generally, every bank has to set up a (near) accident database for opRisk events, reaching back at least 5-10 years, and to use scenario analysis and peer group data thoroughly discussed in their SA (self-assessments) procedure by opRisk analysts (European Commission, 2019; Witzany, 2017, p. 14). Maybe more than in any other risk category and adapted to the fact that risks like cyber risk and reputational risk are extremely hard to quantify and predict precisely, the implementation of clear and strict processes, human education, and training including whistleblowing and white-hat ethical hacking as well as a technical protection mechanism, counter strategies, and business continuity management (BCM) measures are a must (Hull, 2015). That complexity might be also a reason the regulators abolished the possibility to use internal measurement approaches (AMA) and since the finalization of Basel III only a standardized measurement approach (SMA) is allowed to be employed in the future for external opRisk reporting (BCBS, 2022; European Commission, 2019). There is furthermore a substantial need for proper insurance for the mentioned kind of operational risks where available and affordable, as well as tight control of outsourcing and risk-mitigating in that field (Hannemann et al., 2022; Hull, 2015).

When considering the different risk models and their application suitability one should regard the general characteristics of the underlying risks as well (Bouteillé & Coogan-Pushner, 2021, p. 53; Hull, 2015). First, the relevant risk factors and drivers of risk have to be identified, for instance, interest rate risk and currency risk as well as counterparty credit risk when considering a dollar-denominated cross-currency swap. A thorough analysis of the impact of the various single factors has to be done, in an optimal case with quantitative incremental Value-at-Risk for position and risk changes as well as a selection of the data for them, e .g. a time series of a world-wide stock index fund like the MSCI® World All Countries as common market-wide risk driver in a multi-factor

[172] An introduction to opRisk and utilized concepts can be found in (Society of Actuaries in Ireland, 2015).

model for stocks, or on the other hand default data from the own loan portfolio (Bouteillé & Coogan-Pushner, 2021; p. 220; Hull, 2015).

The number of risk factors should be high enough to explain and differentiate the risks – for unexplained components an item-specific idiosyncratic component, residual dummy variable or even a broader nonmodelable risk factor ought to be introduced – however unnecessary factors which solely add complexity and non-parsimonious components and possibly even further reduce interpretability should be omitted (Hull, 2015; Witzany, 2017). This balancing act can be either done in an expert-based heuristic procedure or more quantitatively using principal component analysis (PCA), goodness-of-fit measures or just a margin of conservatism for the models in use (de Laurentis et al., 2010, pp. 95-112; Hull, 2015). Regarding rating models, the corresponding balancing approach for the risk factors was described in detail in Chapter 3 (de Laurentis et al., 2010, pp. 95-112; Hull, 2015).

Data principally should be preferred from internal sources if available, representative and if they form a statistically sufficient large sample or otherwise highly attainable and validated price-liquid external sources like major public index ETFs, major stock exchanges with daily quotes like the NYSE or accredited credit rating agencies like Standard & Poor's ® (de Laurentis et al., 2010). Nevertheless, even with assumed high-quality external data internal due diligence and plausibility tests are recommended and also required by the CRR III for the use of ratings for banks concerning the use of credit risk models (European Commission, 2019). The second-best alternative is pooled data as common in mutual banks, sharing their loss data concerning such portfolios, which however requires a representative test or argumentation before applying them for one own's case or peer group data with similar structures (Witzany, 2017). Sometimes also risk factors or parameters are attainable implicitly or indirectly which is for instance the case for implicit LGD rates for large, listed client companies stemming from bond spread market data – as opposed to directly handled cases after a default of a client (Daldrup, 2006, pp. 91-93). The least preferable possibility is the use of pure expert or model-based estimated data, rare data possibly with gaps – as often the case with new alternative funds whose net asset values are composed of appraisals by fund managers on a low-frequent quarterly basis (Daldrup, 2006; de Laurentis et al., 2010). Thorough data management and coherent data quality processes have to be implemented, data and time series should be updated and validated, also, e.g., the handling of missing values or outliers/extremals should be reviewed concerning for instance adjustments/revisions of former GDP forecasts, and they ought to be synchronized (Guericke, 2018). Clear responsibilities for data steward- and ownership, as well as access management rights, are essential (Hull, 2015).

Depending on the model's aim, the information available and the underlying data different model selections are possible. Regarding for example market risk models in the case of normally distributed data, linear risks as the main driver and stable, highly time-independent markets one might consider a variance-covariance model as adequate. In the case of dealing with not normally distributed (log) returns, unknown prior distributions and an assumed representative past portfolio one would select a model making use of modern historical simulation. When considering new scenarios, e.g., market crashes not included in historical data or more extreme cases or a known distribution like the normal distribution or a uniform transformed one in high-dimensional cases a Monte Carlo simulation is frequently the model of choice (Hull, 2015).

Concerning (single) risk factors and corresponding models thereof, additional structural characteristics have to be clarified and can be used in adapted models. This impacts for instance the question if a time series is or should be modeled as autocorrelated or not, e.g., by time series with i.i.d. random variables for a white-noise or even strictly white-noise process or on the other hand with, e.g., GARCH(1,1) or VARIMA processes if it shows volatility clustering as in stock markets (Engle, 1982; Bollerslev et al., 1992). Certain properties like Markov chain adherence or the mentioned autocorrelation – in practice deciding for (Vector-)ARIMA models for the task of pricing time series and for the GARCH models for volatility heteroscedasticity – should be utilized (Witzany, 2017). Furthermore, the stationarity property which is detected and used by the ARDL, the Watson-cointegration model, the Engle-Granger[173], or the Johanssen-cointegration models and corresponding vector error-correcting models, as an alternative to VAR models by avoiding spurious regression, should be exploited (Bilgili, 1998; Engle & Granger, 1987, pp. 251-276; Hjalmarsson & Österholm, 2007; Johanssen, 1991; Pesaran et al., 2001). Also, the topic of considering conditional or unconditional factors and risk measures has to be decided (Brigo & Mercurio, 2006; Hull, 2015). This holds especially when regarding expectational values and variances or higher moments.

[173] As in https://warwick.ac.uk/fac/soc/economics/staff/gboero/personal/hand2_cointeg.pdf (Retrieved Mai 14, 2022).

Undisputable is the exact calibration of the model – after one has chosen the risks and risk factors involved, their interplay, the data and time series used, including the structural characteristics as autocorrelation and impacts – with the help of a suitable IT implementation (e.g., in R or Java).

As before an inter-department view from end-users and the market side to the model developers, the risk controller and validation side to the IT programmers and designers are crucial. Risks have to be considered front-to-end and human actions and attitudes, including knowledge and risk awareness, processes, systemic controls and limits as well as risk models and the risk cover capital provisioned ought to be optimized and streamlined in a coherent fashion. Furthermore, the execution performance and reliability are essential factors to be considered – apart from the necessary up-to-date hardware thus requiring fast programming languages suitable for the purpose (like Python®, Julia®, SAS®, or R®). These are accompanied by their matching GUI[174]-based studios like the R® studio with their libraries, installments, and (statistical) packages. Adequately educated, staffed, and equipped IT departments are therefore a must-have and distinction criterion among competitors (de Laurentis et al., 2010; Witzany, 2017). Of even higher importance than performance and execution speed is a proper calibration of the models and the setting of the right empirical variables and start parameters. Only a well-calibrated model yields the expected results and is reliable in other aspects.

The calibration has to be challenged and backtested regularly (e.g., once a year in the ordinary validation cycle) and as well on certain occasions, like when major model changes have occurred, as outlined before. In this area, the quality and strict control of the calibration is essential and more important than formal, time considering or documentation issues.

Apart from exact calibration of the models and a thorough validation they must be embedded in a bank. This means the models have to be an integrated part of the overall bank controlling and included in risk management processes and relevant frameworks (Reinwald, 2022b). Risk Management and model maintenance and policies, denoted as model change policies, are becoming an ever greater part of banking and – as already seen – have to be "lived" by the whole organization (Witzany, 2017).

[174] Graphical user interface

Generally, in respect to the quantitative model, structural or empirical derived stochastic processes, multi-factor models with factor (proxy) time series and Monte Carlo methods, especially for exotic risk areas and instruments like path-dependent Asian barrier options, are the chosen utilities able to price (test, forecast) and risk-quantify nearly every asset.

For very advanced purposes, copula linked models employing for instance d-vine copulas might be utilized (Martin et al., 2014; Witzany, 2017, pp. 190-202).

CHAPTER 4
DEFINITION AND CLASSIFICATION OF THE "CLASSICAL" CREDIT PORTFOLIO RISK MODELS

4.1 Modeling of Default Correlations in CPMs and Hazard Rate Models (Reduced Form Models)

In this chapter classical credit portfolio models (CPM)[175] are presented. Following the literature in the field, it is mainly differentiated between two state-of-the-art types of models and hence their most prominent representatives are introduced. This view is shared by regulators as the Bank of England (e.g., Chatterjee) or the Federal Reserve (e.g., Gordy) as well as in the academic field of credit risk research (Bielecki & Rutkowski, 2004; Chatterjee & Chatterjee, 2015; Embrechts, 2003; Gordy, 1998b; Rutkowski et al., 2003). In the late 1990s (1997/1998) most of these models were developed or brought to industrial application[176] – shedding light on a (whole) portfolio context and incorporating correlations not to be considered by classical scoring and credit risk models like Altman's before (Altman & Saunders, 1998; Credit Suisse First Boston, 1997; Elizalde, 2005; Gordy, 1998b; J. P. Morgan, 1997; Wilson, 1998). By regulatory terms, an institute possesses freedom of choice concerning the selection of internal credit risk models, as long as the general regulatory requirements are fulfilled (BCBS, 1999a; European Commission, 2019).

Unlike in the case of market risk, where daily, liquid price observations offer a convenient possibility to calculate the (historical) Value-at-Risk (VaR), in the area of credit risk one needs to construct what one often cannot directly observe – namely the "loss of value" (VaR) due to credit quality migration or even default (Giesecke, 2003; J. P. Morgan, 1997). The first common step among all the credit portfolio risk models is to construct the credit risk framework for a single obligor and its bond[177] and then, in the

[175] The terms credit portfolio risk (CPR) models and credit portfolio models are, as usual, also used interchangeably here.
[176] Yet some structural model versions like Merton's already existed in the academic literature since 1976 and KMV® was also already founded in 1989.
[177] In the primary case the synthetically totally aggregated debt incorporated in one synthetical bond is regarded, an alternative is to consider the (time-and-money-weighted average of the) Senior Unsecured Bonds which long-term ratings are normally considered equal to the issuer's rating as in (Bielecki & Rutkowski, 2004, pp. 77, 85-86).

second step, to extend it to a whole portfolio, i.e., a collection of multiple obligors connected through default correlations (Frey & McNeil, 2001; Gordy, 1998b; Hickman & Koyluoglu, 1998; Li, 2016). This implies defaults that are (generally) not independent of each other and therefore considering correlations and "concentration" of obligors as well as marginal contributions to portfolio risk are necessary (de Laurentis et al., 2010, pp. 25-26).

Furthermore, in the area of credit risk, a normal distribution of losses cannot be expected as opposed to an assumption often credible for market price risks, which makes the risk type "credit" generally more complex (Chatterjee & Chatterjee, 2015, p. 6; Di Biase, 2017, p. 102; J. P. Morgan, 1997, p. 6; Wilson, 1998, pp. 72–79).

As shown, credit risk can arise from ordinary loans and bond holdings, as well as from, e.g., leasing contracts, receivables in trade finance, or from securitizations or derivative contracts (Bouteillé & Coogan-Pushner, 2021; Hull, 2015; J. P. Morgan, 2017, pp. 17-20). The models are generally applicable to all kinds of credit, however originally intended for loans and bonds and that use case is implicitly (overall) also assumed here (Hull, 2015; J. P. Morgan, 2007, pp. 17-20).

At first, the origins and developments of the decisive models are briefly discussed, followed by a thorough examination in own sub-chapters.

One type of credit portfolio risk models are the so-called hazard rate models. In the literature they are also referred to as actuarial models, intensity rate models, or (more general as) reduced form[178] models (Bielecki & Rutkowski, 2004; Chatterjee & Chatterjee, 2015, p. 13). As their name indicates they emerged from an insurance (actuarial) background and insurance mathematics, where damaging events or accidents are often described by Poisson models and the linked common factor is modeled with the help of a Gamma distribution (Chatterjee & Chatterjee, 2015, p. 21; Credit Suisse First Boston, 1997). These credit risk models are also called default models as they (originally) just model defaults and jumps-to-default not credit migration changes, e.g., from a rating class Aa to A[179]. Default in that regard is defined as in Chapter 2, as the first case when a company cannot fully pay back its outstanding debt at the originally agreed time deadline

[178] Sometimes also denoted reduced-form models.
[179] According to Moody's ® terminology. For more on Moody's ® terminology and ratings refer to (Moody's, 2022; Moody's, 2022b).

and conditions anymore. However, the formal 90-days-past due criterion or bank internal unlikeliness to pay conditions (UTP) are not primarily considered here.

It is possible to extend these models further to include rating migration changes and intensities, though not originally feasible, cf. later in this chapter and see (Bielecki & Rutkowski, 2004, p. 90). The defaults are assumed to occur randomly and are modeled by stochastic (non-)survival processes similar as in insurance mathematics (Bielecki & Rutkowski, 2004; Stepanova & Thomas, 2002). The aim is to determine the likelihood of a default on credit obligations from a corporation or sovereign entity and even a whole portfolio thereof. The most famous commercial representative of a default model is CreditRisk+®, which was created by Wilde and his team in a branch of the bank Credit Suisse® (Credit Suisse First Boston, 1997; Gundlach & Lehrbass, 2004). The original academic developers of this type of credit risk model are Jarrow in the year 1995, together with Lando and Turnbull (Jarrow, 2009; Jarrow et al., 1997). Jarrow was a student of Merton. Later Duffie and Singleton, as well as Hull and White, influenced the further development of reduced form models (Duffie & Singleton, 1998; Geyer et al., 2004; Hull et al., 2000).

The second type of credit portfolio models constitutes the class of structural models or enterprise-value-based models, occasionally also in a simplified fashion referred to as Merton models, named by its original founder Merton (Merton, 1974). However, also Galai and Masulis made – often forgotten – important early contributions (Bielecki & Rutkowski, 2004, p. 27; Galai & Masulis, 1976). Later, there were extensions and further developments by Black and Cox resulting in the Black-Cox model and furthermore mainly by Leland, Jarrow, Vašíček, Kealhofer, Longstaff, and Schwartz (Black & Cox, 1976; Kealhofer, 1997; Leland & Toft, 1996; Longstaff & Schwartz, 1995; Vašíček, 1991). The underlying idea in all cases is to assess credit risk by modeling the company's value as an option on its asset price (Bielecki & Rutkowski, 2004; Merton, 1976). If the company's value falls under a certain threshold, e.g., the debt of the company, in the basic version artificially accumulated in one single bond V, it may default or migrates at least to a worse rating grade (Bielecki & Rutkowski, 2004). Therefore, in Merton's original model a company defaults if, at the point in time of its debt service, its assets have a lower value than the outstanding debt (Elizalde, 2006, p. 1; Merton, 1974). The correlation of assets can be dependent on just one or on more factors, in which case the models are labeled multi-factor models. The most prominent example of an implementation of a structural model is called CreditMetrics®, and was developed by Gupton, Finger, and Bhatia from J. P. Morgan Chase®[180] (Gordy, 1998; J. P. Morgan, 1997; RiskMetrics Group, 2007). Another example is KMV® by Kealhofer, McQuown, and Vašíček, presented in more detail later (Kealhofer, 1997; Vašíček, 2012).

A less sophisticated model is o. a. the already mentioned Gordy model. It is a special case of a one-factor Merton-based model, this factor is representing, e.g., the business cycle (Gordy, 1998). The single-factor model is hence, as mentioned, also known as an ASRF model (Gordy, 1998; Gordy, 1998b; Gordy, 2002). In its original form, it requires infinite granularity within the portfolios, which in reality can only be achieved asymptotically when dealing with large portfolios. However, this prerequisite may be weakened by using a Herfindahl-Hirschman-Index(HHI)-based extension, an index that measures and considers the concentration of a portfolio (Gordy, 2003; Gordy & Lütkebohmert, 2007; Kaltofen et al., 2006; Kelly Jr., 1981; Martin & Wilde, 2003). In contrast to CreditMetrics® the Gordy model[181] and the HHI-extended version both do not take rating transitions or multi-factor ideas into account (Márquez Diez-Canedo, 2002). They are denoted as default models only modeling the default component of credit risk (Gordy, 1998). Therefore, hazard rate (reduced form) as well as structural models, can appear in the form of default mode models. As mentioned before, the Gordy model is implemented in the “IRB formula”, which has to be used by IRB-F/A institutes[182] within the finalizing of Basel III framework and also the European version in the CRR III (BIS, 2017; BIS-Statement 2020; European Commission, §153 f.; Hull, 2015, p. 384; Wilde, 2001).

There are various kinds of extensions and improved versions of these models. CreditRisk+® is extended by also taking rating migrations into consideration and by adjusting the default correlation considering heavy (“fat”) tails (Diaz & Gemmill, 2002, p. 1; Engel, 2008, p. 93; Wilson, 1998). CreditMetrics® is extended by multi-factor extensions or recently in an important fashion by Li, who incorporates the default dependencies of asset values and returns and by various mixed-method forms, which are commonly used for pricing CDOs (Koopman & Lucas, 2005; Li, 2016). Tasche also extended the Gordy model by a multi-factor approach (Tasche, 2005).

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[180] The same source as (J. P. Morgan, 1997) is hence (Gupton et al., 1997).
[181] Introduced then by Gordy but building on works of Vašíček as well (Hull, 2015, p. 283).
[182] Used synonymously to (F)AIRBA

A newer and slightly extended version of a structural credit portfolio risk model, known as Zero Price Probability (ZPP), was introduced by de Giuli, Fantazzini, and Maggi (de Giuli et al., 2007).

There exist also macro-econometric parameter-based models like CreditPortfolioView® (CPV®), which is used by the DSGV®[183] – the German association of the Sparkassen, hence the regional loans and savings institutions (McKinsey and Company, 1998). CPV® is built on a multi-factor regressive approach, though operating in a similar fashion as Merton-based ones (Hickman & Koyluoglu, 1998). The three models are the most established ones in the market (Diaz & Gemmill, 2002, pp. 7–8).

In addition, validation techniques of ratings and of credit portfolio models (CPMs) like accuracy measurements (applying the discriminatory measures CAP, ROC, and AR, Gini respectively, from Chapter 3 to see how precise obligors defaults and migrations are differentiated) and backtesting[184] are state-of-the-art, when checking the models' performances (Blöchwitz, 2016; Coppens et al., 2010; Scandizzo, 2016; Sobehart et al., 2000; Tasche, 2003; Tasche, 2006). They are briefly touched on in this chapter as well. There is a problem with backtesting with sparse data and low-default portfolios as there are not enough data points available for statistical purposes, preferring models which are not purely default-mode-based and (also) in that context the unique and decisive role of correlation of defaults is mentioned in that chapter (BCBS, 2005; Flórez-López & Ramón-Gerónimo, 2014). The most important performance criterion for model benchmarking, later utilized in Chapter 5, will be the VaR difference or root mean square error[185] (RMSE), when compared to a given real portfolio, its VaR and its real loss distribution.

On the following pages hazard rate models and hazard rate model-based frameworks are first introduced more precisely. The terminology is in accordance with for instance Brigo or sometimes Bielecki (Bielecki & Rutkowski, 2004; Brigo & Mercurio, 2006; Brigo et al., 2011).

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[183] Deutscher Sparkassen- und Giroverband ®

[184] with various statistical tests as binomial tests, *t*-tests, and Hosmer-Lemeshow tests, rank correlation tests like again Spearman's rank correlation or Kendall's tau and stability tests (PDs and migrations over time, Kullback-Leibler method).

[185] By some researchers also denoted as root-mean-square error.

One can also find a comprehensive overview and introduction to credit portfolio models, including both structural and hazard rate models, in Lando's monograph "Credit Risk Modeling" or by Duffie and Singleton[186], who especially focus on reduced form models and their own groundbreaking contributions to these theories (Duffie & Singleton, 2003; Lando, 2004). Further contributions were achieved by Jarrow and Turnbull (Heath et al., 1992; Jarrow et al., 1997; Jarrow & Turnbull, 2000a; Jarrow & Turnbull, 2000b). These two researchers moreover discussed interesting (general) approaches to link market and credit risk and to differentiate them (Jarrow & Turnbull, 2000b). Bielecki, Jeanblanc, and Rutkowski probably present the most mathematically rigorous approach to credit portfolio risk models in their book and publications (Bielecki & Rutkowski, 2004; Rutkowski et al., 2003). Wilde and his colleagues at Credit Suisse® developed the most famous commercial version of a hazard rate or reduced form model, namely CreditRisk+® (Credit Suisse First Boston, 1997; Emmer & Tasche, 2016).

In the most elementary form of a hazard rate model, the default time is modeled as the "first jump" of a time-homogeneous Poisson process (Brigo & Mercurio, 2006; Bielecki & Rutkowski, 2004, p. 123; Elizalde, 2006, p. 6). Hence, the idea is to model the default event as an exogenous factor that follows a random process (Bielecki & Rutkowski, 2004). The Poisson process is suitable as it is also used in survival analysis and actuarial mathematics, in the CPM context utilized to model the "survival" of a company over time instead of individuals as in life insurance applications (Bielecki & Rutkowski, 2004, p. 123; Stepanova & Thomas, 2002; Witzany, 2017, p. 68). With the help of a Poisson process the number of defaults – the same company might default multiple times in this modeling approach – are counted. The (general) counting process hence follows the special form of a Poisson process (Credit Suisse First Boston, 1997; Frey & McNeil, 2001). One can heuristically imagine it as a company being in a "living state" throughout the time in the first phase and then "suddenly" jumping to a default state. As it is also possible that after a default or even without a default, after a restructuring process, a company (and its debt) is cured one might also consider cures in intensity processes, e.g., with the help of mixture cure models and survival analysis, as in (Tong et al. 2012, Wycinka, 2015). In the thesis, cure processes are not taken into account (apart from potential multiple defaults) and the concentration is on a pure Poisson process in the following.

As this process describes certain one-dimensional points in time it is also known as a Poisson point process and considered homogeneous first, which means the points are being uniformly distributed in any given set (Kingman, 1993). It is then finally called a homogeneous Poisson (point) process - HPP (Credit Suisse First Boston, 1997; Habibi, 2018; Kingman, 1993). Consider a counting process $\{N_t, t \geq 0\}$,

$$N_t = \sum_{k=1}^{\infty} Ind(S_k \leq t) \tag{119}$$

where *Ind* denotes the indicator function where the value of the indicator function is equal to one, if the argument in parentheses $(\dots)$ is true, else it is equal to zero (Bielecki & Rutkowski, p. 123). One can consider that as an (infinite) series of number of defaults k, and S_k denoting the first point in time when k defaults have occurred. If at some point t in time k defaults already happened until t or exactly at t, then S_k is equal to one otherwise zero ("counting if a default occurred for it up to time t"). Hence, the total number of defaults up to and including time t is summed up through the formula by adding "one-after-one". Then one can additionally write

$$S_N = T_1 + \cdots T_n \,;\; T_1 \,, T_2 \; \dots : \Omega \;\rightarrow\; [0, \infty) \tag{120}$$

with the T_i denoting the time (difference) after which the i-th default occurs, when i-1 defaults are already counted and happened up to time S_{i-1} (Credit Suisse First Boston, 1997; Gupton et al., 2001). Therefore, S_N is a sequence of independent, identically distributed (i.i.d) random variables T_i summed up until T_N. As all the T-components map to nonnegative numbers as "time-distances" the time S_N is increasing with N, as heuristically expected when considering an increasing number of defaults.

If further $T_n \sim \exp(\lambda)$, for a $\alpha > 0$ then the process $\{N_t, t \geq 0\}$ is called a homogeneous Poisson (counting) process (HPP) with intensity rate λ (Bielecki & Rutkowski, 2004, pp. 155, 186; Kingman, 1993; Witzany, 2017; p. 68). In short form: $N_t \sim \mathrm{Poi}(\lambda t)$. This means the counting process follows a homogeneous Poisson process with intensity λ. This intensity (rate) λ is also known as hazard rate (Bielecki & Rutkowski, 2004, p. 155). This induces that $\{N_t\}$ has independent and stationary[187] increases, and

[187] $(N_{t(i)} - N_{t(i-1)})$, $i \geq 1$ are independent. Stationary: $N_{t(i)+s} - N_{t(i-1)+s} = N_{t(i)} - N_{t(i-1)}$ for all i. $0 \leq t_0 < t_1 < \cdots < t_n$.

$$\mathrm{E}[N_t] = \lambda t \tag{121}$$

as the expectation value just integrates to *t* or sums up in the discrete case (Bielecki & Rutkowski, 2004). Hence, the expectation value of the counting process at time *t* equals *t* times the intensity. For all $t \geq 0$, $\{N_t\}$ is a Poi(λt) distributed random variable. Technically $\{N_t\}$ is right-continuous and counts the "jumps" (e.g., from zero to one).

For the distribution at point *t* or the probability of the random variable $N(t)$ of being equal to *n*, with *n* denoting the number of accidents or defaults in this case, one can write (Bielecki & Rutkowski, 2004, pp. 186-187, 193):

$$\mathrm{P}\,(N(t) = n) = \frac{(\lambda t)^n}{n!}\, e^{-\lambda t} \tag{122}$$

Evidently, the cumulative probability distribution is then

$$\mathrm{F}(n; t) = \sum_{k=1}^{n} \frac{(\lambda t)^k}{k!}\, e^{-\lambda t} \tag{123}$$

This again shows the expectation by the Taylor expansion definition of *e*,

$$\begin{aligned} \mathrm{E}[HPP] &= \sum_{k=1}^{\infty} k \left(\frac{(\lambda t)^k}{k!}\, e^{-\lambda t} \right) = \sum_{k=1}^{\infty} \frac{(\lambda t)^{k-1}}{(k-1)!}\, e^{-\lambda t} \lambda t \\ &= \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!}\, \lambda t\, e^{-\lambda t} = e^{\lambda t}\, \lambda t\, e^{-\lambda t} = \lambda t \end{aligned} \tag{124}$$

(Kingman, 1993). In the same way

$$\mathrm{Var}(N_t) = \lambda t \tag{125}$$

by the definition of the variance and hence

$$\lambda = \frac{\mathrm{Var}(N_t)}{t} \tag{126}$$

For the special case $t = 1$ the variance and mean are both λ. An alternative way to define a Poisson counting process can be reached by the requirement that the time differences between the events of the counting process are exponential variables having a mean of 1 / λ (Jorda, 2010; Kingman, 1993).

If one defines the default time τ[188] as before as the first jump of a Poisson process, one can calculate its survival probability as in (Jorda, 2010, p. 3):

$$Q(\tau > t) = e^{-\lambda t} \tag{127}$$

[188] More formally: A default time τ is assumed to be an arbitrary positive random variable that is defined on an underlying probability space (Ω, *A*, P) as in (Bielecki & Rutkowski, 2004, p. 222).

This result yields that survival probabilities have the same structure as continuous discount factors in finance (Brigo & Mercurio, 2006, p. 698). The default intensity λ as shown, e.g., by Jorda or Bielecki hence astonishingly plays a comparable role as interest rates do (Bielecki & Rutkowski, 2004; Jorda, 2010, p. 7; Lan, 2011; Lando, 2004).

This observation makes it reasonable to view default intensities as some kind of credit spreads – coming back later on that in the thesis when introduced as a means for calculating implied PDs from credit spread data. Kusuoka and Laurent further enhanced reduced form models also in the credit spread (and swaps) regard (Kusuoka, 1999; Laurent, 2020). Generally, higher default intensities (rates) are correlated with higher credit spreads and hence when the interest rate curve including these spreads is used for discounting purposes of future cash flows to receive a present (fair) value, this value is consequently lower – which is intuitively logical: Because of the higher risk, the risk premium is higher and the present value ceterus paribus hence "worth less".

The inhomogeneous (nonhomogeneous) Poisson point process then is a Poisson point process, where the Poisson parameter is set as a location/time-dependent function (Bielecki & Rutkowski, 2004). That means the intensity depends on the time t and the points are in many cases not uniformly distributed. As a result, "extra dynamic" can be illustrated through the process. The same results as above however are still true for a time-varying intensity $\lambda(t)$, when defined in the way

$$\lambda(t) := \int_0^t \gamma(u)du \tag{128}$$

as the cumulated intensity, cumulated hazard rate, or Hazard function (Garcia et al., 2005, p. 2; Jorda, 2010). A similar result is true for Cox processes (Black & Cox, 1976).

A Cox Process $\{N_t, t \geq 0\}$ is defined as a Poisson process with stochastic intensity λ_t, i.e., $\{\lambda_t, t \geq 0\}$ in that setting defines a stochastic process. Hence, whereas the intensity of a nonhomogeneous Poisson process depends deterministically on the time and is time-dependent the Cox Process even assigns a random variable for the intensity at every point in time, hence is a random process with also random intensity process λ_t in it (Drazek, 2013; Lando, 1998). In practice, an intensity process can be chosen and calibrated to various settings, when one, e.g., intends to illustrate a process in which the intensity at some point in time is dependent on former intensities it is natural to use an autoregressive approach and an AR(1) model as for instance Duffie (Duffie, 1999).

An equivalent introduction of these processes, yet mathematically founded on so-called martingale measures and the connected valuation of assets, is available in the monograph “Credit risk modeling” by Bielecki and Rutkowski (Bielecki & Rutkowski, 2004, pp. 166, 222).

Until here the number of defaults up to a time t was considered. The second factor which has to be taken into account is, similarly to accidents in insurance cases, modeling the “severity” of the defaults, e.g., the exposure at default and ultimately the absolute recovery rates. This aim can be directly achieved by modifying the formula above with the desired EAD numbers or functions while if necessary – and common – building segments of obligors (Credit Suisse First Boston, 1997; Gupton et al., 2001). Hence, certain “bands of obligors” with the same EAD are considered and grouped.[189] The grouping is extended by further including correlations between different obligors to measure the default impact of obligors on other ones (Gupton et al., 2001; Tasche et al., 2004). Especially during times of economic crisis, it is essential that these correlations and hence contagion risks are quantified in a correct manner (Brunnermeier, 2008; Schiavone, 2018). One can also formulate the ideas described in this paragraph by switching from a single asset or obligor perspective to a multi-asset- or portfolio-based one.

There are mainly three ways of introducing default correlations among obligors into intensity models in the literature (Hickman & Koyluoglu, 1998; Huschens, 2004). The first one – conditionally independent defaults (CID) models – generate credit risk dependence among the various obligors through the “dependence of the firms’ intensity processes on a common set of state variables” (Bielecki & Rutkowski, 2004, pp. 265-282; Elizalde, 2006, p. 2). Hence the default rates (themselves) can be treated as independent when the realizations of state variables are fixed (Elizalde, 2006, p. 2). The problem of CID models explicitly regarded when comparing the various credit portfolio risk models in that chapter, are the low levels of default correlation resulting from the models compared with empirical levels (Diaz & Gemmill, 2002). Duffie and Singleton, therefore, tried to extend CID models by introducing common default events (with common triggers) or “joint jumps” in the default processes of the different companies (Duffie & Singleton, 1999; Elizalde, 2006, p. 2). A problem with these extensions is the correct

[189] To divert the distribution of loss for a diversified portfolio with many different assets, the losses are simply divided into different bands with the level of exposure in every band.

calibration of these models and again a tendency of underestimation of correlations as the authors' state (Duffie & Singleton, 1999; Elizalde, 2006, p. 2; Geyer et al., 2004).

The second way – contagion models – include the empirical observation of default clustering. They change default intensities by adding clustering functions, but appear rather arbitrary and adjusted to very specific settings and are hence not useful for generalized settings (Davis, 2011; Davis & Lo, 2001; Hickman & Koyluoglu, 1998; Jiang et al., 2018; Schönbucher & Schubert, 2001). Davis and Lo also coined the term infectious defaults for contagions (Bielecki & Rutkowski, 2004, p. 294).

The latest and most sophisticated approach is the use of copula functions (Nelsen, 1999; Witzany, 2017, pp. 188-202). This type of model separates the estimation of the individual default probabilities, i.e., the default intensity processes from the estimation of the function that incorporates the credit risk dependence (correlation) and links the obligors, this link is done with the mentioned so-called copula function (Embrechts et al., 2008; Frey & McNeil, 2001; Martin et al., 2014; Sempi, 2011). Also other approaches, which are able to model marginal default probabilities, such as the structural approach mentioned before and detailed in the following, can use copulas to model the joint probabilities then (de Giuli et al., 2007). While these functions are quite powerful tools, the calibration and application of the "right" copula function is far from easy and difficult to generalize (Frey & McNeil, 2003; Panchenko, 2005; Zeevi & Mashal, 2002). The most important theorem concerning copulas is the one of Sklar (Sklar, 1959; Witzany, 2017, pp. 191-192).

Sklar's Theorem:

Let $y_1, \ldots, y_n$ be random variables, having marginal distribution functions $F_1, \ldots, F_n$ and a joint distribution function F such that

$$\mathrm{F}(y_1, \ldots, y_n) = C(\mathrm{F}_1(y_1), \ldots, \mathrm{F}_n(y_n)) \text{ for all } (y_1, \ldots, y_n) \text{ in } \mathbb{R}^n \qquad (129)$$

If each F_i, $1 \leq i \leq n$, is continuous, then the copula C is even unique (Elizalde, 2006, p. 29; Martin et al., 2014; Sempi, 2011; Sklar, 1959).

Hence, the joint distribution can be completely defined by the marginal distributions. This is an immensely powerful theorem as it suffices to use marginal distributions for every dimension and then connect them via a central dependence structure, the copula C (Martin et al., 2014).

Copulas that are commonly used are so-called elliptic copulas, Archimedean, Gaussian, Clayton or t-copulas, Frank or Gumpton ones, and the Fréchet-Hoeffding-copulas (Elizalde, 2006; Martin et al., 2014; Meyer & Strulovici, 2015). The Frank, Gumpton and t-copula are especially feasible for credit risk structures (Frey & McNeil, 2001; Witzany, 2017).

Generalization problems, the (missing) quantity of available data and "the nonexistence of a liquid and transparent market for default correlation products which would allow [to distil] the dynamics of default contagion mechanisms, either through a copula or through a contagion model", as Elizalde points out, are leading to the preferred use of CID models in the class of reduced form models (Elizalde, 2006, p. 45). However, the use of copulas also in other settings became increasingly popular during the last 25 years (Meyer & Strulovici, 2015; Nelsen, 1999).

As described, a Poisson distribution X is used to model the number of defaults k and besides bands for different exposures or losses given defaults are introduced and then aggregated. Furthermore, it is common to produce X with the help of a generative function, which is a series such that

$$G(z) = \sum_{k=0}^{\infty} \mathrm{P}(X = k)\, z^k \tag{130}$$

where $\mathrm{P}(X = k)$ is the probability that $X = k$ (Bielecki & Rutkowski, 2004). Hence, the function G is generated by the base function P, forming a series that is converting under certain assumptions (similar to the well-known Taylor series). One necessary assumption from analysis, that

$$|\mathrm{P}(X = k)| \leq 1 \tag{131}$$

is fulfilled as probabilities by definition always have values between zero and one. On the other hand, to obtain $\mathrm{P}(X = k)$ one may write:

$$\mathrm{P}(X = k) = \frac{1}{k!} \frac{d^k(G(0))}{dz^k} \tag{132}$$

d denoting the derivative and d^k the k-th derivative, which is simply the Taylor expansion for G in zero. So, for the Poisson distribution, one gets again the scheme (Bielecki & Rutkowski, 2004)

$$G(z) = \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} z^k = e^{-\lambda} e^{z\lambda} = e^{(z-1)\lambda} \quad (133)$$

As the exponential function is multiplicative for several obligators one can represent

$$G(z) = \Pi_A G_A(z), \quad (134)$$

where $G_A(z)$ is the generative function of a portfolio with just a single bond A without any correlations. This result is grounded in the fact, that for independent events common probabilities may be multiplied. It can be therefore applied to the mentioned homogeneous bands which are roughly independent. Yet it only yields for independent ones.

But credit risks are generally not independent, as just described with the three options of linking defaults and hence in the one-factor model one assumes that there is a certain common macro-economic impact that influences all obligators and their bonds in the same way. As shown before, one normally chooses a common CID type of link. Conditioned on the macro-economic-factor the model behaves then as independent treatments (Credit Suisse First Boston, 1997; Gupton et al., 2001). As a result, one ought to model the following assumptions:

(1) The macro-economic factor is normally chosen as a $\Gamma(\alpha, \beta)$ distributed random variable X, a so-called Gamma distribution with two parameters, as that turned out to be empirically feasible as within insurance mathematics (Credit Suisse First Boston, 1997). With this function, one has a defined density

$$f_{\alpha,\beta}(x) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} x^{\alpha-1} e^{-\beta x} \quad (135)$$

where $x > 0, \alpha > 0, \beta > 0$ and

$$\mathrm{E}[X] = \frac{\alpha}{\beta}, \quad (136)$$

$$\mathrm{Var}[X] = \frac{\alpha}{ß^2} \quad (137)$$

(2) The different losses $L_1,\ldots, L_n$ are conditionally stochastically independent given $X = \lambda$ with the "conditional distribution"

$$P(L_i|X = \lambda) = Poi\ (\lambda_i \frac{\lambda}{E[X]}), \tag{138}$$

for all $1 \leq i \leq n,\ \lambda > 0.$

Hence, the intensity parameter is in a sense "random" and is defined by the random variable *X*, which follows a Gamma distribution modeling the macro-economic factor.

As a result, one deduces the unconditional loss distribution by "mixing" the constituents with the distribution of *X*. Therefore, the model is called "the Poisson mixture model" or "the Poisson-Gamma mixture" (Credit Suisse First Boston, 1997; Fischer, 2019). The result is a negative binomial distribution (NBD), which is an analytically closed solution and can be regarded as a Poisson distribution for which the Poisson parameter is itself again a random variable with a Gamma distribution (Credit Suisse First Boston, 1997; Gupton et al., 2001).

The underlying steps from above can be summarized in a compact form as follows, e.g., preceding a possible implementation in the programming language R[190]. The parameter λ is regarded as a random variable *X*.

1. The conditional distribution of the random variable N_t (conditional on $X = \lambda$) can be written as:

$$P(N = n|X = \lambda) = e^{-\lambda}\frac{\lambda^n}{n!} \tag{139}$$

$$n = 0, 1, 2 \ldots . \lambda > 0$$

2. Now *X* is Gamma distributed with shape parameter α and scale parameter β. The probability density function of *X* is:

$$g(x) = \frac{1}{\Gamma(\alpha)}\ \beta^\alpha x^{\alpha-1}e^{-\beta x} \tag{140}$$

Thus, the joint density of N_t and *X* is:

$$P(N = n|X = \lambda)g(\lambda) = e^{-\lambda}\frac{\lambda^n}{n!}\frac{1}{\Gamma(\alpha)}\ \beta^\alpha \lambda^{\alpha-1}e^{-\beta\lambda} \tag{141}$$

3. The unconditional distribution of N_t is therefore – as calculated similarly in (Witzany, 2017, pp. 134-135).

[190] Or any other (statistical) programming language, e.g., SAS®, Matlab® or Python®.

$$\mathrm{P}(N = n) = \int_0^\infty \mathrm{P}(N = n|X = \lambda) g(\lambda)\ d\lambda$$

$$\begin{aligned}
&= \int_0^\infty e^{-\lambda} \frac{\lambda^n}{n!} \frac{1}{\Gamma(\alpha)}\ \beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda} d\lambda = \int_0^\infty \frac{1}{n!\,\Gamma(\alpha)}\ \beta^\alpha \lambda^{n+\alpha-1} e^{-(\beta+1)\lambda} d\lambda \\
&= \frac{1}{n!\,\Gamma(\alpha)}\ \beta^\alpha \frac{\Gamma(n+\alpha)}{(\beta+1)^{n+\alpha}} \int_0^\infty \frac{(\beta+1)^{n+\alpha}}{\Gamma(n+\alpha)}\ \lambda^{n+\alpha-1} e^{-(\beta+1)\lambda} d\lambda \\
&= \frac{1}{n!\,\Gamma(\alpha)}\ \beta^\alpha \frac{\Gamma(n+\alpha)}{(\beta+1)^{n+\alpha}} = \binom{n+\alpha-1}{n} \left(\frac{\beta}{\beta+1}\right)^\alpha \frac{1}{(\beta+1)^n}
\end{aligned} \tag{142}$$

$$n = 0, 1, 2 \ldots$$

Using the fact that the cdf from zero to infinity of the Gamma function (with ß* := ß + 1 and α* := α + *n*) equals one in equation line three and the functional property of the Gamma function that

$$\alpha\, \Gamma(\alpha) = \Gamma(\alpha + 1) \tag{143}$$

(Bielecki & Rutkowski, 2004; Credit Suisse First Boston, 1997; Hickman & Koyluoglu, 1998). This formula, the "Poisson-Gamma mixture", is hence the exact form of the probability function of a negative binomial distribution (Credit Suisse First Boston, 1997).

Finally, by having derived a closed-form analytical solution for credit risk, it is possible to calibrate a suitable portfolio default model by setting the intensity rate through the Gamma function and its parameters α and β. This is further covered on empirical examples by Li (Li, 1998).

4.2 Structural Models (Asset-Value Models)

The structural models in credit portfolio management were first introduced by Merton (1974) and later developed by Leland, Anderson, and Jarrow among others as already shown (Anderson & Sundaresan, 1996; Jarrow, 2009; Leland, 1994; Merton, 1974). Jarrow, influenced by Merton, was interestingly also one of the founders of reduced form or hazard rate models as seen.

The term "structural" was coined by the fact that there is an underlying economic structure characterizing the movements of an asset of an obligor and, in contrast to hazard rate models, a default or migration event is not something entirely random but pre-determined by a company's value, its movement and volatility and its debt (Hickman & Koyluoglu, 1998; Merton, 1974). It is hence considered an endogenous model. This restriction and the fact that the original model just allowed for default at the time of a debt's maturity were however solved through techniques labeled first-passage time approaches by Cox (Bielecki & Rutkowski, 2004, p. 65). Hence, default could be also considered exogenously and as unpredictable within that same framework, Zhou introduced, e.g., geometric jump-diffusion processes extending the normal price process by jumps, which model the possible "jump-to-default" (Hanson, 2008; Lamba, 2018; Zhou, 1997). The structural model is also intricately linked to the capital structure and the relation of assets and debts of a company as will be seen in this sub-chapter. Therefore, it sets a spotlight to (the occasionally conflicting interests and) the relationship of creditors and shareholders of a company, especially the different risk averseness levels (Bouteillé & Coogan-Pushner, 2021). Management and the board of directors (installed by shareholders and executing largely their interests) are more incentivized to take risk than creditors, who are more interested in a stable, conservative running of a company without taking too much risk as then the risk of losing the given credit/loan is lower, whereas they do not profit from higher dividends or stock prices directly, from which shareholders profit, while not caring too much about not being able to serve creditors once they might have lost their shares already and being the first ones taking losses with their equity anyway (Bouteillé & Coogan-Pushner, 2021; Galai & Masulis, 1976). The different return and risk distribution of equity (and normal market risk) versus not normally distributed credit returns and risk comes here into play again.

Especially short-term interest or expansion interest (e.g., shifting plants in a more risky yet more yield-promising emerging market country or a hostile takeover or risky yet promising appearing acquisition of another company) can differ substantially between the two groups of the liability (funding) side of a company (Bouteillé & Coogan-Pushner, 2021).

There are some crucial preconditions for the structural models. The structure of the liabilities of the company and the management of its assets is a priori known by all participants in the market. This is called full information in efficient markets (Fama & MacBeth, 1973). The markets are additionally – at first – assumed to be frictionless, i.e., there are no transaction costs, fees, dividends, or taxes and the market is liquid (Bielecki & Rutkowski, 2004, p. 51; Brigo & Mercurio, 2006). Furthermore, the daily asset/enterprise[191] value returns of the company follow a certain structure like, most commonly, a normal distribution (J. P. Morgan, 1997). In practice, as company values and their volatility are not directly observable, one is using stock returns as a proxy for the assets as they are daily observable and fully transparent; this proxy is working often astonishingly well empirically (Anderson & Sundaresan, 2000; Diaz & Gemmill, 2002; Gordy, 1998; J. P. Morgan, 1997; RiskMetrics Group, 1997).

The problem of unobservable asset prices or lack of tradability of the firm's value (bonds and stocks) can be simplified under feasible market completeness circumstances or by including further economic factors (such as profits and defaultable bonds characterized in terms of the state variables firm value and profit) to the case were at least one of them is traded (Bielecki & Rutkowski, 2004, p. 64; Buffett, 2000; Ericsson & Reneby, 1999). A further approach by mainly Fantazzini, de Giuli, and Maggi using copulas is described later (de Giuli et al., 2007).

All the debt of a company is further synthetically, by replicating (the duration of) the bonds and loans portfolio of the firm, held in one bond *D*, the "value of its debt" (Bielecki & Rutkowski, 2004; J. P. Morgan, 1997; RiskMetrics Group, 2007). Hence, the company defaults, as regarded before, when the value of its assets is lower than the value of its total debt or equivalently when the (accumulated) returns are lower than a certain transformed threshold (the mentioned exogenous extensions are not considered in that case).

One can think of this setting as an European option[192][193] (Bielecki & Rutkowski, 2004; p. 52; Geske, 1977, pp. 541-552; Gordy, 1998b; Pitts & Selby, 1983, pp. 1311-1313). Thus, the credit risk component of a company's debt can be valued like a put option on the value of the underlying assets of the company (J.P. Morgan, 1997). It is imagined to be bought and just exercised at maturity when the company's assets value falls below the debt value which equals the so-called strike price (Geske, 1977; Gordy, 1998b).

Then the probability of default of a single asset, equals the probability that the

[192] Contrary to an American option which can be exercised at every time in (0,T], T denoting the maturity, a European option can just be exercised at maturity T. An option with many discrete possible exercising dates during (0,T] is called a Bermudian option as it lies "in-between" these two possibilities, cf. for example (Bingham & Kiesel, 2004).
[193] Even the possibility of vulnerable claims in that context, e.g., options with a counterparty default risk were modeled in the literature (Hull & White, 1995, pp. 299-322; Klein & Inglis, 2001).

value of the assets (V) is lower than the value of the debt (D) of the company. The price movement or motion of the assets, or stocks in the case of listed companies and their proxy-view, is generally described by a (geometric) Brownian motion (GBM), the "normal distribution equivalent" for exponential stochastic processes. This means when the returns or (dynamic relative) changes in price are normally distributed, the movement itself is log-normal distributed. Formally, this stochastic differential equation (and price change dynamic) is written as (with payout ratio first assumed to be zero):

$$dV_t = \mu \, V_t \, dt \; + \; \sigma \, V_t \, dW_t \tag{144}$$

(Bielecki & Rutkowski, 2004, p. 51)

This for non-zero V_t equals

$$\frac{dV_t}{V_t} = \mu \, dt \; + \; \sigma \, dW_t \tag{145}$$

with mean μ, often equals the risk-free interest rate r to prevent arbitrage in the market, volatility or statistically the standard-deviation σ, and a fluctuation which is known in the area of financial mathematics as Wiener process W_t (Gordy, 1998b; J. P. Morgan, 1997; RiskMetrics Group, 2007). It means that the relative change in V_t, incorporating the dynamics of it, denoted as dV_t / V_t can be expressed as a trend component changing with time t (multiplied) by the mean factor μ and a random process component and its dynamic dW_t with multiplied volatility σ (Bielecki & Rutkowski, 2004).

The Wiener process is adding "randomness" to the stock or asset price movement and encrypts an equivalent of the Brownian motion of particles in physics. In this case, instead of particles, just the typical "shacking"-motion of stocks is illustrated with that process. One then derives

$$d \; (\ln V_t) \; = \left(\mu - \frac{\sigma^2}{2} \right) dt \; + \; \sigma \, dW_t \tag{146}$$

by Itô's Lemma cf. (Hassler, 2007, pp. 191-192; Itô, 1951; Zhang, 2015). Itô's Lemma can be seen as a chain rule for stochastic calculus and hence as known from (standard) calculus applied, e.g., when taking the derivation from a (natural) logarithm, in standard calculus

$$\ln f(x)\,' = \frac{1}{f(x)} \; f'(x) \tag{147}$$

hence the product of the outer and inner derivative, by the chain rule. In stochastic calculus, it can be seen in a similar way and be heuristically derived by taking the Taylor series expansion of the function up to its second derivatives. The resulting terms are then the ones seen in the formula above and the correcting factor $\sigma^2 / 2$ for the defining solution is explained directly by that expansion and the Itô integration definition (Bielecki & Rutkowski, 2004; Itô, 1951).

The solution of the formula above is then by first (stochastically) integrating the right-hand side to "get rid of d" and then taking the exponential process to "get rid of ln". In the graph $A_0 := V_0$ and

$$V_t = V_0 e^{\int (\mu - \frac{\sigma^2}{2}) \, dt + \sigma \, dW_t} \tag{148}$$

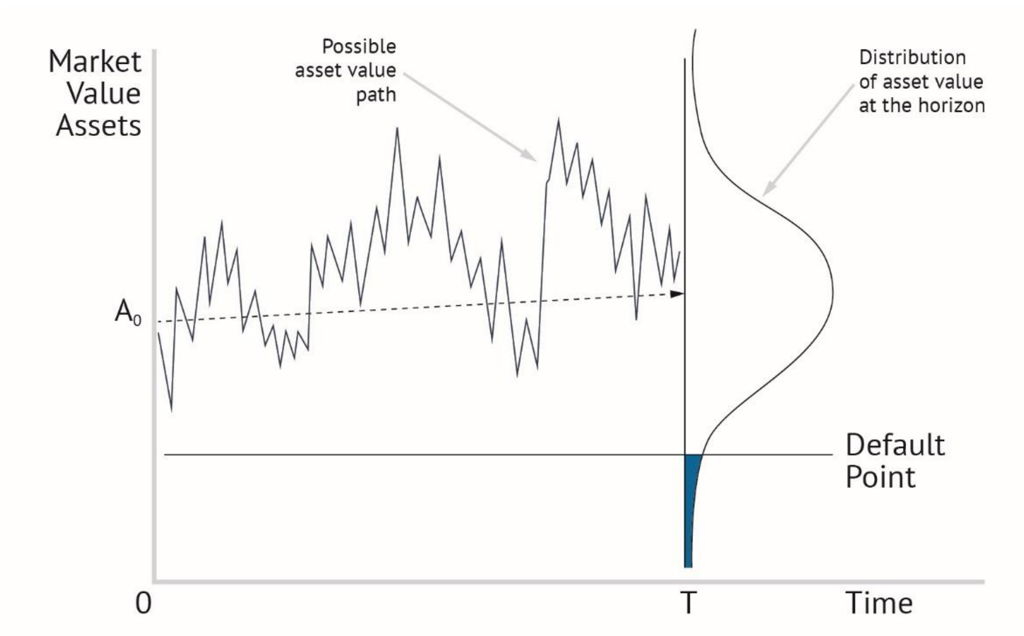

Figure 37 Asset value path in asset-value models (structural models).

Sources: Own graph according to: (Vašíček, 2012, p. 16).

Hence by the definition of the variance and expectation value (integration) as in (Bielecki & Rutkowski, 2004, pp. 51-55):

$$\mathrm{Var}[\ln V_t] = \sigma^2 \text{ and } \sigma^2(T-t) := \sigma^2[\ln V_t] \text{ and } \mathrm{E}[\ln V_t] = \ln V_0 + (\mu - \frac{\sigma^2}{2})(T-t) \tag{149}$$

Furthermore then applying the normal distribution

$$\ln V_t \sim \mathrm{N}[\ln V_0 + \left(\mu - \frac{\sigma^2}{2}\right)(T-t), \sigma^2(T-t)] \tag{150}$$

Thus the conclusion is:

$$P\,(V_t < D_t) = \phi(\frac{D_t - \mathrm{E}[V_t]}{\sigma[V_t]}) = P(\ln V_t < \ln D_t) \qquad (151)$$

as the ln function is monotone increasing. One gets

$$PD = \phi(\frac{\ln D_T - \ln V_T - (\mu - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}) \qquad (152)$$

(Bielecki & Rutkowski, 2004, p. 53).

So, one could derive the *PD* once one would get to know the parameters V_T, D_T, σ and $\mu = r$ (Bielecki & Rutkowski, 2004). Yet, all these parameters are already known in advance by the company's capital structure and market data (σ, r) and V_T is substituted by the stock price motion.

More precisely the following relationship, which links equity and asset (start) values, is exploited, utilizing their volatilities and "the 'hedge ratio' of the Black Scholes formula" (de Laurentis et al., 2010, p. 60; Itô, 1951):

$$\sigma_{equity}\, E_0 = \sigma_{asset}\, \mathrm{N}(d_1)\, V_0 \qquad (153)$$

For the volatility normally the historical standard deviation of returns is chosen, which can be directly seen from price information (Anderson & Sundaresan, 2000; Merton, 1974).

One can hence, as shown, directly draw the link to option price theory, which was also introduced in the early 1970s by Merton, Black and Scholes, and the structure of the capital of a company, cf. the original works in (Black & Scholes, 1973; Merton, 1973; Merton, 1973b).

Furthermore, as in CAPM, the company's credit risk is decomposed into two components. One is the systematic risk, which is depending on the industry sector and the country of the obligor, and the other one is the non-systematic or idiosyncratic risk, which depends solely on the specific company itself. One should not confuse systematic risk with systemic risk, the risk which lies in the entire financial system ("systemically") and is an observation object for prudential regulation.

These systematic risk and common systematic risk (and asset price) factors for various assets are also connected with a joint default probability (Lando, 2000a; Lando, 2000b), as will be explained.

By construction, Merton's first model does not allow for default before the maturity of the claim (Bielecki & Rutkowski, 2004; Black & Cox, 1976). As mentioned, however, various structural models were developed – inter alia a famous variant by Black and Cox – for a valuation of corporate debt where this restriction was eliminated and the default is defined as the first time the debt threshold is crossed as in (Black & Cox, 1976; Briys & de Varenne, 1997; Li, 2016).

The original motivation of Cox was to find an optimal capital structure of a company in terms of debt service and (choice of an) optimal default (Bielecki & Rutkowski, 2004, p. 82). The work was extended by Leland in the context of levered firms on an infinite time horizon and by Leland and Toft on a finite time horizon, taking bankruptcy costs and tax issues into account (Leland, 1994; Leland & Toft, 1996). Strategic debt service in that regard and debt negotiations are an important part before (or once starting) restructuring or defaulting in practice. Hence, also these components were further included with ideas, e.g., from the Scandinavian researchers Anderson, Sundaresan, and Ericsson (Anderson & Sundaresan, 1996; Ericsson, 2000).

Many other restrictions and assumptions could be lifted and extensions with subordinated debt capital, dividends paid, safety covenants for the debt, optimal permanent capital, or variable interest rates have been incorporated in structural models (Black & Cox, 1976; Goldstein et al., 2001; Ho & Singer, 1982; Ho & Singer, 1984, pp. 315-336; Jou & Lee, 2009; Leland, 1994; Lettau & Wachter, 2011; Longstaff & Schwartz, 1995; Vašíček, 1984). An overview is also given by Bielecki and Rutkowski in (Bielecki & Rutkowski, 2004, pp. 58, 88-89).

Even a variable time-dependent threshold of default as by Collin-Dufresne and Goldstein and the risk of the events of defect were introduced leading to realistic, well-calibrated models (Collin-Dufresne & Goldstein, 2001; Driessen, 2005).

The most important improvement was that the original model was soon extended from defaults to credit migrations depending on, e.g., ratings of a company, related to their assets by Jarrow et al. (Jarrow & Protter, 2015; J. P. Morgan, 1997; RiskMetrics Group, 2007, pp. 60, 62, 65-76). Mathematically, the migrations are often described by Markov chains and Markovian models (Bielecki & Rutkowski, 2004, pp. 352-385; Jarrow et al., 1997). It is a stochastic process for which a state at time t just depends on the one at time $t-1$ and inherent factors (i.e., the "history of the process" and a memory is not needed). As these migrations are leading to higher credit spreads through the means of higher credit migration and default risks once a downward rating appears, empirically different forward rate curves for discounting the bond value have to be applied (J. P. Morgan, 1997; RiskMetrics Group, 2007). This implies lower (present) values of the future (discounted) cash flows and ultimately migrations downward also lead to portfolio losses.

One can collect default probabilities from historical default rates of similar companies, i.e., within the same industry, country, market cap and rating. Then supplementary calibrating the threshold of the described structural models as described before as

$$PD = \mathrm{P}(V < D) \tag{154}$$

where V is again the face value of the assets and D the value of the debt of a company, one can also get historical transition matrices (Gordy, 1998b; J. P. Morgan, 1997; RiskMetrics Group, 2007). These are called "rating migration matrices" and denote the probability of (similar) assets of a certain class and rating to migrate to another rating.

Original rating	Probability of rating after five years (in Percent)							
	AAA	AA	A	BBB	BB	B	CCC	D (Default)
AAA	80.71	17.00	1.77	0.18	0.16	0.08	0.08	0.04
AA	0.94	82.35	14.97	1.37	0.14	0.13	0.13	0.06
A	0.07	3.24	85.51	9.96	0.76	0.28	0.28	0.14
BBB	0.02	0.24	6.71	84.33	6.88	1.17	1.17	0.43
BB	0.02	0.07	0.42	9.51	74.24	12.70	12.70	1.83
B	0.00	0.04	0.18	0.66	9.66	73.61	73.61	8.92
CCC	0.00	0.01	0.21	0.41	1.85	21.40	21.40	48.82
D (Default)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Figure 38 Rating transition matrix.

Source: Own illustration (from the empirical comparison calculation in Chapter 5).

Similar as before, one then links the returns of a stock and the migration probabilities. However, not only the default threshold is set, but also the different rating migration thresholds. Technically, as returns are modeled by a normal distribution, the inverse cumulative normal distribution and its quantiles are considered. To exemplify this one may take a bond with rating BBB and get:

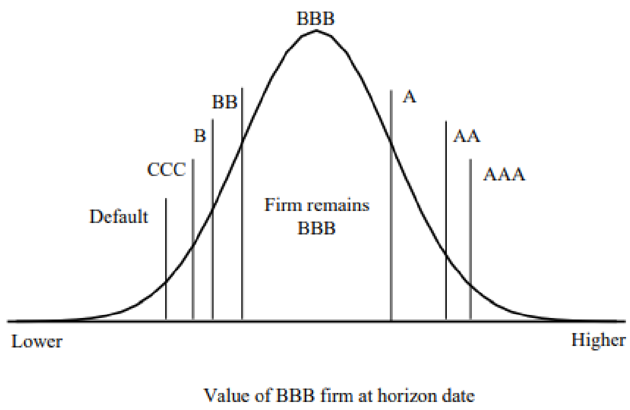

Figure 39 Quantiles of the normal distribution with corresponding ratings (example).

Source: Illustration from CreditMetrics® Technical Document (RiskMetrics Group, 2007, p. 67).

The different quantiles show the probability of a rating migration to that class. One can see that in this example and the related figure above. The probability of the bond to stay in the BBB rating is the highest, the second-highest probability is an upgrade to grade A, BB (down) or AA (two notches up) are a bit less likely, etc. When implementing a structural model in, e.g., CreditMetrics® one then has to look at simultaneous migration matrices of many assets – as the probability that for three stocks S1, S2, S3 with initial ratings R(S1) = AAA, R(S2) = BB, R(S3) = C after one period the ratings are R*(S1) = AA, R*(S2) = BB (same), R*(S3) = BBB, this has to be done for all possible rating and migration combinations. As a consequence, in the case of very many assets, Monte Carlo simulations are performed in practice instead of analytical calculations as the former are less complex and much faster (Erlenmaier & Gerbach, 2000). However, in either case the correlation of assets and corresponding joint movements have to be included in the model (Lando, 2000a; Lando, 2000b; J. P. Morgan, 1997; RiskMetrics Group, 2007, pp. 85-91, 92-102). Especially for low-default portfolios, the modeling of migration risk is even the most important component in practice, e.g., for bond portfolios that are containing high-rated obligors (J. P. Morgan, 1997; RiskMetrics Group, 2007).

Apart from the probability of default and of migration (PD, PM), CreditMetrics® allows for convenient classification of the losses which occur in case of a default. These are the LGDs or the recovery rates RR (or R, which again equals one minus the LGD). They denote the rate of a bond that is (typically) possible to receive back after a default. As for the rating classes, a segmentation of loan and debt types is made according to the collateralization or securitization and seniority class of credit and tables of average recovery rates are considered then (J. P. Morgan, 1997; RiskMetrics Group, 2007). Intuitively the higher ranked (in case of seniority) and more secured or collateralized a loan is the higher the recovery rate is as well. For the individually different so-called workout-LGD, first, the collateralization structure has to be regarded, and the amount after bringing the collateral into the market is considered (Daldrup, 2006, pp. 91-93; de Laurentis et al., 2010). Afterward, the loss in book or market value is included. Finally, direct costs, e.g., for lawyers or incisor services and indirect ones residing in the NPL[194] or loan resolution department of a bank as administration, IT, workout-related costs, etc., are taken into account. An empirical analysis of recovery rates and hence LGD was, e.g., done by Altman et al., Gupton, and by Grunert and Weber (Altman et al., 2004; Grunert & Weber, 2005; Grunert & Weber, 2007; Gupton, 2005; Privara et al., 2013; Witzany, 2009). CreditMetrics® is using an LGD classification of different classes of debtors like the following table from (Paulsen, 2009, p. 6). A very similar one with recovery rates is to be found in (Witzany, 2017, p. 122).

Table 2 LGDs for certain classes of debt with mean and standard deviation.

Class of debt	**LGD mean**	**LGD standard deviation**
Senior Secured	46.20	26.86
Senior Unsecured	48.87	25.45
Senior Subordinated	51.48	23.81
Subordinated	67.26	20.18
Junior Subordinated	82.91	10.90

Source: (Paulsen, 2009, p. 6).

As the table shows the loss given default (on average) increases with less seniority of the debt and the variability in terms of the standard deviation becomes less, i.e., individual recovery factors are less important.

Furthermore, similar to the intensity rate for reduced form models it is possible to use deterministic, time-dependent LGDs (or RRs) – hence a function LGD(t) – or even stochastic recovery rates (Böttger et al., 2008; Witzany, 2009b). Time-dependent or

[194] Non-performing loan

stochastic interest rates are commonly involved in the model already (Bielecki & Rutkowski, 2004; pp. 51, 59; Jamshidian, 1989; Shimko et al. 1993). Models by Kim or similar Longstaff and Schwartz regard interest rate and credit risk in their models together (Bielecki & Rutkowski, 2004, pp. 96, 98; Kim, 1993a; Kim, 1993b; Longstaff & Schwartz, 1995). The observation of stochastic interest rates may also include special forms like credit spreads, e.g., dealt with in a structural framework by Shirakawa or dispense independence assumptions between the value process and stochastic interest rates altogether (Bielecki & Rutkowski, 2004, p. 58; Saa-Requejo & Santa-Clara, 1999; Shirakawa, 1999, pp. 83-97; Szatzschneider, 2000).

Even extensions of CreditMetrics® which model dependence between the (stochastic) LGD and the default indicators, commonly called “PD-LGD correlation”, within a factor model were developed recently. First ideas in this regard were developed by Altman et al. (Altman, 2002). Frye later finds that different approaches in that by Pykhtin, Giese, Tasche, and Hillebrand yield remarkably similar results (Frye, 2013, pp. 3-4; Giese, 2005, pp. 79-84; Hillebrand, 2006, pp. 120-125; Pykhtin, 2003, pp.74-78; Tasche, 2004). Frye introduces a certain LGD function and concludes risk managers are better served by using the LGD function as by utilizing (noisy) statistical models which are calibrated to the available data (Frye, 2013, p. 2, 12). The LGD function “connects the conditionally expected LGD rate (cLGD) to the conditionally expected default rate (cDR)” treating conditionality the same way that underlies the conditional default in hazard rate models and using a Vasicek model for the dynamic (Frye, 2013, p. 2). Tasche, by improving Pykthin and others, models the LGD on the basis of a single risk factor modeling of the LGD and the default event using a Vasicek process and integration via Legendre polynomials (Emmer & Tasche, 2016, p. 10). Witzany improved these models by introducing a further systematic LGD-factor and hence a two-factor PD-LGD model, with ARIMA(p, q) processes describing the (autocorrelated) PD and LGD time series and subsequently solves the equations via Markov chain Monte Carlo simulations (MCMC) as shown in (Witzany, 2011; Witzany, 2017, pp. 130-132). He showed that an increase of the unexpected loss (economic capital) of 30 % might occur (Witzany, 2011; Witzany, 2017, p. 132).

Ozdemir and Miu show that the lack of correlation can be controlled, as common in risk management, by utilizing a higher degree of conservatism[195] in the cyclical LGDs when employing their PIT framework. (Davis, 2011; Miu & Ozdemir, 2016). Interestingly, they extend the one-year horizon of A-IRB models for PD, LGD, and EAD to a three-year horizon and different scenarios, the methodology expected from the IFRS 9 framework – for IFRS stage 2, hence the expected (credit) loss over lifetime (ECL/ELL) – in their paper (Miu & Ozdemir, 2016). Whereas differences are generally small and ought to be completely ruled out by consistent standards over time one should mention that there still exist minor differences in the accounting point of view (IFRS and GAAP) concerning loan provisions and the economic capital or Basel III-based point of view (Witzany, 2017, pp. 6-7, 90).

Another intuitive possibility to model the PD and LGD correlation would be by a straightforward differentiating of the various components of correlations in question, e.g., when modeled by a linear multi-factor framework, and then adding again the mentioned margins of conservatism. This means the mean PD and LGD need to be increased for not taking the correlations into account (Miu & Ozdemir, 2016).

Lee, Rösch, and Scheule examined the "smile formed" dependency, especially for mortgages (Lee et al., 2014). Another model extension of Ozdemir and Miu by German researchers was – instead of capturing "only" the dependence between the default indicator and the LGD and here also including the secured and unsecured recovery rates – additionally taking into account the remaining risk parameter EAD by means of the utilization rate at default and hence models all dependencies (Eckert et al., 2016).

Each of these variables in the model is driven by an individual risk driver, which in turn again depends on systematic factors as well as on idiosyncratic factors. Risk factors are coupled within a multi-factor framework, then connected through a linear regressive structure and equipped with their individual (and in general a priori unknown) weights. A multivariate extension of Heckman's selection model of 1979 was developed for the model in order to estimate the unknown parameters in an unbiased fashion within a maximum likelihood framework (Eckert et al., 2016).

As a conclusion the Tasche model, having the advantage of a not necessary empirical calibration, and especially the latest extension by Fischer, Eckert, and Jakob,

[195] Cf. MoCs as mentioned in Chapter 1.

which is empirically calibrated, however, covers all risk parameters, are the most developed extensions in the recent years. To summarize, for structural credit portfolio models with rating transitions as in CreditMetrics®, the following steps have to be executed.

1. Determination of the PD and PM for every single credit of the portfolio and the derivation of the (rating) transition matrix.
2. The valuation of the assets of the portfolio according to the correlated scenarios, normally using empirical stock correlations, and hence proxy joint probabilities of transition from one rating to another finally using the different rating-corresponding forward curves for discounting, is done. The applied scenarios are, e.g., for big portfolios, then created with Monte Carlo simulations and in that case not analytically traceable.
3. The calculation of the CreditVaR as the unexpected loss is executed.

If feasible then the steps are accompanied by a PD-LGD link as in the Tasche or Eckert models.

The most common industry implementations of structural models are KMV® by the mentioned authors Kealhofer, McQuown, and Vašíček of KMV® (founded in 1989, later acquired by Moody's ®) and the CreditMetrics® model of J. P. Morgan® in 1997 (J. P. Morgan, 1997; Kealhofer, 1997; Vašíček, 2012). The original proposal and the technical document on CreditMetrics® are valuable sources for learning how the ideas for constructing CreditMetrics® were developed and how it was assembled (J. P. Morgan, 1997; RiskMetrics Group, 2007).

KMV® is a slight modification of CreditMetrics®, where especially the default threshold is set in a different manner (Lu, 2005). It uses a so-called distance to default (in terms of asset price minus debt price as well as time) originally introduced by Vašíček and an empirical relationship between that distance and estimated default rates or frequencies, the Expected Default Frequency EDF (Kealhofer, 1997; Vašíček, 1984). It was the first popular industrial implementation of a structural model and was sold to Moody's ® after some time (Bouteillé & Coogan-Pushner, 2021). For the empirical relationship to be built it needed samples from firms.

KMV® has observed in an empirical fashion from a sample of companies that these are generally more likely to default in case their asset values breach a critical level. This level generally lies at some point between the value of its total liabilities and total short-term debt (Alexander & Sheedy, 2014; Ong, 2005). For the KMV® model the Default Point (DPT), in some terminologies the default threshold, is approximated as follows (Bouteillé & Coogan-Pushner, 2021; Kealhofer, 1997; Witzany, 2017, pp. 143-148):

$$DPT = STD + 0.5\,LTD \tag{155}$$

where it is declared STD: Short-Term Debt and LTD: Long-Term Debt. The default point amount is hence approximated via the total sum of short-term debt and half of the long-term one.

Furthermore, before computing the probability of default KMV® models calculate an index called Distance to Default (DD) as in (Bouteillé & Coogan-Pushner, 2021).

$$DD = \frac{\mathrm{E}[V_T] - DPT}{\sigma} \tag{156}$$

This is the difference between the expectation value of a firm's assets and the Default Point, normalized by the standard deviation of the firm's future asset returns (Bielecki & Rutkowski, 2004; Ong, 2005; Vašíček, 1984). As the expected value is taken, it is a forward-looking measure.

A further difference is that KMV® models use historical sets of frequencies of default (the mentioned EDF, Expected Default Frequencies) instead of a theoretical normal or log-normal distribution and are therefore slightly more flexible when calibrating the models (Bouteillé & Coogan-Pushner, 2021, pp. 88-90; Kealhofer, 1997; Witzany, 2017, pp. 143-148). This plays especially out for special small companies (SMEs) as shown by Zhang et al. (Zhang et al., 2007). However, it is economically less feasible, and the exact calibration is a business secret. Bohn together with Crosbie introduced some extensions of the KMV® model that soften the condition that the relationship between asset volatility and equity volatility is fixed, and they justify the use of a KMV model in that regard (Crosbie, 2002; Kamali et al., 2020, p. 35). A further enhancement of the Merton model was done by estimating volatility directly from market-observable returns on a company's value (Charitou et al., 2013).

Differences between the model KMV®[196] and the CreditMetrics® model are generally of minor importance, in practice CreditMetrics® is preferred by much more banks and better to calibrate empirically (Frey & Mcneil, 2003; Hahnenstein, 2003; Hickman & Koyluoglu, 1998; Kealhofer & Kurbat, 2001). Various researchers like Su & Huang, or Li et al. also showed the superiority of the "CreditMetrics®-extension" ZPP compared to the KMV® approach and similar variants (Fantazzini & Zimin, 2020, p. 1; Li et al., 2016; Su & Huang, 2010). ZPP is based on a copula approach to link stock and bond prices in the asset-value model (de Giuli et al., 2007). It also uses a Monte Carlo simulation for generating the results. It assumes that a company defaults, once the price level is nonpositive (≤ 0), as a price process the authors used however a (student-*t* process including) GARCH method (de Giuli et al., 2007; Kamali et al., 2020). Fantazzini further found a method to include his ZPP model for credit risk in a general multivariate framework to measure market and credit risk simultaneously, e.g., for cryptos (Fantazzini & Zimin, 2020).

Another possibility to measure the migrations in credit risk is to measure the credit spread change occurring directly after a migration ceterus paribus. It is useful to predict credit spreads with the help of so-called credit spread models (CSM).

As mentioned, several credit spread models exist, are however considerably less often employed than CPMs and are mainly related to problems arising in traded markets. Fons and Foss mainly contributed to the early development of spread models (Fons, 1987; Fons, 1994; Foss, 1995). The most famous proponents are however the Das or the Nielsen and Ronn model (Das & Tufano, 1996; Das 1997; Nielsen & Ronn, 1997, pp. 175-196). Both types of models are two-factor models, which describe the stochastic default-free (short-term) interest rate and the stochastic movement of a short-term credit spread, which are usually both correlated via Brownian motions (Bielecki & Rutkowski, 2004, p. 264; Das 1995; Nielsen & Ronn, 1997).

There is literature for structural as well as reduced form models linking them to credit spreads data and subsequently full credit spread models (Das & Tufano, 1996; Giesecke & Goldberg, 2005; Jarrow et al., 1997). In a similar context, Das and Tufano

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[196] Moody's ® derived and developed further versions like RiskCalc® (Kocagil et al., 2001). Also RiskFrontier® and other variants were built, the newest one being the cloud-based software PortfolioManager ® from 2021 (Moody's, 2021).

also introduced model extensions of CPMs with stochastic recovery rates (Bielecki & Rutkowski, 2004, p. 269; Das & Tufano, 1996).

The most promising approach is extracting the credit spreads via the payout ratio (Grass, 2013). The utilized measure CS_{POR} then denotes the increase in continuous interest payments to creditors which is necessary to exactly offset the impact of an increase in the asset variance on the option value of the debt (Grass, 2013, p. 28). The measure used by Grass in his work is applied to the credit spreads of corporate bonds and credit default swaps (CDS) and clearly outperforms the used benchmark. Its associated predictors outperform the ones from a powerful reduced form model (Grass, 2013).

Before that result, Mason and Rosenfeld (1984) showed how these types of credit risk models "systematically underestimated observed spreads" (Arora et al., 2005, p. 1; Mason & Rosenfeld, 1984; Zhu et al., 2005). Their research observed companies during the period 1977 to 1981, as described in (Arora et. al, 2005, p. 1; Mason & Rosenfeld, 1984). Ogden later also confirmed that, finding that the structural models "under-predicted spreads over U.S. treasuries by an average of 104 basis points" (Arora et al., 2005, p. 1; Ogden, 1987). Furthermore, Lyden and Saraniti executed a comparative analysis of the Merton and the Longstaff-Schwartz models, which are an extension of the Merton model with stochastic interest rates, and find that both models still under-estimated the credit spreads by around 50-100 basis points (Lyden & Saraniti, 2001; Ogden, 1987).

The later introduced KMV® model, as explained before, "appears to produce unbiased, robust predictions of corporate bond credit spreads" (Agrawal et al., 2004; Arora et al., 2005, p. 2).

Hence as a result, structural (Merton) models were also modified. Until then, the typical structural models estimated a "corporate-risk-free reference curve" instead of utilizing the treasury curve (Arora et al., 2005, p. 2). Arora, Bohn, and Zhu found that the under-prediction which seemed to appear for the Merton model resulted from the selection of a "wrong" benchmark curve,

> in the sense that the spread over U.S. treasuries includes more than compensation for just corporate credit risk. The assumption here is that the appropriate corporate default risk-free curve is closer to the U.S. swap curve (typical estimates are ten to twenty basis points less than the U.S. swap curve.). (Arora et al., 2005, p. 2)

Correcting for that choice and selecting the appropriate curve yields plausible results for Merton models as well (Arora et al., 2005, pp. 3, 19). Further extensions for it were, e.g., the assumption of a liquidity or funding premium in connection with a company's direct access to capital markets, time-varying market confidence premiums and an expected recovery amount which is time-varying – the latter already described before (Arora et al., 2005, p. 3; Bielecki & Rutkowski, 2003; Bielecki & Rutkowski, 2004; Grass, 2013; RiskMetrics Group, 2007).

All these modifications, of which many are commonly used today as a post-crisis multi-curve approach are state-of-the-art in current CreditMetrics®-implementations, contributing to finally realistic credit spread (CS) estimates as shown by Grass. Reduced form models on the contrary are generally worse in that regard on a broader scale (Grass, 2013). The advantage however is, as they are atheoretic (economically), they are quite abstract and flexible in their functional form and application. Unfortunately, however, as Duffie proved, this flexibility results in a model fitting rather strong in-sample, if carefully calibrated to a certain set of obligors and spreads, but with poor out-of-sample properties and hence general predictive ability (Arora et al., 2005, p. 4; Duffie, 1999). Duffie further showed that the parameter estimates can be fairly unstable (Duffie, 1999; Geyer et al., 2001).

These results are already shedding some light on the overall picture when structural and reduced form models are compared.

Before comparing these two models the third type of Credit Portfolio models used in the industry is briefly introduced, which is known as the econometrically-based model approach, like CreditPortfolioView®. As econometric models are fairly similar to structural models, solely a brief, compact overview is provided. In the banking industry, many savings and loans associations like the (rating) service providers S-Rating® and RSU®[197] for the German Sparkassen and Landesbanken[198] or American S&L[199] associations apply that model.[200] The econometric models' aim is to provide an empirically estimated connection between each obligor's default rate and a normally

[197] Which is the RSU Rating Service Unit GmbH & Co. KG in Munich.
[198] State-owned banks in Germany, shareholders are mainly the connected federal states like the free state of Bavaria for the Bayern LB®, Baden-Württemberg for LBBW®, Lower-Saxony for NORD/LB® and further Sparkassen associations of the states.
[199] Savings & loans
[200] As can be seen (in German) on https://www.s-rating-risikosysteme.de/Unser_Aufgabenspektrum/CreditPortfolioView.html?msclkid=fb215a49ac1c11ec8c29519f390b0b7d (Retrieved Mai 7, 2022)

distributed index of macroeconomic parameters linked by a “link function”. Standard parameters include, e.g., inflation, GDP growth, or unemployment figures (Bluhm et al., 2003; Bucay & Rosen, 2001; Wilson, 1998).

As common in the rating industry the index $y_{i,t}$ of macro-parameters can be transformed to a default probability utilizing a Logit (or Probit) function (Wilson, 1998; Witzany, 2017, p. 138):

$$p_{i,t} = \frac{1}{1 + e^{y(i,t)}} \tag{157}$$

The macroeconomic index $y_{i,t}$ is composed of different normally distributed macro-variables with lagged dependency $z_{j,t}$, such that for all j (in the index) and discrete times t:

$$z_{j,t} = a_{j,0} + a_{j,1} z_{j,t-1} + a_{j,2} z_{j,t-2} + \cdots + \varepsilon_{j,t} \tag{158}$$

with $\varepsilon_{j,t}$ denoting a normally distributed random shock (or innovation). This is due to the fact that macro-economic shocks might transfer to the target variable with a certain lag in time (Wilson, 1998). So, the macro-variables are autoregressive as in other ARIMA models (Witzany, 2017, p. 139). As result, one derives

$$y_{i,t} = b_{i,0} + b_{i,1} z_{1,t} + b_{i,2} z_{2,t} + \cdots + \varepsilon_{j,t}{}^{*} \tag{159}$$

with $\varepsilon^*_{j,t}$ being a normally distributed random shock, also called “innovation” in economics (Bluhm et al., 2003; Witzany, 2017).

The factor loadings for the index are then completely determined by the empirical relationship between the default rates and the macroeconomic variables calculated by – as the formula further above indicates – logistic regression (Hickman & Koyluoglu, 1998; Wilson, 1998). The macro variables’ sensitivities (“betas”) themselves are also calculated by regression of the empirical model. A test for the calibration of the logistic regression can be the Hosmer-Lemeshow one as seen before.

The use of the model and the calculation of the loss distribution by Monte Carlo (MC) simulation is summarized by the following five steps (Wilson, 1998):

1. Depending on their covariance structures, random innovations are drawn to both, each macroeconomic variable and the index value, by using empirical time series and hence include the covariances of the data
2. Calculation of:

2.1 the macroeconomic variables’ results received from the lagged (past) values as well as random shocks (building it up from past values and 1.)

2.2 index values from the macroeconomic values and the index random shocks (like in 2.1)

2.3 The default probabilities are derived as a result.

3. The distribution of defaults is calculated by convoluting each debtor's distribution of results (consisting of two states, which are then used for convolution). This is done for every iteration.

4. A distribution of portfolio losses by calculating ten thousand of paths for the MC simulation is executed as next step four.

5. Extraction of the VaR for a given time interval and confidence level is implemented finally.

The original founder of CreditPortfolioView® is Wilson and the company McKinsey® (Wilson, 1998). It is now widely used in various derivative forms within the banking industry. Apart from the empirical regression and link function via logistical regression, which has to be estimated at the beginning, it is quite similar to structural models also exploiting a Monte Carlo simulation for calculating the VaR (Bielecki & Rutkowski, 2004; Witzany, 2017, p. 140). However, such a forecast model induces additional uncertainty (Hamerle et al., 2003, p. 14). Furthermore, it is not as widely used in the industry and with more shortcomings regarding, i.e., the parameter estimations and regressions as will be shown in the following sub-chapter.

4.3 Reduced Form Versus Structural Models in Measuring and Managing Credit Portfolio Risks

Both types of Credit Portfolio models structural and reduced form models have their inherent strengths and shortcomings. There is extensive literature comparing these models as shown in this chapter and many different ways of categorization.

The first and most evident differences are inherent in the models themselves and are encoded in the way a default is described and on the assumptions the models have. The following table is featuring these differences in a direct manner, stemming from a literature review and practical use-cases (Bélanger et al., 2003; Bluhm et al., 2003; Diaz & Gemmill, 2002; Gordy, 1998b; Hickman & Koyluoglu, 1998; Jarrow & Protter, 2015):

Table 3 Differences between hazard rate models and structural models, with different properties.

Model: type and equivalent used names	**Intensity rate models (or hazard rate models or reduced form models, actuarial models, (mixed) Poisson-Gamma distribution models, credit default models[201])**	**Structural models (or Merton models, EV/Enterprise-value models, asset-value models, credit migration (portfolio) models)**
Economic background, accessibility	No direct economic derivative, harder to access and imagine, atheoretic but flexible	Direct economic derivative, dependence on one (in case of a one-factor model, “business cycle”) or more economically feasible factors, good to access
Default characteristics	Defaults appear randomly, exogenously defined, no prior hypothesis on the causes of default of a company	Defaults and its causes are (totally) pre-defined by a company’s debt, its asset structure and its value’s movement
Information (requirement)	No further information more than general market information is required	(Full) information about the company’s capital is required and transparent in the market in the standard version
Origins	Valuation of companies’ assets and capital-structure, option price theory	Insurance mathematics, actuarial, and stochastic background
Generality	Not too general, often “overfitting”	Very high and general

[201] Which is however not entirely correct as also, e.g., structural models can be assembled as pure default (mode) models as the Gordy model or CreditMetrics® in a simplified version without migrations, etc. As seen on the other hand also extended migration mode versions of, e.g., CreditRisk+® exist, yet both not in its standard version.

Calibration	Very exactly	Very exactly
Extensions (as illustrated in detail in the sub-chapters above)	Migration mode models, stochastic interest rates, correlations – in the basic version the number of defaults over a period is independent from that of any other period, stochastic recoveries and linkage through copulas (though not common and CID is still the most popular)	Multi-factor models, randomness in defaults, multi-curve approaches, stochastic interest rates, correlations – in the basic version the number of defaults over a period is independent from that of any other period, stochastic recoveries and linkage through copulas as, e.g., in ZPP (though not common yet)

Source: Own illustration, based upon an own collection including the named sources in parentheses above.

Having laid out these differences one can directly derive the following results:

- The structural models are more approachable and economically feasible compared to reduced form models whereas
- Reduced form models are easier to implement, they need less data, storage, and computer performance.

The first important comparison of credit portfolio models, enriched by ideas from Gordy, was done by Hickman and Koyluoglu (Gordy, 1998b; Hickman & Koyluoglu, 1998). The primary aim of this comparison was to show that the underlying ideas, theories and results of the three credit portfolio models CreditMetrics®, CreditRisk+® and CreditPortfolioView® are similar and by putting them within a single general framework and harmonizing them, as well as adjusting their parameters, Koyluoglu and Hickman could prove that. Wong and later Bélanger developed similar ideas of generalization and unification (Bélanger, 2001; Wong, 1998). They also use these three components (indirectly) and utilize a generalized default time (Bélanger, 2001; Bielecki & Rutkowski, 2004, p. 250; Wong, 1998).

The core principle is to set up a generalized framework consisting of three main components, in which regard the division of conditional defaults and joint defaults is similar to CreditRisk+®:

The joint default distribution is again describing the correlation in the portfolio, i.e., how strongly obligors' conditional default rates vary together in various cases or "states" (Lando, 2000b). It is also referred to as dependent defaults (Bielecki & Rutkowski, 2004, pp. 293- 313).

The second is the conditional distribution of the portfolio default rate. For each "case" and its corresponding obligors' conditional default rates, one can derive the common conditional distribution of a homogeneous sub-portfolio default rate in the same way as if individual defaults would be independent (Bielecki & Rutkowski, 2004; Hickman & Koyluoglu, 1998). One speaks of conditionally independent defaults (Bielecki & Rutkowski, 2004, pp. 265-292).

The third component is then the final aggregation of the data. The aim is to receive the unconditional portfolio default distribution. This distribution is calculated by aggregating and averaging the homogeneous sub-portfolios' conditional distributions of defaults in each "state" weighted by the corresponding probability of the given state (Wong, 1998).

This heuristically direct feasible scheme can be represented as follows:

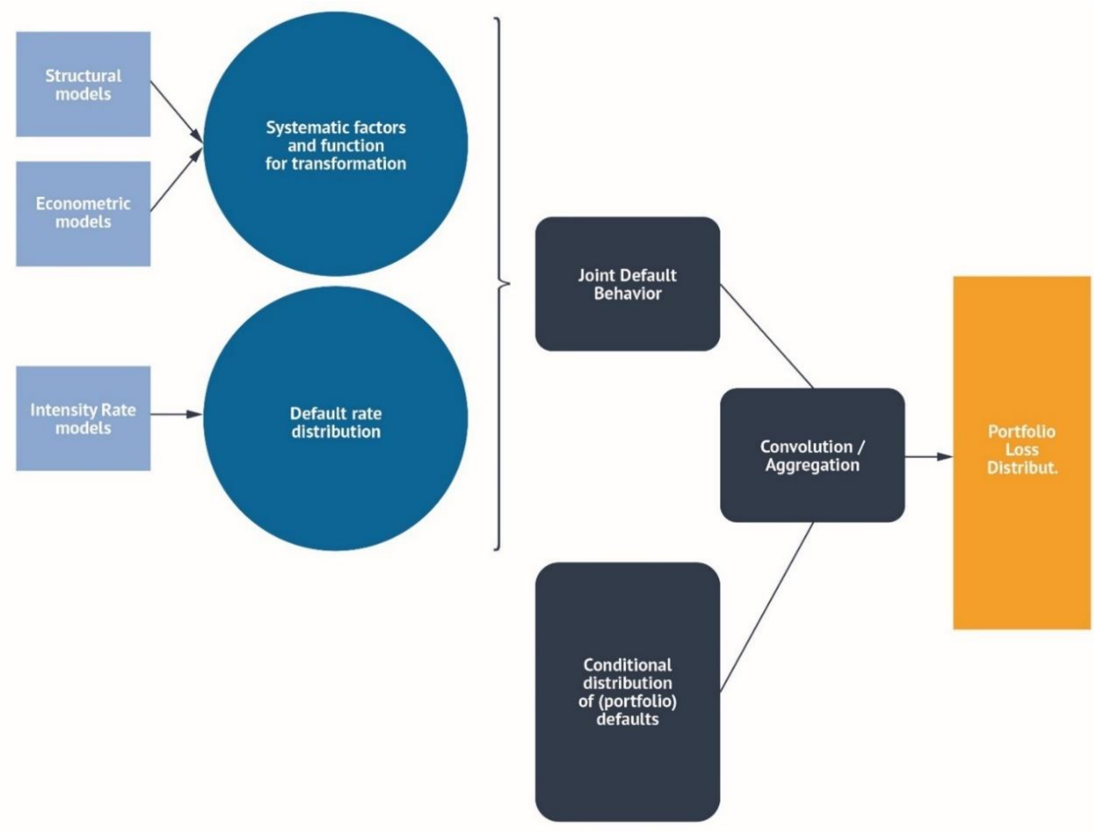

Figure 40 Common framework for structural, hazard rate and econometric credit risk models.

Source: Own illustration.

The default rate distribution is given explicitly in hazard rate models and implicitly in structural (or econometric) ones. All three models mentioned above further link their defaults rates to variables describing the economic cycle or state ("systematic factors"), and this fact can be described by an underlying transformative conditional default rate function (Hickman & Koyluoglu, 1998; Wong, 1998).

For deriving the transformative function in the structural model the asset value change has to be disassembled, as common in the context of multi-linear models, into a set of orthonormal systematic factors x_k, which are all governed by a normal distribution in the same way as the further idiosyncratic factor ε_i (Hickman & Koyluoglu, 1998, p. 5).

$$\Delta V_i = a_{i,1}x_1 + a_{i,2}x_2 + \cdots + \sqrt{(1 - \sum_k a_{i,k}{}^2 \varepsilon_i)} \tag{160}$$

with factor loadings $a_{i,k}$ and x_k, ε_i iid ~ N [1,0], and ΔV_i i.i.d ~ N [1,0] (Hickman & Koyluoglu, 1998, p. 5). Hence, the complete asset value change is normally distributed.

The mean of the distribution is determined by the factor loadings and values, the standard deviation as a second necessary parameter for a normal distribution is further determined by the weight of the idiosyncratic factor (Hickman & Koyluoglu, 1998, p. 5).

This transformation resembles the multi-factor approach of the econometric models as will be discovered soon.

In practice, the systematic factors and their loadings introduced above are chosen in such a manner that they are able to replicate the empirically given asset correlations (Gordy, 1998b). The correlations are generally derived by a Cholesky decomposition or principal components analysis – PCA (Hull, 2015, p. 230; RiskMetrics Group, 2007, p. 115). For uniqueness, as known from linear algebra, *N* one-dimensional factors for *N* obligors are required, otherwise less.

Given a default threshold *c* and the condition $\Delta V_i \leq c$ for defaults as before (Gordy, 1998b). Then the unconditional default probability, denoted as $p^\star$, is given by the calibration $p^\star = \Phi(c)$, where Φ is labeling the cumulative density function (cdf) of the normal distribution as common in statistics (Gordy, 1998b; Hickman & Koyluglu, 1998; J.P. Morgan, 1997). The default rate, which is “conditioned on the values of systematic factors”, is then by the arguments of factor loadings defining mean (*m*) and deviation (*d*) directly written as

$$p_{i|X} = \Phi\left(\frac{c-\sum_k a_{i,k}\,\mathrm{x}_k)}{\sqrt{1-\sum_k {a_{i,k}}^2}}\right) = \Phi\left(\frac{c-m}{d}\right) \tag{161}$$

(Bluhm et al., 2003; Hickman & Koyluoglu, 1998, p. 5).

For a homogeneous portfolio, the systematic factors can be conveniently expressed by one single variable *y*, with $\mu = \sum_k {a_{i,k}}^2$ describing the asset correlation in the homogeneous portfolio (Hickman & Koyluoglu, 1998, p. 5). Then the searched probability density function for the default rate $f(p)$, which is related to the probability density function of normally distributed systematic factors $\phi(y)$ as seen, can be derived by means of using the Radon-Nikodym-derivative (Björk, 2009, pp. 478-479). That yields

$$f(p) = \frac{\sqrt{1-\mu}\ \phi\left(\dfrac{c-\Phi^{-1}(p)\sqrt{1-\mu}}{\sqrt{\mu}}\right)}{\sqrt{\mu}\,\phi\left(\Phi^{-1}(p)\right)} \tag{162}$$

as seen in (Björk, 2009; Hickman & Koyluoglu, 1998, p. 5).

The RN-derivative is basically a means and accompanying theory to change (substitute) different measures and densities of probability distributions in a pre-defined way to simplify them (Björk, 2009).

In the hazard rate model, the default rate distribution $f\,(p;\ \mu;\ \sigma)$ is Gamma distributed as described before. Hence, to account for a normally distributed systematic factor the following equation for the transformation function and all points ($a;\ b$) must be upheld (Hickman & Koylugolu, 1998, p. 6; Wong, 1998):

$$\int_0^b \Gamma(p;\ \alpha;\ \beta)\, dp = \int_a^\infty \phi(m)\, dm \qquad (163)$$

Hence, the function for the transformation is

$$p|_m \ = \ \Theta^{-1}(1 - \ \Phi(m);\ \alpha;\ \beta) \qquad (164)$$

In that expression, Θ is denoting the cumulative density function of the Gamma function. While the approach is described with respect to normally distributed systematic factors, the normal distribution is not a necessary assumption as seen in the derivation of formulas – nonnormality would still make them comparable in the same manner, as long as standard conditions like the existence of inverse CDFs is upheld, it may just change the specific results (Hickman & Koyluoglu, 1998, p. 7).

After describing the joint default behavior in the three models the next step as referenced above is the conditional distribution of portfolio default rates (Wong, 1998). Conditional means again that a homogeneous sub-portfolio is considered, in which all borrowers' debts are independent when fixed (conditional) default rates are given as described above. As already illustrated the probability of k defaults in an n obligor-portfolio then follows a Binomial distribution. While some models explicitly use a Binomial distribution, e.g., econometric ones, and some implicitly like structural ones the use of the distribution in the various models can be summed up as follows, see also (Wong, 1998):

In structural models, one calculates asset value change for each obligor and is then checking for default (with result yes/no), which is at the end equivalent to the two states yes/no of a Binomial distribution.

CreditPortfolioView and other econometric models convolute the individual obligor's distributions in an iterative procedure as mentioned and these individual ones are all directly Binomial distributions.

CreditRisk+® approximates the Binomial with the Poisson distribution as described before and as the Poisson distribution is the limiting distribution for the Binomial distribution as shown in (Eberlein, 2007), for reasonable portfolios with few defaults, hence where the probability that multiple defaults occur is low, there is no significant difference as shown by Stuart and Ord (Stuart & Ord, 1994).

Therefore, one can describe the conditional probability (CP) of default rates by a uniform Binomial setting and finally needs an aggregation function as the third necessary part of the framework. As shown before, the unconditional probability distribution of defaults is derived by aggregating, i.e., by "averaging across the conditional distributions of portfolio defaults" for all various "states of the world", which are weighted by the probability of a given state with the help of a convolution integral (Frey & McNeil, 2001; Hickman & Koyluoglu, 1998, p. 7).

For a homogeneous sub-portfolio with n obligors and with a single systematic factor, which is (roughly) normally distributed, in a structural or econometric model one subsequently obtains for the convolution integral the formula below (Duffie & Singleton, 1999; Hickman & Koyluoglu, 1998; Wong et al., 1998).

$$\mathrm{P}\ (k\ default|\ n\ obligors) = \int_{-\infty}^{\infty} \mathrm{B}(k;\ n;\ p|_m)\ \phi(m)\ dm \qquad (165)$$

For a homogeneous sub-portfolio in the case of a hazard rate model the integral, as shown before, is consisting of Poisson distributed independent obligor default rates convoluted with a Gamma function:

$$\mathrm{P}\ (k\ default|\ n\ obligors) = \int_{0}^{\infty} \mathrm{P}(k; np)\ \Gamma(p; \alpha; \beta)\ dp \qquad (166)$$

Here the convolution of the Poisson distribution with the Gamma distribution yields the Negative Binomial Distribution as mentioned in the CreditRisk+® description in this chapter.

The integrals are directly calculable and in all of the three modeling approaches "the procedures are (theoretically) exact in the limit", using enough Monte Carlo simulations in the structural or econometric cases respectively iterations and small band sizes in the hazard rate case of models (Hickman & Koyluoglu, 1998, p. 8). The extension is straightforward.

An overview of the 3-step-process executed can be seen in the following figure:

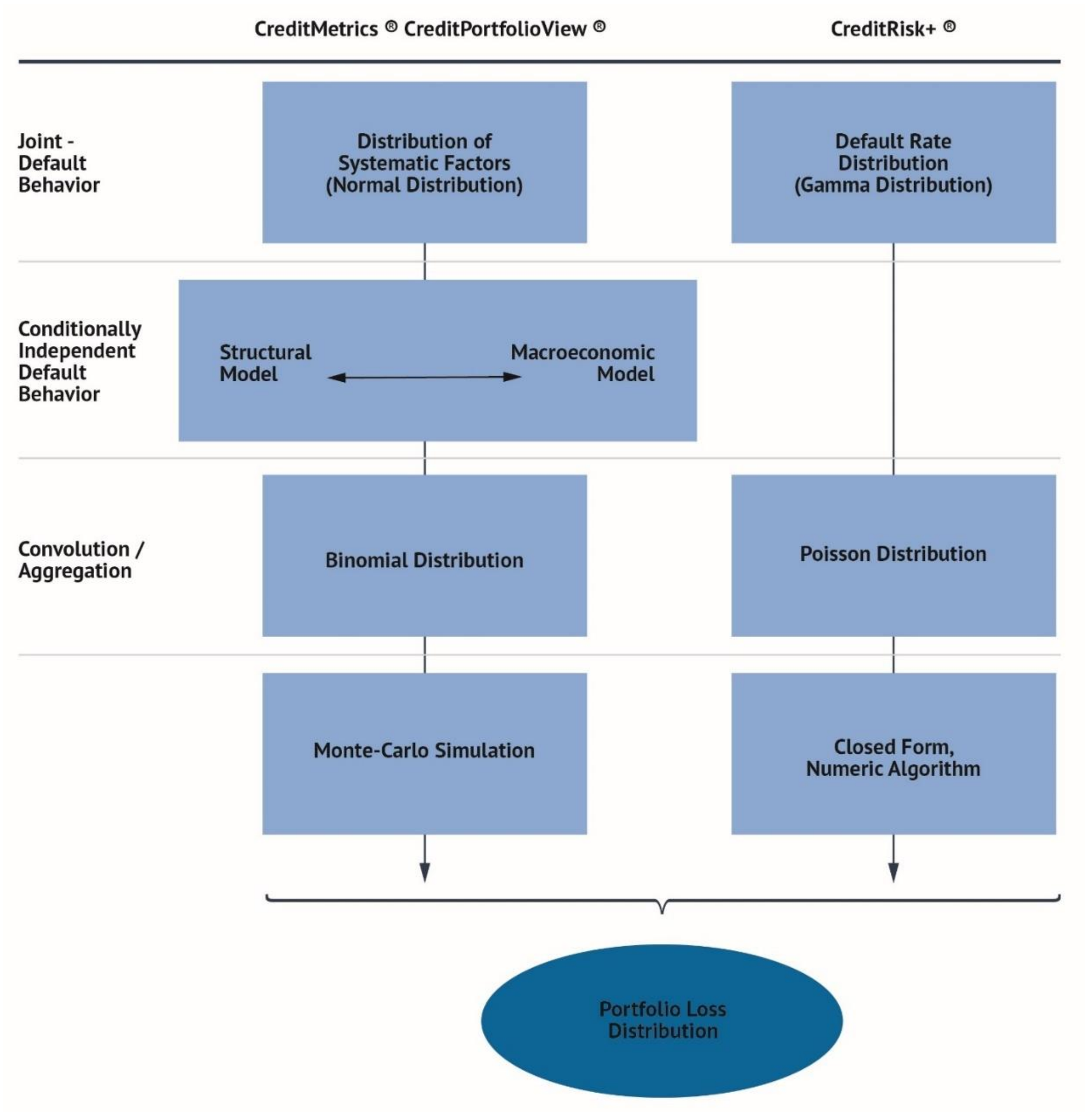

Figure 41 Unified framework and components of commercial credit portfolio models.

Source: Own illustration in line with (Hickman & Koyluoglu, 1998, p. 9).

As all models rely on the parameters of unconditional default probability and joint-default behavior the last parameter is the decisive one diverging in the different models and appearing in various forms. Structural models utilize asset correlations, econometric models calculate regression coefficients for macroeconomic factors incorporating correlations amongst the factors and actuarial models exploit the default rate volatilities of different sectors (Frey & McNeil, 2001; Gordy, 1998b; RiskMetrics Group, 2007, pp. 92-102). An extension of CreditRisk+, the CreditRisk++ model by Han and Kang mentioned again later, exists which uses explicit asset correlations (with a risk factor decomposition to a systematic and an idiosyncratic factor) as well and is in that case classified with structural ones, yet performs very similar as CreditRisk+ (Han, 2014; Han & Kang, 2008).

All these parameters for joint-default behavior are related and yield equivalent information for describing joint-default behavior (Bélanger et al., 2003; Wong, 1998).

To link coefficients and correlations, as described before, the "joint-default behavior" is represented in structural models as an asset correlation matrix of the pairwise asset correlations, or in an equivalent fashion represented by a set of asset factor loadings in the following way (Hickman & Koyluoglu, 1998, p. 9; RiskMetrics Group, 2007, pp. 92-102):

$$\Delta V_i = a_{i,1}x_1 + a_{i,2}x_2 + \cdots + \sqrt{(1 - \sum_k a_{i,k}{}^2 \varepsilon_i)} \tag{167}$$

with factor loadings $a_{i,k}$ and x_k, ε_i i.i.d ~ N[1,0], and ΔV_i i.i.d ~ N[1,0]. In the next step, because the systematic factors were defined above to be orthonormal[202], one can obtain:

$$\begin{aligned} &Correlation\left[\Delta V_i, \Delta V_j\right] \\ &= \frac{\left(\mathrm{E}\left[\Delta V_i\, \Delta V_j\right] - \mathrm{E}[\Delta V_i]\mathrm{E}\left[\Delta V_j\right]\right)}{\sqrt{\left(\mathrm{E}\left[\Delta V_i{}^2\right] - \mathrm{E}[\Delta V_i\,]^2\right) - \left(\mathrm{E}\left[\Delta V_j{}^2\right] - \mathrm{E}\left[\Delta V_j\,\right]^2\right)}} \\ &= a_{i,1}a_{j,1} + a_{i,2}a_{j,2} + \ \ldots \end{aligned} \tag{168}$$

Hence, a correlation matrix is directly calculated given the factor loadings of assets and on the other hand, factor loadings are derived from the correlation matrix (Bluhm et al., 2003; Hickman & Koyluoglu, 1998, p. 9; Wilson, 1998).

The econometric models' logistic regression coefficients, characterizing the relationship of the default rate "index" to macroeconomic variables, apparently have a very strong similarity to the asset factor loadings of the structural models and therefore an index correlation is described in the same fashion (Bluhm et al., 2003).

Now the default rate volatility s is just calculated by the standard integral formula for the variance s^2:

$$s^2 = \int_0^\infty (p - p^*)^2 f(p)dp \tag{169}$$

and for a structural model (with r: asset correlation):

[202] This means orthogonal (the scalar product is equal to zero) and normalized with Euclidean norm ("length") one.

$$s^2 = \int_0^\infty (\Phi(\frac{\Phi^{-1}(p^*) - m\sqrt{r}}{\sqrt{1-r}}) - p^*)^2 \phi(m)\, dm \qquad (170)$$

Consequently, the default rate volatility is calculated as a function of *r* (correlation) and *p** (unconditional default rate) (Gordy, 1998b; Hickman & Koyluoglu, 1998).

For the special case of a homogeneous sub-portfolio, the correspondence between default correlation and variance of the default rate is even directly feasible and approaches

$$s^2 = (1 - p^*)p^* r_{default} \qquad (171)$$

(Hickman & Koyluoglu, 1998, p. 11).

Generally, all models utilize a two-parameter-defined default rate distribution. Therefore, the mean and standard deviation (or first and second moments or respectively here the unconditional default rate and the standard deviation of the unconditional default rate) are “sufficient statistics to define the parameters for any of the models” (Hickman & Koyluoglu, 1998, p. 12; Witte & Witte, 2010).

Hence, by setting the unconditional default rate *p** and standard deviation of default rates, one can derive the necessary parameters for all three kinds of models to describe them within the same presented framework and transform them into each other.

The structural model needs the parameters *c* (threshold) and *r* (asset correlation) as seen. With

$$c = \Phi^{-1}(p^*) \qquad (172)$$

and then

$$s^2 = \int_0^\infty (\Phi(\frac{\Phi^{-1}(p^*) - m\sqrt{r}}{\sqrt{1-r}}) - p^*)^2 \phi(m)\, dm \qquad (173)$$

one can derive *c* and *r* by setting *p** and *s*.

For the econometric model, the factor loadings and coefficients are defined and by regrouping and setting

$$y_{i,t} = U_i + V_i m, \qquad (174)$$

where

$$U_i = a_{i,0} + \sum_k a_{i,k} \ (b_{k,0} + \sum_j a_{k,j} x_{k,t-j}) \qquad (175)$$

and V_i is the residual parameter

$$V = \sqrt{var(\zeta_{i,t}) + \sum_k (2\, a_{i,k}\, cov(\zeta_{i,t},\, e_{k,t}) + a_{i,k}{}^2\, var(e_{k,t}) + \sum_{k \neq m} a_{i,k}\, a_{i,m}\, cov(\, e_{m,t}, e_{k,t}))} \quad (176)$$

with m ~ N[0,1], condensing the macroeconomic variables and index into a single equation for the index $y_{i,t}$, such that

$$y_{i,t} = (a_{i,0} + \sum_k a_{i,k} \;\; (b_{k,0} + \sum_j a_{k,j} x_{k,t-j}\,)) + \sum_k a_{i,k}\;\, e_{k,t} + \zeta_{i,t} \quad (177)$$

it is required to derive just U and V (Engel, 2008; Hickman & Koyluoglu, 1998, p. 10; Wilson, 1998).

This can be achieved with the two equations (given p^* and s):

$$p^* = \int_{-\infty}^{\infty} \frac{1}{1 + e^{U+Vm}}\, \phi(m)\, dm \quad (178)$$

$$s^2 = \int_{-\infty}^{\infty} \left(\frac{1}{1 + e^{U+Vm}} - p^*\right)^2 \phi(m)\, dm \quad (179)$$

The parameters of the actuarial model are directly derived from p^* and s by definition (cf. above) by

$$\alpha = \frac{p^{*2}}{s^2} \quad (180)$$

and

$$\beta = \frac{s^2}{p^*} \quad (181)$$

For all of the presented models the probability density functions which are describing the default rate look “very similar”, with only minor discrepancies at the tail within extreme rates (Hickman & Koyluoglu, 1998, pp. 13–18; Wong, 1998).

Hickman and Koyluoglu conclude that “the models are virtually indistinguishable when the systematic factor is greater than negative two standard deviations, which accounts for almost 98% of the probability mass” (Hickman & Koyluoglu, 1998, p. 13).

All models then finally belong to a single general framework, which consists of three components

- the (joint) default rate distribution
- the conditional default distribution
- the convolution/aggregation technique to receive an unconditional default distribution

Differences between the various models only "arise from differences in modeling joint-default behavior which manifest in the default rate distribution" (Hickman & Koyluoglu, 1998, pp. 17-20; Overbeck & Schmidt, 2009). Hence, after the joint default parameters are harmonized (by setting p^* and s) the default rate distributions are remarkably similar and comparable. This fundamental result is apparently mathematically replicable and was confirmed by many studies like Wahrenburg and Niethen, by Schwarz, or Gordy (Gordy, 1998b; Schwarz, 2006; Wahrenburg & Niethen, 2000, pp. 14 - 20).

Differences in the results of the three models can therefore be attributed to parameter value estimates implicating the default rate behavior.

These estimates, however, can induce significant differences between the models with factors or multiples larger than 3, as Wahrenburg and Niethen showed with an example of n homogeneous loans, all from the German building sector (Wahrenburg & Niethen, 2000). They made use of same sized-loans, as to avoid size concentration effects on the VaR estimator, with estimators for empirical input data derived from the insolvency time series (1980-1994) of the German official federal statistics office for the building sector ("Statistisches Bundesamt für das Baugewerbe") as well as stock returns from building companies in the German stock index DAX 100® from that time (Wahrenburg & Niethen, 2000). Gordy in another study, which was concentrating on default correlations, assumes that all loans are ordinary term loans, the distribution in size and S & P ® (covering a 17 year period from 1981 to 1997) as well as Moody's ® (it covers the 29 year period from 1970 to 1998) rating grades are used; the loans are further characterized according to data from two large samples consisting of middle-sized and large-sized corporate loans from the Federal Reserve Board surveys of large bank organizations (Gordy, 1998b; Gordy & Heitfield, 2002). The number of obligors is increasing in three different scenarios, and the concentration is calibrated by dividing the number of obligors across the rating grades and then determining how the exposure within these grades is distributed across the number of all obligors in this grade – denoted as "brackets" (Gordy & Heitfield, 2002).

The models also perform in a similar way for average quality commercial loan portfolios when σ is small for CreditRisk+®. Gordy generally stresses the extreme sensitivity of CreditRisk+® to σ and in these cases seems to prefer CreditMetrics® as more robust (Gordy, 1998b, pp. 23-24).

Also in other respects, especially for so-called low-default portfolios[203] and large obligors, structural models seem to perform better with the result of systematically higher default rates, and hazard rate ones may underestimate default correlations as shown in the following (BCBS, 2005; Gordy, 1998b; Kolman, 2010).

For instance, Diaz is extending the analysis carried out by Koyluoglu and Hickman. As seen before, their framework – similar unified frameworks were also developed by Bélanger et al. and Wong – allowed comparing the default distributions of both models under equivalent parameters (Bélanger et al., 2003; Gordy, 1998b; Hickman & Koyluoglu, 1998; Wong, 1998). Diaz is extending this study by comparing an enhanced CreditRisk+® and the full version of CreditMetrics® considering migration risks for both and by setting up a slightly extended mathematical framework to compare the loss distributions. The conclusion is that for internal purposes CreditMetrics® is more precise and thus preferred, the CVaR difference is up to 19 % (Diaz & Gemmill, 2002; Diaz & Gemmill, 2011, pp. 3, 33-37). Han found that for Han's and Kang's proposed CreditRisk+ extension CreditRisk++, which allows for explicit asset correlation modeling and has high flexibility, the result is still similar (not better for CreditRisk++) for standard risk weights even when just compared to a two-state CreditMetrics® (Han, 2014, p. 16; Han & Kang, 2008).

Just for low-quality retail portfolios, when migration risk accounts for a truly little proportion of the overall CVaR or ratings are not available, the results between CreditMetrics® and CreditRisk+® are similar, then CreditRisk+® is in some cases a faster and less expensive choice (Diaz & Gemmill, 2011 p. 35).

Hamerle et al. consider both models valuable and comparable (rather equivalent) in their empirical analysis (Hamerle et al., 2004; Hamerle et al., 2003).

Two of the most comprehensive studies on the subject of comparing credit portfolio models, an older one by Crouhy, Galai, and Mark and a more recent one by Kollár and Gondžárová, also conclude that CreditMetrics® is the preferred choice, with the latter (study) saying that the "biggest disadvantage of CreditRisk+® model comes from Poisson distribution, because it underestimates the probability of default for all

[203] Were the probability of default being << 1 % and therefore so low that it is hard to measure and observe (as for high-quality triple-A companies or states like Norway or Germany) and hence nearly impossible to historically backtest as mentioned before.

rating grades" (Crouhy et al., 2000; Kollár & Gondžárová, 2014, p. 346). A similar concern is shared by Stein (Stein, 2002).

A study by Arora, Bohn, and Zhu confirms the preference for structural models citing that "a[sic] (HW) reduced-form model largely underperforms a sophisticated structural model like that of the VK model (as implemented by MKMV)" and also confirms that the "performance of the VK model is more consistent across large and small firms, while the performance of the HW and Merton models worsens considerably across larger firms" (Arora et al., 2005, p. 13). In another large study, with an empirical part as well as based on a supervisory credit risk model applied in practice, Hamerle et al. showed the superiority of models with economic (structural) factors and furthermore that PDs and default correlations can be linked as lagged variables (Hamerle, 2004). In terms of realistic default correlations structural models – even though they appear in few cases to be slightly too conservative, while hazard rate ones too less – seem to clearly outperform hazard rate ones (Zhang et al., 2008; Zeng & Zhang, 2001). A further advantage of CreditMetrics® is visible, when considering low-default portfolios with high ratings. As the main part of the credit VaR is then attributed from the migration component and much less from the defaults, the inherently rating migration focused CreditMetrics® can play out its strength (Kollár & Gondžárová, 2014; RiskMetrics Group, 2007; Zhang et al., 2008).

As mentioned before, however, CreditRisk+® is convenient to implement and requires fewer data.

One can also compare the models from an informational point of view, i.e., in terms of the information which is assumed to be known a priori by the modeler or market. Structural models generally assume that the modeler has complete information concerning all the debtor's assets and liabilities, i.e., similar to the management of the company (Jarrow & Cenit, 2004). Hence, one derives a predictable default time (Merton, 1974). That is a distinctive characteristic of structural models at first. Contrary to that, hazard rate models operate under the assumption that the modeler has just the same information set as the market has – in practice thus an incomplete knowledge of the company's assets and liabilities (Duffie & Lando, 2001). This leads to an inaccessible default time (Credit Suisse First Boston, 1997).

Hence, the distinction between structural and hazard rate models, in this case, takes place along the information set criterion, whether the information set is observable by the market participants or whether this is not the case (Jarrow & Protter, 2004; Jarrow & Protter, 2015). Therefore – at first – according to Jarrow and Protter, for pricing and hedging purposes in incomplete markets, reduced form models might be regarded as the method of choice (Jarrow & Protter, 2004).

However, Giesecke and Goldberg showed in 2004 that "it is possible to develop a structural model in which the modeler also has incomplete information about the default point, making the time-to-default inaccessible even in a structural model" (Arora et al., 2005, p. 24; Giesecke & Goldberg, 2004). Duffie and Lando 2001 proposed a hybrid model[204], which is incorporating noisy accounting information, leading to an inaccessible default time in the realm of structural models as well (Duffie and Lando, 2001, p. 633-664). Later Jarrow himself proved the result in an alternative form (Jarrow, 2004). Madan and Unal and later Davydov et al. also developed hybrid models, which contain structural and actuarial/hazard rate components exploiting the Feynman-Kac formula[205] (Davydov et al., 1999; Madan & Unal, 1998, pp. 43-65; Madan & Unal, 2000, pp. 141-160).

Finally, Kolman showed in a large and realistically calibrated empirical study that, when the models CreditRisk+®, KMV® and CreditMetrics® are applied to a portfolio containing a rather limited number of bonds or rated or large corporate exposures as in the study, the latter two clearly outperform the first method and yield similar and more realistic results, not underestimating the VaR (Kolman, 2010; Witzany, 2017, pp. 149-151).

Table 4 Results of the comparison of CPMs by Kolman.

Model	99 % quantile	Unexpected Loss
CreditMetrics®	180.755	45.931
CreditRisk+®	168.000	24.867
KMV®	174.506	39.827

Source: (Kolman, 2010).

[204] Hybrid models are models combining structural and actuarial, hazard rate elements. Furthermore, as described before also CreditRisk+® is extensible by introducing multiple migration stages instead of a pure default modeling and CreditMetrics® by multi-factor modeling – yielding comparable results. Cf. Li as shown.

[205] The famous formula is linking parabolic partial differential equations of deterministic functions with the expectation value of stochastic processes. The Black-Scholes formula can be derived from a straightforward application (Björk, 2009).

Hence overall and in reply to the crucial question, which modeling approach is better in terms of identifying relative value in the predicted VaR compared to real portfolio data and its related credit losses, CreditMetrics® (or similar KMV®) as a structural model is the preferred choice for a credit portfolio model in general. It especially achieves to discriminate defaulters from non-defaulters more precisely than the alternatives, as seen. CreditRisk+® is, regarding its straightforward implementation and equivalent results for the segment of homogeneous unrated (unscored) retail portfolios, only preferred for these retail portfolios.

As it is the case for specific copula-based models with their data-fitted marginal distributions like the ZPP, also multi-factor extensions of CreditMetrics® coupling all risk parameters or hybrid models can be yet – as shown – further valuable for specific tasks and special situations.

CHAPTER 5
AI TECHNIQUES, THE SVM IN CREDIT PORTFOLIO RISK AND COMPARISON WITH TRADITIONAL MODELS

5.1 Categories of Artificial Intelligence, Artificial Neural Networks

In Chapter 5 a new approach to credit portfolio risk measurement with the help of support vector machines (SVM) is represented in a detailed fashion. While techniques such as the SVM (not the SVR) are already in use for single customer scoring and pre-classification, they are not applied to portfolios and dependent structures of debt yet. Support vector Machines are denoted as a classifier or regressor technique in the field of machine learning, and the aim is to categorize data in two (or iteratively more) classes like "0"/"1" or in the case of credit risk in "default"/"non-default" or, e.g., "VaR > 5%"/"VaR < 5 %". In the regressor case, SVM regression minimizes the distance between "observations" and calculated values, it acts in that regard like a "normal" regression (Schölkopf et al., 1998). A kind of tube having minimal radius is put – primarily symmetrically – around the estimated function, where all points outside it are penalized and all within a smaller radius not, hence minimalizing errors (Awad & Khanna, 2015, p. 67; Glover et al., 1990). SVM regression is the method and concept used for the final model in this thesis, it is often abbreviated as SVR, support vector regression (Awad & Khanna, 2015; Smola & Schölkopf, 1998b; Vapnik, 1998). The classifier-related idea is to put hyperplanes into the point space to separate the (two or more) point sets. This, under optimization conditions, ultimately leads to the "support(ing) vectors" of the hyperplanes and the name of the technique (Boser et al. 1992). If separation is not possible in the present dimension a transformation in higher dimensions (up to infinity) followed by separation of the point sets and backtransformation to the original dimension (space) is performed (Awad & Khanna, 2015b; Boser et al., 1992; Schölkopf et al., 1998). To prevent the calculations to run for too long, but instead to be suitable for computers and to perform a special kind of "re-transformation" needed, a form of scalar products and so-called kernels are utilized (Schölkopf et al., 1998; Vapnik, 1998). These kernels are well known in functional analysis and similarly also in nonparametric statistics and decode "density" in a certain sense (Conway, 2019; Hollander et al., 2015; Werner, 2007).

At the decisive stage, much training data is necessary to be found and with the help of penalty variables or so-called slack variables and a suitable kernel function an SVM is then separating as many point sets as exact as possible (Vapnik, 1998). Thus, the aim is to avoid over- as well as underfitting (Schölkopf et al., 1998). Overfitting means that the model is particularly good calibrated to the specific training data but performs much poorer when applied to new, different out-of-sample data. Hence, the method is not general enough in such a case. The other (extreme) case is a too general model like just a linear kernel approach, which is not precise enough (calibrated) for the training set. A "middle-ground" has to be found as will be shown later – directly linked with the so-called "bias-variance" trade-off (Dixon et al., 2020; Vapnik, 1998). A brief (general) introduction to machine learning and basic concepts is given now, to see which important approaches exist and in which area of AI the SVM belongs.

Especially since the 2010s AI-based techniques and herein mainly machine learning and deep learning techniques experienced a revival as computing power (cf. "Moore's law"[206], though meanwhile disputed) and the amount of available data enormously increased. It is commonly labeled as the rise of "big data" (Dixon et al., 2020, pp. 4-6). Furthermore, research breakthroughs as a seminal paper by Hinton, Krizhevsky, and Sutskever, practical applications and software tools such as IBM's Watson® by Ferrucci et al., Amazon's Alexa/Echo® box originally by Osowski[207], Google Analytics® by Chen and Clifton, self-driving cars like the ones from Waymo®[208] inspired by Thrun[209] and Levandovski, or traffic control research by Bayen[210] et al., lead to a rapidly evolving interest in artificial intelligence methods (Bayen et al., 2022; Clifton, 2008; Ferrucci et al., 2013; Harris, 2014; Krizhevsky et al., 2012).

The original ideas of artificial intelligence were already researched in the 1950s, inter alia by McCarthy or Samuel in his 1959 paper (Nilsson, 1998, Samuel, 1959).[211]

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[206] Stating that computing power doubles nearly every two years and was "valid" until the mid-2010s. Afterward, technological limits of leaking currents, lithography and sub-atomic structures made it impossible to uphold for the time being.
[207] Lukasz Osowski from the University of Gdańsk founded the company IVONA®, developing the text-to-speech and speech recognition engines that now run Alexa®. The company was later bought by and integrated into Amazon®. Cf. (Westerby, 2020).
[208] See Waymo® 's website at https://waymo.com/ (Retrieved Mai 12, 2022).
[209] See Harris, M. (2014). The Unknown Start-up That Built Google's First Self-Driving Car. IEEE Spectrum: Technology, Engineering, and Science News.
[210] Much of his research on autonomous cars and trucks as well as traffic control was initially developed from general fluid dynamics control techniques and also applied to river flows and air traffic control as seen in https://bayen.berkeley.edu/research/overview
[211] See (Gabel, 2019).

Ideas to connect a neuron's activity with logical functions were even already introduced by McCulloch and Pitts a decade before (McCulloch & Pitts, 1943). Rosenblatt built on that work and introduced the first "supervised learning" classifier as an artificial neural network, the perceptron, in 1957 (Rosenblatt, 1957). In 1955 Newell, Simon, and Shaw programmed the "Logic Theorist" commonly regarded as the first implementation of an "AI program" (Russell & Norvig, 2010). All of the named researchers, along with further scientists as especially Minsky, but also More, Rochester, or Shannon took part in the so-called Dartmouth Workshop of 1956 and shaped the development of AI for the next decades (Newquist, 1994, pp. 91-104).

The general idea of AI is the use of machines to solve problems with some kind of – to some degree – "replicated human intelligence" and by "learning", i.e., sensitivity to the data. This is distinct from mechanical automation, which is a machine following a set of pre-programmed static algorithms on (pre-defined) data to accomplish a rather repetitive task (Russell & Norvig, 2010).

AI happens in a spectrum ranging from "weak" or "narrow" AI to "strong" AI – with the goal of strong AI being a total or even better replication of human intelligence and reasoning. A test for "intelligence" for machines the so-called Turing test, abbreviated TT and originally labeled "imitation game", which is now a special case of a TT, dates back to the 1950s (Traiger, 2000, p. 99; Turing, A., 1950, p. 433). To date and obey the enormous progress made during the last years all AI is still considered "weak" (Rebala et al., 2019). It is widely used in (quantitative) asset and risk management firms and also in the Robo advisory (portfolio management) and fintech or insurance fintech ("insurtech") market (Blackrock, 2019; Dixon et al., 2020). Many models stem from hedge funds originally (Dixon & Halperlin, 2019, p. 1; Dori et al., 2018).

Machine learning (ML) as a special form of an AI technique means "extracting knowledge from data by identifying correlated relationships or patterns without receiving prior information about what causal dependencies to look for" (Dori et al., 2018, p. 3). Hence, as patterns and dependencies are detected which are not known before, this technique can be fruitful when applied to a sizeable portion of previously unordered data stemming, e.g., from automatized processing with the aim of categorizing them and "learning more" about their inherent structures (Dixon et al., 2012; Shanmuganathan & Samarasinghe, 2016).

Classification algorithms as a discrete sort of ML divide observations into a finite number of categories, and regression algorithms in ML estimate outcomes to problem settings that have an infinite (countable, overcountable or continuous) number of solutions (Dixon et al., 2020).

When thinking of rating classes and grades, PD ranges, and loss distributions these methods intuitively might work for credit risk as well. This is indeed shown later in that chapter. Furthermore, a common classification of the distinct types of machine learning can be done as follows (Dori et al., 2018; Rebala et al., 2019):

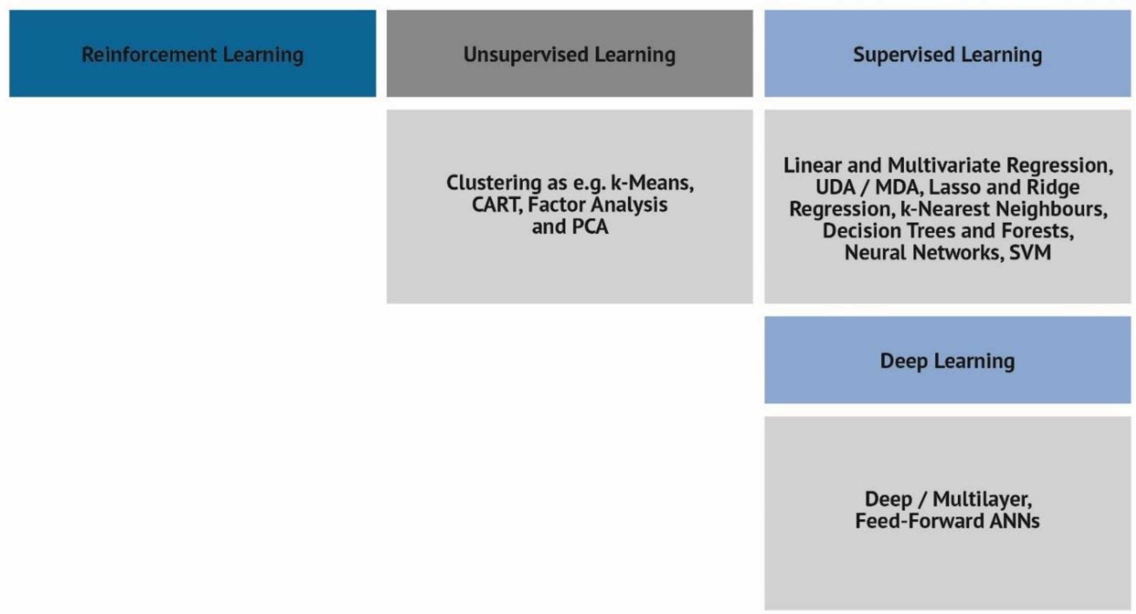

Figure 42 The three types of machine learning.

Sources: Own illustration in line with (Dori et al., 2018).

Supervised learning means that an algorithm learns based on training data (input and output) that expose known "relationships" (Dori et al., 2018). Those formalized relationships (denoted as models) are subsequently applied to test data and if applicable validation data, e.g., in *k*-fold cross-validation[212] (Dixon et al., 2020, pp. 140, 213). Unsupervised learning means that algorithms only learn from input data but have no further information about the output data or relationships (Dixon & Halperlin, 2019, pp.

[212] A validation technique where a certain set is divided (equally and) randomly into *k* chunks and first the training is done on 1-(*k*-1) and the test on *k*, then training on 1-(*k*-2) plus *k* and test on (*k*-1) and so forth, iteratively *k* times. Then one has *k* validation samples (and can, e.g., take the average results).

3-4). Therefore, these algorithms "detect patterns in the data by identifying clusters of observations that depend on similar characteristics" (Dori et al, 2018, p. 4). Combining in a certain sense such methods of unsupervised and supervised learning is known as reinforcement or reinforced learning (Shanmuganathan & Samarasinghe, 2016). It can be also viewed as "in-between" unsupervised and supervised learning, within the extremes of no output data and the exact output data label (Dixon et al., 2020, p. 22). These algorithms first detect patterns on their own, then additionally receive feedback from an exogenous source (e.g., a "teacher", "trainer", or "instructor") to further validate the learning process (Dori et al., 2018).

Therefore, one can regard the learning processes as guided by feedback. Hence, a reward feedback impulse for the algorithm is a necessity to learn the desired behavior (Dixon et al., 2020, p. 22).

A general trade-off exists between "optimising the fit of a model on the in-sample training and the true out-of-sample prediction" – the mentioned over-/underfitting balance (Dori et al., 2018, p. 3; Rebala et al., 2019).

In research as well as practical applications, most time and effort are indeed spent on properly selecting, preparing, using, and interpreting data (Ng et al., 2012; Ridzuan & Zainon, 2019). The models themselves are efficiently implemented in many statistical programming languages like Matlab®, SAS®, Python®, or R® and their corresponding packages and are readily available (Innes et al., 2018).

Especially the interpretation in statistical interference and the conclusions drawn by the samples have to be carried out in a prudent, conservative, and replicable manner. As usually in statistics, a (traditional) frequentist estimation, as well as Bayesian estimations, are possible (Dixon et al., 2020, p. 48). However, whereas in traditional statistics measures like R^2 (or goodness-of-fit generally), *p*-value, and *t*-value are crucial for statistical significance (and often unbiased estimators at hand) in terms of AI-based statistics and methods the focus (while also trying to minimize the error measure like *MSE*) lies on balancing out the Bias-Variance trade-off (Dixon et al., p. 112). Some of the differences can be found in the following tables, as well as an overview of differences when regarding dynamical processes and (time series) econometrics (AI in Orange).

Table 5 Statistical interference vs. AI-based methods as supervised learning compared.

Property	Statistical interference	AI-based methods as supervised learning
Goal	Causal (or at least correlated) models with explanatory power	Prediction performance, and explainability is of limited importance (though explainable AI exists and increases)
Data	The data is generated by a model (mostly parametric)	The data generation process is “unknown”
Framework	Probabilistic	Algorithmic and Probabilistic
Diagnostics	Extensive	Differs
Robustness	Prone-to Overfitting	Designed for out-of-sample usage
Model selection	Based on information criteria (e.g., AIC; BIC)	Numerical optimization
Expressabilty, Scalability	Often linear, best for low-dimensional data	Nonlinear, prone to high-dimensional data

Source: Own illustration in line with (Dori et al., 2018).

Sometimes traditional techniques like multivariate regression or logistic regression are denoted as (simple) AI as well – this is not true generally and only in (the rare) case of an unknown (nonparametric) error process – as can be seen from the table above (Dixon, 2020).

Different methods within one modeling framework might be applied by “widening the amount of available data” (e.g., through bootstrapping methods) and especially averaging them through changed settings (e.g., the use of distinct kernels in the case of SVMs and taking the arithmetic average of the results) to improve performance – the latter one called bagging. Bagging methods proved to be fruitful in several scientific areas, also for instance in the realm of financial applications (Witzany, 2017). Furthermore, boosting approaches, procedures that are applied sequentially and one model learns from the results of the former one, might be exploited (Bhavsar & Ganatra, 2012; Dixon, 2020; Witzany, 2017). The possibility to use several different AI-based models, apply them to the same problem, and then take an average or weighted result of them is finally known as ensemble, is even more sophisticated, and increasingly applied in the area of AI as well (Dixon, 2020).

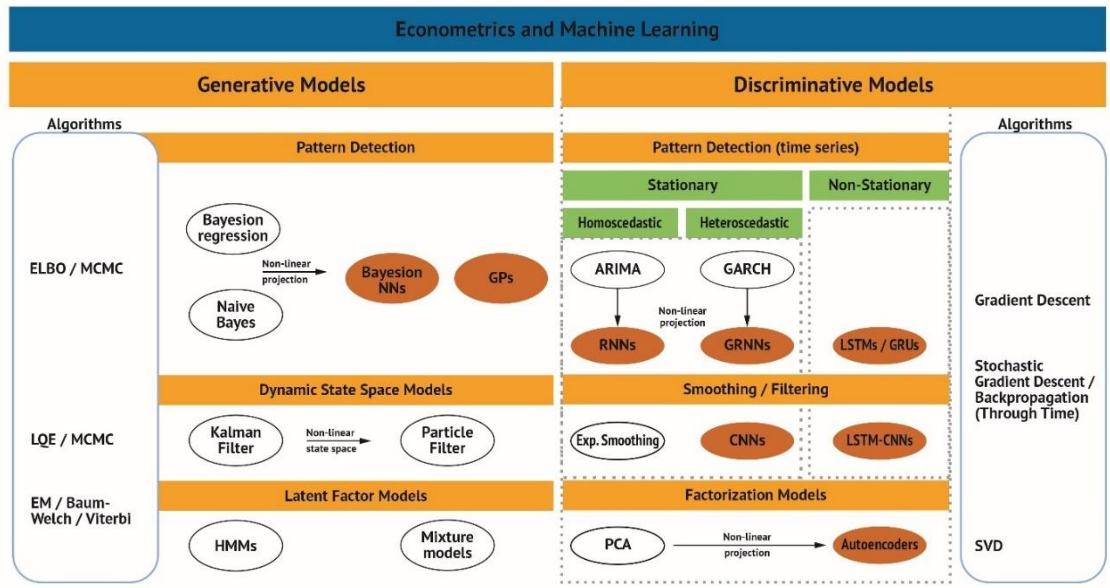

Figure 43 Overview of model types in econometrics and machine learning.

Source: Own illustration, directly based on (Dixon, 2020, p. 18).

For credit risk, (historical) data and expected results (labels) in regard to it are available and can be structured, hence supervised learning methods may be preferably utilized, instead of a "passive" unsupervised one, which is, e.g., already the case for individual scoring (de Laurentis et al. 2010; Dixon et al., 2020). Therefore, as this "information advantage" still exists and can be exploited for the realm of credit portfolios, supervised methods appear to be superior for portfolios as well and are chosen for the scope of the thesis.

When selecting a specific AI method in many complex applications for supervised learning most researchers might tend to choose between artificial neural networks (ANNs, often in the form of deep neural networks, DNNs, with many layers) and SVMs as in fault detection (Fuqing et al., 2013, p. 49; Lessmann et al., 2008).

The question of which one is "better" in terms of classification performance is highly debated in recent years in the literature and as often the answer is: It depends on the application.

For IT purposes like intrusion detection systems and classification of malware, etc. The SVM method seems to outperform the ANN one, as Chen, Hsu, and Shen showed, in mammography and some other clinical environments the ANN concept first seemed to outperform the SVM approach– as later studies yet showed a balanced data approach combined with the SVM seems to be even more promising (Chen et al., 2004; Huang et al. 2017; Ren, 2012; Shen et al., 2008). In detecting food contamination and public health risks SVMs outperformed ANNs in a big cross-border Nature study (Bisgin et al., 2018). Generally, SVMs performs rather well (Meyer et al., 2003). When different data-mining techniques (also in individual credit scoring) are compared the SVM approach (and also the ANN one) is among the very best techniques compared with other AI techniques and the industry standard (benchmark) logistic regression (Baesens et al., 2003; Bhavsar & Ganatra, 2012; Witzany, 2017, pp. 2, 85). A similar though less mild finding for the case of defect detection in software and the case of credit scoring – thereby showing the still very good results of logistic regression – is proven by Lessmann et al. (Lessmann et al., 2008; Lessmann et al., 2015). The latter study is revisiting credit scoring and roughly confirms Baesens et al. support vector machines are hence a very fruitful AI technique and will be utilized for CPM in this chapter.

The support vector machine further is a technique, which is solving an optimization problem in an analytical form, as it gives back exactly the same optimal hyperplane each time – other than genetic algorithms (GAs) or perceptrons (as a form of an ANN), which depend heavily on the initialization and the termination parameters in place (Awad & Khanna, 2015, p. 39).

In the last years in academic research in general, the ANN approach might seem to be slightly more popular and more articles were published related to ANNs, so it is introduced here as well (Jeeva, 2018; Shanmugathan & Samarasinghe, 2016). However, it was not until the 1980s before ANN algorithms reappeared (again) in active research, and as late as 2012, when Hinton proved that it is possible to apply generalized backpropagation algorithms to train multi-layer neural nets and with his paper, dealing especially with image recognition, thus finally reviving the field of deep learning (Krizhevsky et al., 2012). Since then, the use of ANNs became immensely popular within the scientific world (Hinton et al., 2012; Jeeva, 2018; Krizhevsky et al., 2012;

Shanmuganathan & Samarasinghe, 2016). His original work on backpropagation dates back to 1986 (Hinton, 1986; Plaut et al., 1986).[213]

As the name indicates ANNs are networks composed of artificial neurons – in contrast to the biological neurons of the human brain (Kandel et al., 2012). The ANN concept hence simulates the way the human brain processes information and ultimately learns (Rosenblatt, 1958; Shanmuganathan & Samarasinghe, 2016; Zou et al., 2008). The neurons or axions (or units/nodes) are interconnected and hence build up a "web" or network of connections (Rebala et al., 2019).

The neurons of an artificial neural network are arranged in layers and are usually interconnected in a fixed and given hierarchy (Dixon, 2020). The neurons are in most cases connected between two layers the so-called inter-neuron layer connection, but in rare cases also within a layer, denoted as intra-neuron layer connection (Kurková et al., 2018; Shanmuganathan & Samarasinghe, 2016).

These artificial neurons and layers are also lined up between input and output units (Dixon et al., 2020, pp. 114-119; Shanmuganathan & Samarasinghe, 2016). The input units receive information based on an internal system of weights, then the neural network learns from that presented information in order to produce the output (Shanmuganathan & Samarasinghe, 2016). An artificial neural network then makes use of a set of learning rules called backpropagation, more precisely: During the supervision phase, the neural network compares its real, actual output with the one it was supposed to produce - it is then adjusted during backpropagation (Barhoom et al., 2019, p. 8; Rebala et al., 2019; Dixon et al., 2020). In 1985, the "Backpropagation of Error learning procedure" was developed (separately) as a generalization of the delta rule (or chain rule in the context of gradient procedures), mainly by the Parallel Distributed Processing (PDP) Research group and Rumelhart et al. (Dixon et al., 2020; Rumelhart et al., 1986). As a consequence, nonlinearly separable problems are solvable by multilayer perceptrons[214] and not only,

[213] Interestingly, as shown in (Rumelhart et al., 1986) the backpropagation algorithm – similar to Itô's Lemma in stochastic calculus in Chapter 4 of the thesis – can be regarded as a generalization or form of the chain rule as well.

[214] The perceptron is a simplified artificial neural network. In its basic version (simple perceptron), it is built of a single artificial neuron that contains adjustable weights and a threshold function. Today, this term is used to refer to various combinations of the original model, distinguishing between single-layer and multi-layer perceptrons (MLP). Perceptron networks can be viewed as representing a so-called (simple) associative memory by its functionality and the ability "convert an input vector into an output vector" (Osterrieder et al., 2021, p. 7; Rosenblatt, 1958).

for instance, by means of SVMs as will be illustrated later (Kurková et al., 2018; Shanmuganathan & Samarasinghe, 2016). Generally, as briefly mentioned, the idea is to minimize the error between the output received and the correct, expected output and to adapt the weights of the neurons of the last layer accordingly, this procedure is done (neurons-layer-wise) backward until the input layer is reached. Then the network is executed again in the forward direction with the new, optimized weights and the new output is repeatedly minimized against the correct one, etc. (Dixon et al., 2020; Shanmuganathan & Samarasinge, 2016). Therefore, the "true solution" is approximated iteratively better under reasonable assumptions (Dixon et al., 2020). By the idea of further including all possible neurons in layers and then if needed setting them to zero in all directions (i.e., neuron deleted/no weight) or resetting them to a weight larger than zero neurons can be "deleted" or "added" and thresholds be updated by so-called on-neurons (Shanmuganathan & Samarasinge, 2016). Hence, the update of the weights is the decisive parameter (apart from the form of the activation function).

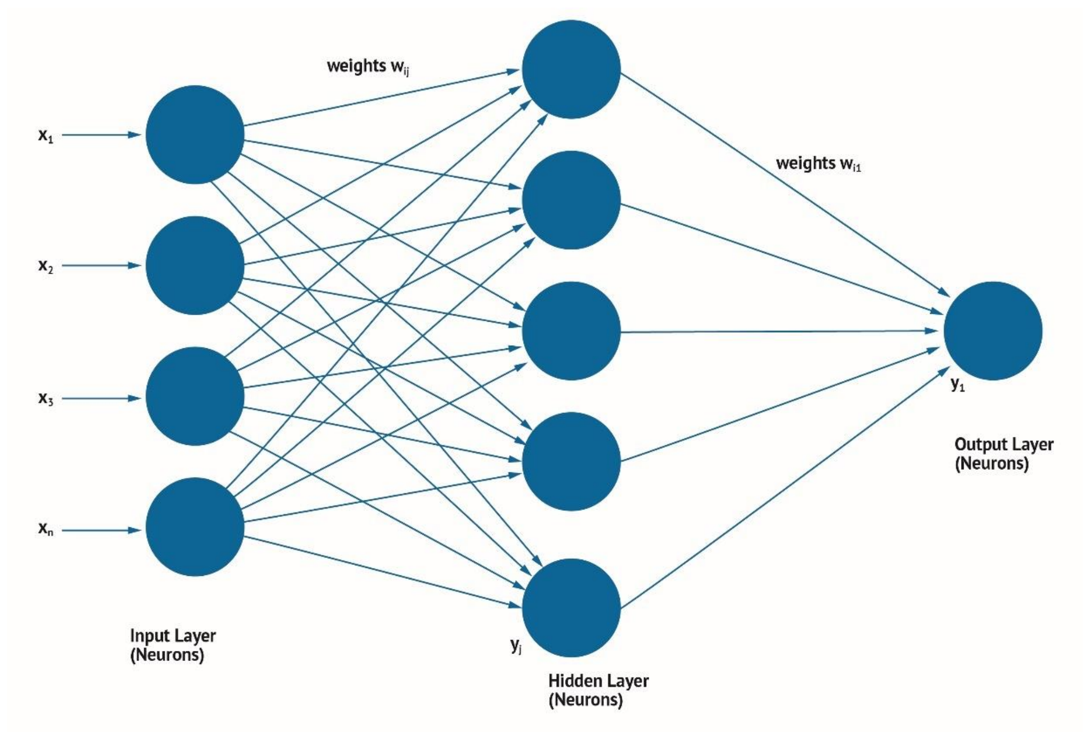

Figure 44 Layers of an artificial neural network.

Source: Own illustration, similar as in (Dixon et al., 2020).

The mentioned rearmost layer of the network, whose neuron outputs are usually the only ones visible outside the network apart from the input neurons and its layer, is commonly called the output layer (Shanmuganathan & Samarasinghe, 2016). The layers in front of it are called the hidden layer (Dixon et al., 2020; Shanmuganathan & Samarasinghe, 2016). If there is just one hidden layer one sometimes denotes that as shallow ANN (Dixon et al., 2020, p. 115; Dixon & Halperlin, 2019). If there are more, the ANN is considered a deep neural network – DNN (Dixon et al., 2020, p. 115). The fundamental mathematical theorem in terms of ANNs is the universal representation theorem, which states that the set or more precisely the functional space of all continuous functions (from an n-dimensional real-valued domain to the one-dimensional real-valued image space) can be arbitrarily approximated by ANNs and even by a one-layer ANN (Dixon et al. 2020, p. 119; Hornik et al., 1989).

In mathematical terms, the corresponding ANNs are dense in the functional space of continuous functions named above (Dixon et al., 2020, p. 119). While DNNs are thus not "necessary", they are often much better in terms of performance and much better explainable (Dixon & Halperlin, 2019; Došilović et al., 2018; Samek et al., 2019). Therefore, DNNs and parallelization techniques like convolutional neural networks (CNNs) are heavily employed in practice (Dixon et al., 2020; Yamashita et al., 2018). The superiority compared to linear models (and linear additive models as linear regression or linear multi-factor models) is stated for instance in (Dixon et al., 2020, pp. 127, 177).

Regarding the network structure again the inputs are often weighted (for each set of neuron/node input) already at the beginning and then – the w_{ij} – aggregated to a net input and often further processed by an activation function before reaching another layer or the output itself (Hornik et al., 1989; McCulloch & Pitts, 1943).

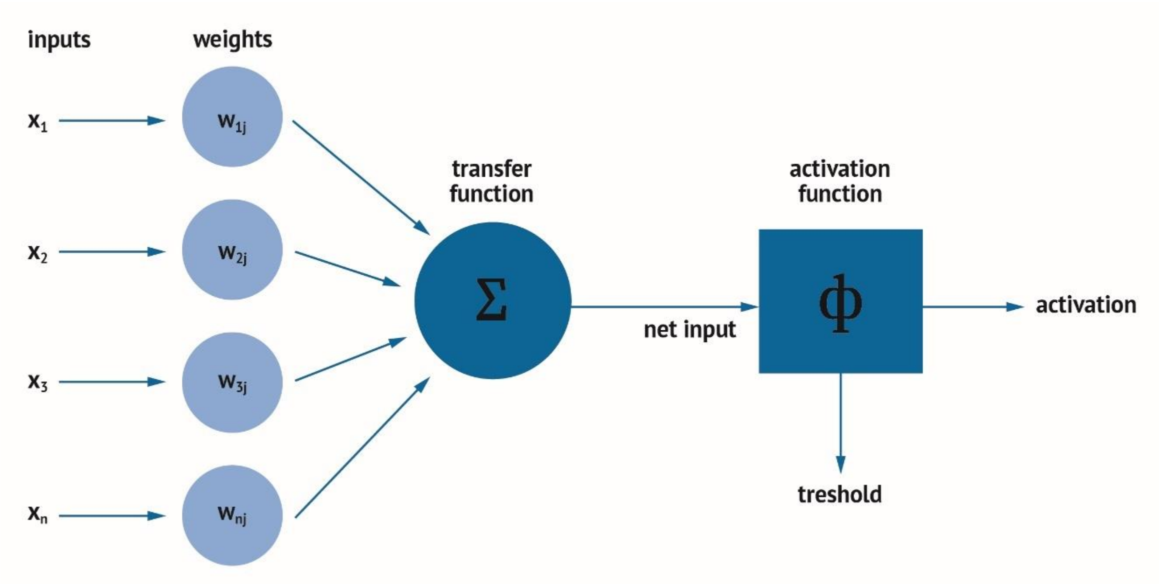

Figure 45 Activation function of an ANN.

Source: Own illustration.

These activation functions are generally just "activated" when a certain numeric threshold or sum is reached and often form multiple cascades of lines of data processing.

Mathematically, activation functions are simply the equations that determine the specific output of the neural network (Dixon et al., 2020; Hornik et al., 1989; Shanmuganathan & Samarasinghe, 2016). They encode the overarching form of such a network. Activation functions further have a decisive impact on a neural network's ability to converge and they normalize the output – between zero and one usually (Dixon et al., 2020). The activation function can be broadly classified into two main categories. There are on the one hand linear and on the other hand nonlinear activation functions (Shanmuganathan & Samarasinghe, 2016). Within linear functions, one differentiates further between:

Binary step functions (BSF) and (traditional) linear activation functions (LAF) as in (Dixon et al., 2020; Shanmuganathan & Samarasinghe, 2016). A binary step function is generally used in the perceptron linear classifier (Rosenblatt, 1957; Rosenblatt, 1958). It thresholds the input values to one and zero, if they are greater or less than zero, respectively. The step function is mainly used in binary classification problems and works well for linearly severable problems. It cannot classify the area of multi-class problems (Dixon et al., 2020).

The equation for the Linear activation function in contrast is a basic function of the form:

$$f(x) = ax \tag{182}$$

as known from normal real-valued analysis (Shanmuganathan & Samarasinghe, 2016). This yields just a linear regression and as the aggregation of linear functions is again linear additive linear functions can be modeled – and with only one layer (Dixon et al., 2020, pp. 114, 119). As however, the application of linear regression is quite limited – and backpropagation is not possible as no nonconstant derivative/gradient exists mathematically – modern neural network models utilize nonlinear types of activation functions. These nonlinear activation functions possess the ability to create more complex, high-dimensional mappings between inputs and outputs of a deep neural network (Dixon et al., 2020). These are the most important nonlinear activation functions.

- Rectified linear units abbreviated as ReLU
- Sigmoid activation functions
- Tanh or hyperbolic activation functions
- Complex nonlinear activation functions

It is widespread to employ a ReLU (rectified linear unit) as the chosen activation function for input and hidden layers in artificial neural networks (Jeeva, 2018; Sharma, 2017).

Mathematically, it has the form

$$\begin{gathered} f(z) = z, z > 0 \\ f(z) = 0, z \leq 0 \end{gathered} \tag{183}$$

hence

$$f(z) = \max(z, 0) \tag{184}$$

as in (Dixon et al., 2020, p. 116). Again (as with SVM) one can think of each manifold as a certain hyperplane where “the neuron gets activated when the observation is on the ‘best’ side of the hyperplane” and “the activation amount is equal to how far from the boundary the given point is” (Dixon et al., 2020, p. 116).

Different extensions of ReLU exist as will be seen. Regarding the output layer, one often employs either a softmax function for activation, if the task is a classification one, or the actual value if it is used for the purpose of prediction (Dixon et al., 2020, p.

157; Jeeva, 2018). Softmax for the i-th node has the form (as often in softmax context, exp instead of e is written for the Euler exponential function, both forms are equivalent):

$$softmax(z_i) = \frac{\exp(\mathrm{z_i})}{\sum_{\mathrm{j}} \exp\,(\mathrm{z_j})} \tag{185}$$

as in (Dixon et al., 2020, p. 141).

When the ReLU activation function is applied for a deep neural network, the backpropagation signal – by taking again the gradient to see the slope and direction of the signal – will either diminish to zero or it may “explode into a large number when it reaches back the input layer” – however, in case there is no properly received backpropagation signal, the weights can never adjust in the lower layers [215] (Jeeva, 2018). That circumstance is known in the literature as vanishing and exploding gradients problems or not saturating activation functions (Dixon et al., 2020, pp. 20, 468-470). Variants of ReLU were introduced to overcome this obstacle. Among these are the so-called leaky ReLU, a further extension known as randomized leaky ReLU, the parametric leaky ReLU, and finally the exponential linear unit, which is commonly abbreviated as ELU (Dixon et al., 2020; Shanmuganathan & Samarasinghe, 2016; Trottier et al., 2017). Leaky ReLU substitutes the side

$$f\,(z) \;=\; 0 \text{ for } z \;\leq\; 0 \tag{186}$$

with

$$f(z) = ß \cdot z \tag{187}$$

for a “very small” beta ß, hence making differentiation (and a gradient application) possible and barely changing the result values.

The sigmoid or logistic activation function is a logistic function, and the output is ranging between zero and one (Saul et al., 2016). It has the form

$$f(z) = \frac{1}{1 + e^{-z}} \tag{188}$$

and is useful to assign probabilities (as output) to a certain input, e.g., default probabilities in credit risk (Saul et al., 2016).

[215] For a quick (informal) overview see (Sharma, 2017). Because the Gradient descent is used as algorithm for the weight updates, if the parameter itself is zero (the derivative of the constant zero is equal to zero), then the gradient of the weight will be also zero and that weight will henceforth never be updated. The gradient is “dying” (sometimes denoted “Dying ReLU” in that context) or “vanishing”.

Hyperbolic functions yield a similar concept using hyperboles (Dixon et al., 2020). They also suffer from the so-called vanishing or exploding gradients problem in their pure versions.

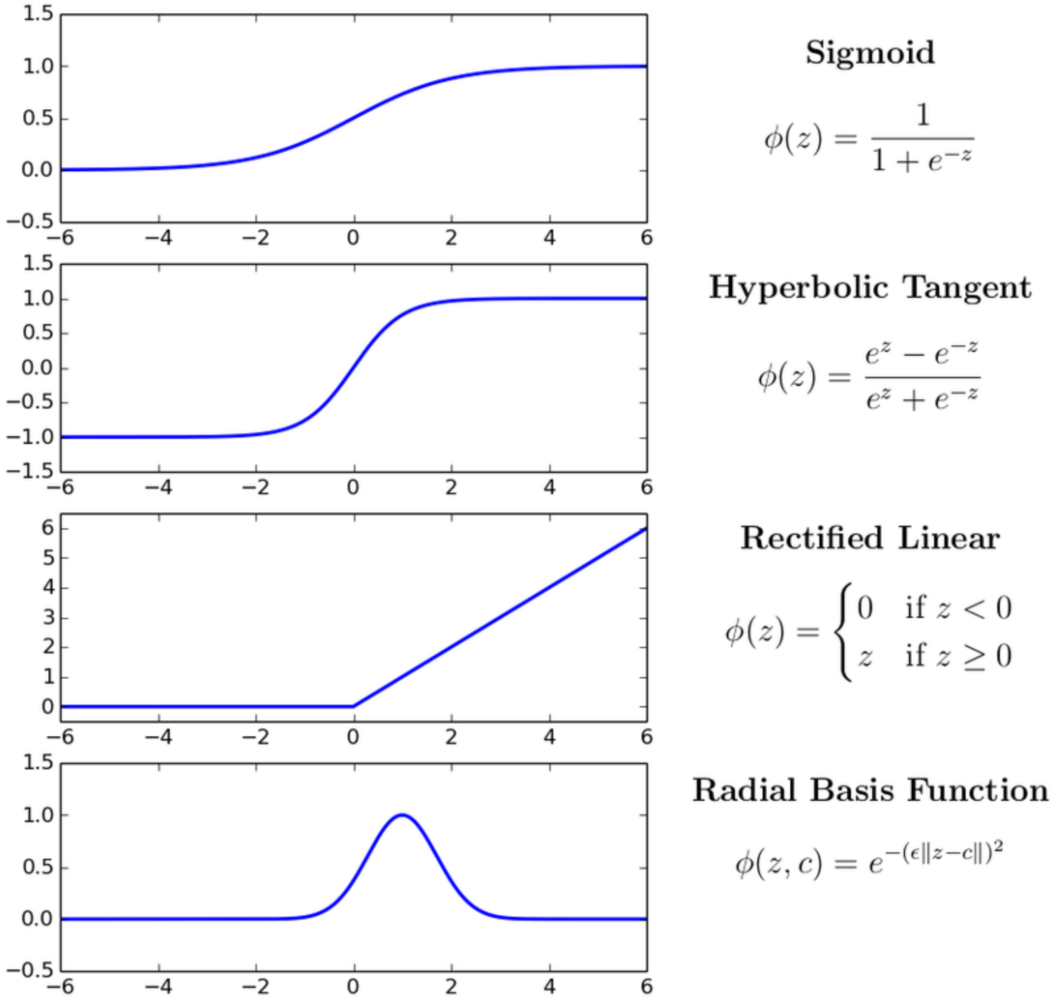

Figure 46 Most common activation functions (and their graphs and formulas).

Source: Own plot and illustration, similar as in (Yamashita et al., 2018), online retrieved Mai 22, 2022 from https://link.springer.com/article/10.1007/s13244-018-0639-9#citeas, figure 5.

An alternative, extended sigmoids, which perform even better than leaky or parametric leaky ReLU is the swish function by Google (Ramachandran et al., 2017). It simply has the form

$$f(z) = z\ sigmoid(\beta z) \tag{189}$$

with a “very small” beta (Ramachandran et al., 2017).

The softmax function is sometimes also labeled as soft argmax function or multi-class logistic regression (Bishop, 2006). This is grounded in the fact that the softmax function is a generalization of logistic regression, which is suitable for multi-class classification. Its formula further resembles the sigmoid function, which is employed for logistic regression (Bishop, 2006; Dixon et al., 2020; Jeeva, 2018, p. 3).

Hence as a summary, the softmax function (often for the output) and mainly leaky/parametric leaky ReLU and the swish function are the advanced activation functions used (without suffering from vanishing gradients) in the industry.

The structure of a network is furthermore directly related to the learning method used and vice versa; thus, only a single-layer network can be trained (directly) with the delta rule, while a slight modification is required for multiple layers (Dixon et al., 2020). Networks do not necessarily have to be homogeneous: there are also combinations of different models in order to combine different advantages.

There are pure feedforward networks (single-or multi-layer), in which layers are always connected to the next higher (hidden) layers (Dixon et al., 2020, p. 111). They are the most common types of networks. In addition, there are networks in which connections are allowed in both directions (and include feedback loops). The appropriate network structure can be found, e.g., by evolutionary algorithms or error feedback measures (Rebala et al., 2019).

(Highly-)Multi-layer neural networks are called deep neural networks (DNN) as seen and hence learning algorithms using DNN are called deep learning algorithms (Dixon et al., 2020). They are, as an application, used in hand-writing recognition (or similar pattern recognition tasks) and also in nonstationary settings with so-called LSTMs (Graves et al., 2009).

Having absorbed the concept, components, structure and working (via activation functions) of neural networks some applications in finance will be further considered now.

In finance, ANNs are due to their structure also useful when one has to deal with big data and with some sort of classification or prediction task (Dixon et al. 2020, p. 16). Natural applications are hence customer search processes like finding an “ideal” customer, i.e., a certain pre-defined risk profile admitting client, who is also having certain behavioral patterns over time (resulting in large “user data/patterns”), which might then, e.g., be used for scoring or rating purposes for credit card companies, for loan applications as well as for BI (business intelligence) analytics and customer strategy (Dixon et al., 2020; Roa et al., 2021). Artificial neural networks are increasingly used in fraud detection (patterns), evaluating secondary information or digital footprints for future company performances like sociological data, shopping patterns or cookies histories, etc. (Dixon et al., 2020; Roa et al., 2021). Combined with smart contracts and documents (blockchain-based in the optimal case) it might revolutionize document process management as well (Sandner et al., 2020). Mentioning blockchain, a natural application of ANNs are also cryptocurrencies (Dixon et al., 2020, p. 7). Furthermore, the main applications of ANNs in finance are in the "classical areas" of asset and derivative pricing, hedging as well as in risk management as detailed in (De Spiegeleer et al., 2018; Dixon et al., 2016; Dixon et al., 2020, p. 111; Dixon & Halperlin, 2019; Feng et al., 2018; Hornik et al., 1989; Hutchinson et al., 1994; Roa et al., 2021). The artificial neural network concept regarding risk and credit risk measurement is mainly used in the area of scoring, rating, and pre-selection not in portfolio risk, however. An introduction to ANNs and their application to scoring is also given in (Witzany, 2017, p. 78-81). A good overview of financial applications is presented by Soramäki and Cook in their new book "Network Theory and Financial Risk" from April 2022 (Soramäki & Cook, 2022).

The software packages most often used for machine learning and ANNs in Python® are PyTorch® and TensorFlow® respectively mnet®/neuralnet or ANN2® within the CRAN-project[216] in R®.

By their nature, all of the applications above are destined to be exploited by machine learning in general and not (necessarily) limited to ANNs (Bishop, 2006; Dixon et al., 2020, p. 117).

Hence – after having shown its superiority in regard to ANNs in many circumstances, as in grading and scoring applications, at the beginning of the chapter – SVMs might be a further valuable candidate for credit risk. Support vector regression was already successfully applied in research to forecast stock returns and real estate prices, so it is utilized in the financial field of credit risk now (Henrique et al., 2018; Li et al., 2009).

Therefore, having introduced machine learning (ML) in general and artificial neural networks (ANNs) as a prominent case of it, as well as some basic ideas and comparisons related to support vector machines (SVMs), these SVMs are now described in a detailed fashion in the following sub-chapter.

[216] See https://cran.r-project.org/web/packages/ANN2/index.html (Retrieved Mai 7, 2022).

5.2 Background and History of the SVM and Related Concepts

Support vector machines are so-called supervised learning models utilizing learning algorithms that analyze data destined for classification as well as regression analysis (Dixon et al., 2020, p. 139; Schölkopf & Smola, 2001; Smola & Schölkop, 1998b). While both applications are widely used, they are quite similar in their underlying ideas and hence especially the most common one – classification – is presented here at first. As the last part of the name already indicates, the SVM is a technique from machine learning (and pattern recognition).

The algorithm was originally invented by Vladimir Naumovich Vapnik at AT & T Laboratories and builds on the 1963 grounding of the Vapnik-Chervonenkis theory (VC theory) of machine learning – later popularized by him and extended for nonlinear classification in 1992/3 (Boser et al., 1992; Cortes & Vapnik, 1995; Vapnik, 1964; Vapnik, 1998).[217] Vapnik also contributed to related fields like clustering unlabeled data – which is just possible with unsupervised learning methods – he wrote the support vector clustering algorithm together with Siegelmann and others (Ben-Hur et al., 2001). Furthermore, the method of Platt scaling is a means of transforming the resulting classes of the SVM classifier then into a corresponding probability distribution over these classes and hence even a probabilistic extension of an SVM is available (Platt, 1999). Finally, the so-called "soft margin" implementation (including linearly inseparable classes as will be shown), which is used in most statistical software packages, e.g., in R, was originally introduced by Cortes and Vapnik in 1993 and then published in 1995 (Cortes & Vapnik, 1995).

The mentioned VC theorem and inequality are mathematically equivalent to an empirical process formulation and the VC inequality can be also proven by applying symmetrization and the Hoeffding inequality (Ben-Hur & Weston, 2010; Hoeffding, 1963; Van der Vaart & Wellner, 2013; Vapnik, 1998).

For binary classification, the VC inequality is stating that with increasing sample size, provided that the filtration F used has a finite VC dimension (cf. next paragraph for a definition), the empirical classification serves as a good proxy (bound) for the expected one (Gyorfi et al., 1996; Vapnik, 1998).

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[217] Large parts of this idea of the “kernel-trick” however date back to Aizerman in the year 1964 (Aizerman et al., 1964).

An alternative introduction to the VC dimension can be found in (Dixon, 2020, pp. 120-124). Vapnik (in line with Chervonenkis) gave name to this dimension, a kind of "learning capacity":

One may regard a set of functions $f(x, \alpha) \in \{-1,1\}$. Consider then a given set of l points, these can be naturally labeled in 2^l ways (Cuiping & Tung, 2021, p. 80; Vapnik, 1998). If any member of that set $\{f(\alpha)\}$ can be found, which is able to correctly assign the labels for all labelings, then the set of functions is said to "shatter" the point set (Cuiping & Tung, 2021, p. 80; Schölkopf & Smola, 2001; Vapnik, 1998). The VC dimension $dimVC$ of $\{f(\alpha)\}$ is then the maximum number of training points the set $\{f(\alpha)\}$ may shatter (Cuiping & Tung, 2021, p. 80; Dixon, 2020, pp. 120-124). In the case of an n-dimensional real vector space $\mathbb{R}^n$ the VC dimension of a set of oriented hyperplanes is $n + 1$ (Dixon, 2020, pp. 120-124; Vapnik, 1998).

Further academic research has generalized these foundational concepts from inter alia Vapnik, Smola, and Schölkopf, has illustrated connections to regularization theory, and shown how SVM-based methods are "incorporated in a wide range of other related algorithms" (Burges, 1998, p. 122; Girosi, 1998; Schölkopf et al., 1998; Schölkopf et al., 1998b; Smola et al., 1998; Smola & Schölkopf, 1998a; Wahba, 1998). A good overview of different SVM methods and applications is also given in (Schölkopf et al., 1999).

The SVM algorithm is now classifying a set of objects into different classes with the aim of maximizing the margin/space around those classes – hence denoted as large margin classifier LMC (Schölkopf & Smola, 2001; Vapnik, 1998). Classification means, that given some points and each of them labeled with either zero or one, the algorithm decides for a new point which label it will get according to the ones already labeled and a "labeling rule" with maximal distance, hence maximal generality. More precisely: A support vector machine at first builds on a set of training objects, for which it is a priori known which class they are classified or assigned to (Harrach, 2019, p. 23; Vapnik, 1998). The training objects are "represented by a vector in a vector space" (Harrach, 2019, p. 23; Schölkopf, 1997; Vapnik, 1998). The idea of the support vector machine is to fit a hyperplane in the vector space, which is serving as a "separating surface" and thereby dividing the training objects into two or iteratively more classes (Cortes & Vapnik, 1995; Schölkopf et al., 1998). The distance between those vectors that are closest to the hyperplane is maximized (Cortes & Vapnik, 1995). As not all vectors are needed to describe the hyperplane uniquely, but only the "nearest" ones which hold the hyperplane, the second part of the methods name – presenting these "support vector(s)" – is explained (Dixon et al., 2020; Vapnik, 1998). However, a clear and clean separation is – at first – just possible if the objects are linearly separable, e.g., by a line in two dimensions or by a plane in three dimensions, which is in the general case not possible (Aizerman, 1964; Boser et al., 1992; Vapnik, 1998). Hence, one uses a transformation – the idea behind the so-called "kernel trick" – of the vector space and thus also the training vectors contained in it to a higher-dimensional space (Aizerman, 1964; Boser et al., 1992; Cortes & Vapnik, 1995). In a space with a sufficiently high number of dimensions, in case of doubt also infinite and the mathematics of functional analysis is applied then, even the most nested set of vectors can be linearly separated (Dixon et al., 2020; Schölkopf & Smola, 2001). In this higher-dimensional space, the separating hyperplane is therefore determined (Schölkopf & Smola, 2001). During the following back-transformation into the lower-dimensional space, the linear hyperplane becomes a nonlinear, possibly even noncontinuous hypersurface, which separates the training vectors into two classes (Cortes & Vapnik, 1995). Hence, nonlinear separation is possible of any given (discrete) set (Werner, 2007). The problem is the high cost-intensive – time and memory-wise – calculation and computational power needed for the (back-)transformation to higher dimensions (Smola et al., 1998). As a consequence, the functions utilized by the SVM method are designed in a way that ensures that just dot products of pairs of input data vectors need to be applied (Vapnik, 1998). These dot products then can be used in a straightforward way and evaluated directly in terms of the variables in the original space without calculating them completely out (Cortes & Vapnik, 1995; Schölkopf & Smola, 2001). A so-called kernel function that suits the original problem is used for that purpose (Cortes & Vapnik, 1995; Vapnik, 1998). Furthermore, "slack variables" are introduced to punish wrong classification, limit the number of support vectors needed and avoid overfitting of the model (Vapnik, 1998). The selection of the right kernel – at its core idea, a function for measuring the density of the classes and re-transforming them into higher-dimensional spaces and back – and of the right slack variables and their "punishing" values is the key to solving underlying classification problems (Schölkopf & Smola, 2001; Vapnik, 1998).

With observations as before and the SVM approach one can define more formally:[218]

Consider l observations consisting of a pair (x_i, y_i), with $x_i \in \mathbb{R}^d$, $i = 1, ..., l$ and the associated "label" $y_i \in \{-1,1\}$ (Ciuping & Tung, 2021, p. 84). In the linearly separable case, one may suppose there is a (separating) hyperplane

$$wx + b = 0 \tag{190}$$

that separates the positive ($\geq +1$) from the negative (≤ -1) points (Ciuping & Tung, 2021, p. 84; Vapnik, 1998). Hence,

$$wx_i + b \geq 1 \quad \text{when} \quad y_i = +1 \tag{191}$$

and

$$wx_i + b \leq -1 \quad \text{when} \quad y_i = -1 \tag{192}$$

Equality holds for the nearest points, as shown in (Vapnik, 1998). The distance between these hyperplanes is therefore $2/||w||$, the margin m

$$m = \frac{1}{||w||} \tag{193}$$

This expression has to be maximized. The SVM method therefore now calculates the hyperplane minimizing $||w||$, the corresponding w-norm is normally just $||w||_2$ the usual Euclidean distance (Schölkopf & Smola, 2001).

Minimizing $||w||$ then maximizes the distance $2/||w||$.

Hence, formally the idea can be expressed as follows – the factor ½ is convenient for derivative purposes and does evidently not change the minimization result (Platt, 1998, pp. 3-4):

$$min_{w,b} \ \frac{1}{2} \ || \ w \ ||^2 \qquad \text{subject to (s.t.)} \ \ y_i(wx_i - b) \geq 1, \text{for all } i \tag{194}$$

Using the usual Lagrangians from economics, this optimization is transformed into a so-called dual form ("just min max and max min are changed"), which is a standard quadratic programming problem and the objective or target function Ψ is only dependent on the set of Lagrange multipliers α_i (Platt, 1998; Vapnik, 1998).

[218] See also http://www.stat.columbia.edu/~madigan/DM08/svm.ppt.pdf (Retrieved Mai 8, 2022).

$$min_{\alpha}\,\Psi(\alpha) = min_{\alpha}\ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} y_i\, y_j\,(x_i x_j)\alpha_i\,\alpha_j - \sum_{i=1}^{N}\alpha_i \qquad (195)$$

where N is the number of training points, s.t. the (linear) inequality constraints,

$$\alpha_i \geq 0,\ \ \forall \text{i (for all i)} \qquad (196)$$

and just one linear equality constraint,

$$\sum_{i=1}^{N}\alpha_i y_i = 0 \qquad (197)$$

As one can directly see in the equation above, a one-by-one relationship between each Lagrange multiplier and each training point exists (Platt, 1998).

Hence, once Lagrange multipliers are calculated the normal vector *w*, as well as the threshold *b*, can subsequently be derived from the Lagrange multipliers (Platt, 1998; Schölkopf & Smola, 2001):

$$\sum_{i=1}^{N}\alpha_i y_i x_i = w$$
$$b = w x_k - y_k \qquad (198)$$

for some $\alpha_k > 0$, k in 1,…, n. As w might be already calculated with this equation from the training data before the application of the SVM optimization, the amount of computation needed for a linear SVM is constant in the number of just the non-zero support vectors (Cortes & Vapnik, 1995; Smola & Schölkopf, 1998a; Vapnik, 1998). Sometimes for the SVM approach one further denotes $d+$ ($d-$) as "the shortest distance from the separating hyperplane to the closest positive (negative) example" – it is however not used in the thesis (Ciumag & Tung, 2021, p. 85; Vapnik, 1998).

As indicated before, Cortes and Vapnik introduced an extension of the original optimization method, which allows, yet penalizes, the failure of points to be off the margin and hence generalizes the SVM method by allowing "soft margins" (Platt, 1998). To reach that goal, one needs further positive slack variables ξ_i, such that

$$w x_i + b \geq 1 - \xi_i \text{ when } y_i = +1 \qquad (199)$$

$$w x_i + b \leq -1 + \xi_i \text{ when } y_i = -1 \qquad (200)$$

and finally (in the same way as above) a modified objective function

$$½\,||w|| + \gamma\sum_{i}\xi_i \qquad (201)$$

where $\gamma = \infty$ is the not separable case (Cortes & Vapnik, 1995). Hence, also the penalties ξ_i have to be minimized.

γ thereby is the decisive setting parameter that controls the trade-off between a large margin and a small sum of "margin failures" (Platt, 1998, p. 3). When the Lagrangian for the dual problem is derived in the same way as above, only the constraint $\alpha_i \geq 0, \forall i$ changes to the box constraint

$$\gamma \geq \alpha_i \geq 0, \forall i \tag{202}$$

and is hence independent of the single ξ_i s (Platt, 1998).

This yields the final Lagrangian for the linear case, allowing (but penalizing) for misclassification.

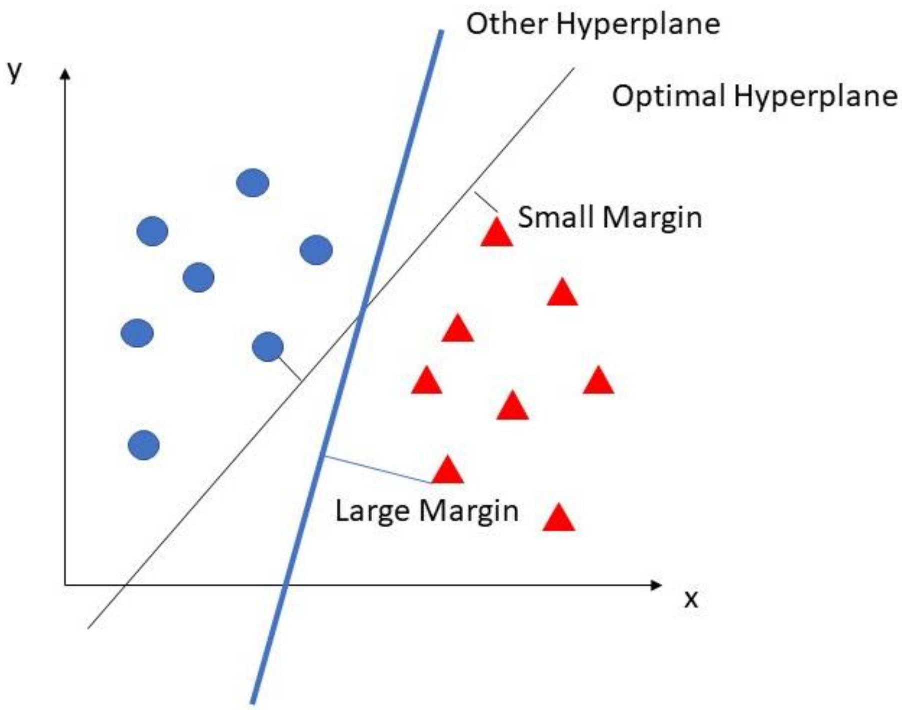

Figure 47 Example of an optimal hyperplane in SVM.

Source: Own illustration

For the nonlinear SVM (the point set is always separable in higher dimensions in the extreme case in a functional infinite-dimensional space), however, one replaces $x_i\, x_j$ with $k(x_i; x_j)$ and k is denoting the crucial kernel function as in (Boser et al., 1992; Cortes & Vapnik, 1995).

These kernels and transformations in higher dimensions allow (linear) separability in the target space then and the Lagrangian is also derived in the same way, by just replacing $x_i\, x_j$ with $k\,(x_i; x_j)$ (Cortes & Vapnik, 1995; Platt, 1998; Vapnik, 1998).

That therefore formally denotes as:

$$min_{\alpha}\, \Psi(\alpha) = min_{\alpha}\ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} y_i\, y_j\, k(x_i; x_j)\alpha_i\, \alpha_j - \sum_{i=1}^{N}\alpha_i \qquad (*) \qquad (203)$$

(where N is the number of training points), s.t. the (linear) inequality constraints,

$$\gamma\ \geq \alpha_i \geq\ 0,\ \forall \text{i (for all i)} \qquad (204)$$

and just one linear equality constraint,

$$\sum_{i=1}^{N}\alpha_i y_i = 0 \qquad (205)$$

as shown again in (Platt, 1998, p. 4).

Prominent examples of these kernel functions are the following ones, cf. (Dixon et al., 2020; Schölkopf & Smola, 2001; Vapnik, 1998):

$$k(x_i; x_j) = 1 + < x_i; x_j >^{d} \qquad (206)$$

- is the d-fold dot product or Polynomial kernel (for $d = 1$: linear kernel)

$$k(x_i; x_j) = e^{\frac{||x_i - x_j||^2}{2\sigma^2}} \qquad (207)$$

- is the radial basis function kernel, RBF, with volatility parameter σ

$$k(x_i; x_j) = \tanh(\,a\, x_i x_j\ + \Theta) \qquad (208)$$

- is the sigmoid kernel with parameters a and Θ

As a reminder in that context, the Karush-Kuhn-Tucker (KKT) conditions possess a significant role in the area of constrained optimization (i.e., constrained linear and nonlinear programming) and hence also SVMs (Jarre & Stoer, 2019; Kuhn & Tucker, 1951; Ruszczyński, 2006).

The KKT conditions[219] are satisfied at the solution of any constrained optimization problem (being it convex or even not) if "the intersection of the set of feasible directions with the set of descent directions coincides with the intersection of the set of feasible directions for linearized constraints with the set of descent directions" (Burges, 1998, p. 131; Fletcher, 1987; McCormick, 1983).

Those regularity assumptions always hold for SVMs, because the constraints in the equations above are, as one directly sees, linear constraints. The tasks posed for support vector machines are even convex – and for convex problems under the regularity condition, the presented KKT conditions are exactly necessary and already sufficient at the same time (Burges, 1998, p. 131; Fletcher, 1987). Therefore, solving the SVM problem is truly equivalent to the task of finding a solution for the KKT conditions, which is an interesting mathematical identity (Burges, 1998).

For the SVM the KKT conditions simplify to (Platt, 1998, p. 4):

$$\gamma > \alpha_i > 0 \text{ if and only if (iff,} \Leftrightarrow) \;\; y_i \, o_i = 1 \tag{209}$$

(where o_i denotes the output of the SVM method of the i-th point)

$$\gamma = \alpha_i \Leftrightarrow y_i \, o_i \leq 1 \tag{210}$$

$$\gamma = 0 \;\; \Leftrightarrow \;\; y_i \, o_i \geq 1 \tag{211}$$

Finding the solution to the quadratic programming in (*) above (or equivalently the KKT conditions) is reached, after having applied the "kernel trick" to the dual Lagrangian problem as seen, by finally using stochastic (Sub-)Gradient methods (SGM) or the Sequential Minimal Optimization (SMO) algorithm as solvers (Platt, 1998, pp. 6-9; Vapnik, 1998).

The main advantages of the SVM approach, also compared to the ANN approach as stated, can be summarized:

- High-Dimensionality: The SVM approach is known as a very effective tool for high-dimensional spaces and the kernel trick combined with scalar product representation allow for efficient computing (Boser et al., 1992; Dixon et al., 2020; Halls-Moore, 2017, p. 263; Schölkopf & Smola, 2001; Vapnik, 1998). High-performance methods such as Sub-Gradients can be applied as solvers (Wilmott, 2007).

[219] These conditions are primal feasibility, dual feasibility, complementary slackness, and stationarity. A good introduction and overview can be found, e.g., in (Ghojogh et al., 2021).

- Flexibility: The possibility to test and apply various SVM kernels allows for extreme flexibility regarding the actual decision boundaries. This leads to versatility and a superior classification performance as mentioned (Awad & Khanna, 2015b; Burges, 1998; Schölkopf & Smola, 2001; Witzany, 2017).
- Memory-Efficiency: There is only a subset of the training points which are used in the actual decision process of assigning new members necessary (Cortes & Vapnik, 1995; Schölkopf & Smola, 2001; Vapnik, 1998). Hence, also only these points have to be stored in the computer's memory when applied for decision making (Halls-Moore, 2017, p. 263).
- Availability in all common statistical software packages like Python and R, hence "ready to use" tool with various fine-tuning possibilities.

In practice, the SVM is used, e.g., in the programming language R® with the libsvm library, and as mentioned the decisive choice is the factor gamma, which is balancing margin width and penalized errors and the kernel function. An SVM introduction and application of SVMs in (solely) credit scoring is shown in (Witzany, 2017, p. 82).

SVR finally just extends the SVM by regression instead of classification, which means that a kind of tube having minimal radius is put – normally symmetrically – around the estimated function (which can be seen as the separator analog to classification), where all points outside it are penalized and all within a smaller radius not, hence minimalizing errors (Awad & Khanna, 2015, p. 67; Smola & Schölkopf, 2004). This is specified also in (Smola & Schölkopf, 2004).

Hence the algorithms in use are basically very similar. For the linear regression case, one needs to find the linear function

$$f(x) = x^T \beta + b \tag{212}$$

as usual and to make sure it is "as flat as possible," hence the norm $\beta^T \beta$ is minimized. As a reminder x^T denotes the transposed x (Awad & Khanna, 2015). This can be seen (again) as a convex optimization problem as well:

$$\min_\beta \; \beta^T \beta \tag{213}$$

subject to all residuals having a value less than ε, therefore:

$$\forall n: | \, y_n - (\, x_n^T \, \beta + b) \, | \leq \, \varepsilon \tag{214}$$

Again slack variables can be introduced such that the inequality does not need to hold for all points (Awad & Khanna, 2015; Smola & Schölkopf, 2004). Furthermore, the transformation with a kernel to nonlinear cases is possible in the same way (Smola & Schölkopf, 1998b).

Then, the mentioned Lagrangian duality problem can be formulated in ways of SVR (sometimes denoted ε-SVR) as :

$$\begin{aligned} L(\alpha) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\, \alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) K(x_i; x_y) \\ + \, \varepsilon \sum_{i=1}^{N} (\alpha_i + \alpha_i^*) - \sum_{i=1}^{N} \, y_i (\alpha_i - \alpha_i^*) \end{aligned} \tag{215}$$

again subject to the constraints

$$\sum_{i=1}^{N} (\alpha_i - \alpha_i^*) = 0, \tag{216}$$

$$0 \, \leq \, \alpha_i \, \leq \, \mathrm{C} \quad \forall \mathrm{i}, \tag{217}$$

$$0 \, \leq \, \alpha_i^* \, \leq \, \mathrm{C} \quad \forall \mathrm{i} \tag{218}$$

The problem is (again) solved iteratively via SGM or SMO (Smola & Schölkopf, 2004). The function used to predict new values is then

$$f(x) = \sum_{i=1}^{N} \, (\alpha_i - \alpha_i^*) K(x_i; x) + b \tag{219}$$

As SVM regression has many applications and advantages compared to other ML methods, it might be especially useful for credit risk as well. Considering the fact that it is generally able to capture nonlinear and nonnormal (even nonparametric) structures and dependencies as shown, it might be even better suitable for credit portfolio risks and to model its dependency structures than, e.g., the normal asset return distribution-based CreditMetrics® (where of course the credit risk returns are also nonnormal but correlations are often taken from that asset returns and therefore strongly restricted methodologically) or the Poisson-Gamma distribution-based CreditRisk+®. The idea of the thesis is to apply it to CPR.

5.3 Application of SVR to Bond Portfolios. Empirical Test and Comparison with Linear Models

In the empirical part of the thesis, the portfolios used are described as the underlying data base, as well as the process of comparing the SVM regression model to industrial models and the results thereof. Hence, the usual (data) scientific path of first explaining the data base, its sources, structure and possibly descriptive statistical properties and then illustrating and comparing the models operating on that data in terms of their design, implementation (cf. annex of the thesis with program code) and underlying methods is generally followed in this thesis as well (Greene, 2003; Dixon et al., 2020). In terms of comparing the SVR model, it is benchmarked against a linear model, first to show the overall usability and superiority in that case and then compared to the industrial models CreditMetrics® and CreditRisk+®. As these two models – their methodical, mathematical background as well as their design and use cases – were already described in detail in the previous chapters it is herewith referred to the corresponding parts to avoid redundancy and preserve a coherent structure and smooth readability.

As outlined above, the first focus is on the data used.

According to common empirical studies' standards (e.g., by SciDAC and by Greene)[220] and programming best practice, the corresponding files are divided into a data file – namely "Portfolio_Data" – in which on the front page the data content and exact formatting is described as well as containing the raw data and formatted data on further tabs in Excel®, then the code files by using the standard data science language R® and finally output files.[221]

The data process cycle or operating scheme can be therefore described as I-P-O, i.e., input, processing, output, where in the input part the data is explained with its steps from raw data to finally formatted data.

The portfolio data used are original raw data extracted from Thomson Reuters Refinitiv® systems at the University of Gdańsk lab. Refinitiv® and Bloomberg® are the

[220] See, e.g., SciDAC Scientific Data Management Center (lbl.gov); Risk Assessment for Scientific Data (codata.org) (Retrieved Mai 17, 2022). For a general overview, see (Greene, 2003).
[221] All these files are supplemented (and made available with) to the thesis and/or in the annex.

standard and most reliable data providers in the financial industry, and prime use for academic studies in finance (Dixon et al., 2020).[222]

For reasons of representativeness and to achieve rather general results diversification among the most important economic regions (and bond issuance locations) – hence the U.S. and the combined E.U. markets – and among risk structure profiles – measured as volatility, i.e., the standard deviation of returns which is the square root of the variance and binary ordered as low vs. high volatility – is applied (BIS, 2022; ICMA, 2020; McEnally & Ferri, 1982; Worldbank, 2022a; Worldbank, 2022b). Considering liquidity concerns and excluding high premia, which could potentially bias the bond spreads solely large cap, high liquid portfolios were selected – hence, e.g., excluding emerging markets or shaping more granular risk structures as the binary ones (BIS, 2022; DeJong & Driessen, 2012; Euroclear, 2020; Goldstein et al., 2017; Kou & Varotto, 2008).

This results in four high-liquid, representative portfolios for bond markets to measure credit risk in (correlated) bonds. A U.S. portfolio with high historical volatility, one with low historical volatility and the same division for European counterparts is employed.

A further reason for excluding emerging markets, apart from liquidity, spreads, and survivorship bias concerns, is the lack reliable data history as a direct Bloomberg terminal search shows. Recent geopolitical developments such as the Russian invasion of Ukraine and the following economic sanctions are a further argument for the necessity of liquidity and deeply developed financial markets as in the EU – considering the way to an ECMU[223] – and the US for bond portfolios. As the thesis further aims to compare credit risk models for large-to-mid-size listed companies and their liquid bonds, emerging countries and BRICS[224] are consciously not in scope. In credit risk management a time period of at least three to optimal ten years (3-10 or 5-10 yrs.) should be considered and for these markets, such a long track period is rarely existing or for the constituents missing values and pricing errors appear too frequently. Coupon bonds, i.e., no zero-bonds with common maturity profiles (5-7 years) and structures are used within the portfolio indices, as they are a common standard and highly available.

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[222] As on https://www.refinitiv.com/en/about-us#our-data, https://www.bloomberg.com/professional/expertise/the-terminal-on-campus/ (Retrieved Mai 16, 2022).
[223] European capital market union
[224] BRICS: Brazil, Russia, Indonesia, China, and South Africa – a term coined by Goldman Sachs analyst Jim Reed.

Therefore, the named geographical regions and their corresponding bond markets with described bond profiles were chosen. Considering the volatility, the historical rolling five-year volatility (with volatility again as standard deviation) is selected as it is the most common type, the industry standard also for index risk measurement and explicit volatility indices like VIX®, VDAX®, additionally also in contrast to measures like VaR (Value-at-Risk), implicit volatility (only available for options on bonds and stocks) or maximum drawdown, directly available and verifiable from the provider. Therefore, the historical volatility with attribute settings “high” and “low” with a predefined constant threshold range by the provider (i.e., in a “middle-range” some corporate bonds can be ordered in low as well as high vola portfolios stemming from a binary decision process) and the corresponding E.U. and U.S. markets are chosen. The currency and denomination are the standard ones for the domestications, e.g., EURO for the European Union and the U.S. dollar ($) for the United States. Currencies from non-eurozone E.U. members like the Złoty (PLN) in Poland or the Koruna česká (CZK) would be converted by the midday actual conversion rate published by the ECB – however, as the current constituents are all large eurozone corporations like Allianz®, Deutsche Telekom®, SAP®, Telefonica®, Axa®, LVMH® or Philips® a conversion is not even necessary in the thesis’ setting.

Following the extraction of these four raw, high-liquid U.S and E.U. bond portfolios ranging from 09/28/2009 to 09/26/2019 – hence a ten-year period – primary formatting is applied. This procedure is executed according to strict data scientific standards and utilizing a minimalistic approach. Hence, the data is kept original and raw, and only the following selections are made to make them statistically feasible.

Bonds from companies with inconvenient time series, because of series which are starting too late or finishing too early compared to the common sub-time frame of the ten-year period, or with an inadequate number of missing values (more than 10%) are excluded. This guarantees a common, synchronous time series data set. As credit risk is measured mainly in terms of rating changes also constituents without ratings or unregular updates are excluded. This yields the pre-formatted portfolios then. For constituents with fewer missing values the gaps are filled conservatively with the last prices available, later similarly with ratings, which is standard practice and preferable to other methods as this method is not depending on further – possibly error containing – calculations or pre-assumptions. Examples of these calculations include the arithmetic mean of pre-/successor values, extrapolation rules or peer benchmark values. Furthermore, the ratings are filled equivalently for a coherent full history and mapped to a common standard master rating scale as usual. The original portfolios have the following constituents (less the constituents without ratings or price quotes):

Table 6 Tables of constituents (companies) of the selected portfolios with low volatility (U.S. and E.U.).

1.1 and 1.2 USLV (U.S. low volatility)	
Company/Constituent	**Ticker/ID**
SOUTHERN CO	842587CM7=
DUKE ENERGY	26441CAF2=
NEXTERA ENERGY	65339F796=
CONSTELL ENERGY	210371AL4=
ALTRIA GROUP	02209SAL7=
MCDONALDS	58013MEJ9=
WAL-MART	931142CU5=
PROCTER GAMBLE	742718DY2=
VERIZON COMMS	92343VAX2=
COCA- COLA	191216AR1=
AT AND T INC	00206RAX0=
PEPSICO	713448BN7=
HJ HEINZ FIN UK	US010768101=
ELI LILLY	532457BC1=
TARGET	87612EAV8=
ALLSTATE	020002AZ4=
STARBUCKS	855244AD1=
PHILIP MORRIS	718172AH2=
PHILIP MORRIS	718172AH2=
MERK	58933YAA3=
BRISTOL MYERS	110122AT5=
JOHNSON JOHNSON	478160AW4=
UNITEDHEAL GRP	91324PBM3=
CVS CARE	126650BW9=
LOCKHEED MARTIN	539830AY5=
WALGREENS BOOTS	US113835869=

1.3 and 1.4 EULV (E.U. low volatility)	
Company/Constituent	**Ticker/ID**
AMADEUS IT GROUP	ES187819105=
DELHAIZE	24668PAE7=
FRANCE TELECOM	35177PAW7=
ESSILOR	FR0011842939=
MUNICH RE	DE060839255=
VIVENDI	FR047096715=
IBERDROLA FIN IE	ES073630363=
ENEL	IT101499707=
DEUTSCHE TELEKOM	DE164448282=
ADIDAS	DE111415528=
DANONE	FR063036137=R
VINCI SA	FR0011225127=
LOREAL	FR0125763344=
GDF SUE	FR0011147305=
ENI	IT052100097=
SANO	80105NAG0=
FRESENIUS FINANC	DE087343251=
AIR LIQUIDE	FR089930723=
ALLIANZ	DE085787250=
TOTAL	FR119520193=
SAP	DE085325329=
SAFRAN	FR105689217=
DEUTSCHE POST	DE086294150=
NOKIA	654902AC9=
SIEMENS FINANCRG	DE082605029=
TLEFONICA EMISNS	87938WAM5=

PFIZER	717081CY7=		**BAYER**	DE102326857=
			LVMH	FR114511315=

Source: Refinitiv®

As intuitively reasonable the low volatility portfolios (LV) contain more conservative branches like provider bonds (energy provider, telecommunication provider or postal services, etc.), consumer staples (less discretionary) and other consumer brands which are (largely) noncyclical like Procter & Gamble® or nutrition brands and chains like McDonalds® and CocaCola®, health and care products like L' Oreal® or pharmaceuticals like CVS®, Pfizer®, or Johnson & Johnson® (Adam & Merkel, 2019; Boudoukh et al., 1994; Yuksel & Bayrak, 2012). A further advantage is that these branches and corporations are relatively stable over time and hence the index or portfolio composition does not have to be adopted often (Adam & Merkel, 2019; Barber & Odean, 2000; Boudoukh et al., 1994). In contrast to that, one expects newer, more tech-affine and – in general, more pro-cyclical – branches like internet streaming and movie platforms (Netflix®), tech companies (Amazon®, Alphabet® as the Google® mother holding company, Intel®, Nvidia®), and banks (MorganStanley®, Bank of America (BofA)®) to be of higher volatility (HV) (Buchbinder et al., 2018; Chaudhary et al., 2020). The same holds for pro-cyclical branches like the car industry (VW®, Daimler®, etc.). Volatile markets and commodities in practice rapidly lead to a contagion (cf. betas and correlations on Bloomberg) of the whole production and supply chain – including sub-contractors and pre-suppliers – of, e.g., car manufacturing (Brunnermeier, 2008; Filbeck et al., 2016; Yuksel & Bayrak, 2012).

Table 7 Tables of constituents (companies) of the selected portfolios with high volatility (U.S. and E.U.).

1.5 and 1.6 EUHV (E.U. high volatility)			**1.7 and 1.8 USHV (U.S. high volatility)**	
Company/Constituent	**Ticker/ID**		**Company/Constituent**	**Ticker/ID**
BBVA	ES41321145=		**INTEL**	458140AJ9=
AIRBUS GROUP	NL125458459=		**BOEING**	097023AZ8=
BANCO SANTANDER	ES41390029=		**GENERAL MOTORS**	37045VAB6=
INTESA SANPAOLO	IT048645453=		**JPMORG CHAS**	46625HHZ6=

VW	XS1984518867=TE
DAIMLER	DE085720546=
ING GROEP	456837AE3=
SCIET GENRLE	FR052681448=
KON PHILIPS	500472AE5=
BMW FINANCE	DE096831625=
BNP PARIBAS	FR053516335=
ABINBEV	BE063302678=
AXA	FR050366529=
KERING	FR111686661=
CRH FIN SERV	FI098144293=
ASML HOLDING NV	NL097253056=
BASF	DE086068311=
NOKIA	654902AC9=
LVMH	FR114511315=
TLEFONICA EMISNS	87938WAM5=
BAYER	DE102326857=
AIR LIQUIDE	FR089930723=
ALLIANZ	DE085787250=
TOTAL	FR119520193=
SAP	DE085325329=

TEXAS INSTRUMENT	882508AW4=
CHEVRON	166764AB6=
ADOBE SYST	00724FAB7=
ALPHABET	02079KAA5=
AMAZON.COM	023135AJ5=
CISCO SYSTEMS	US046683838=
BIOGEN	09062XAC7=
PRICELINE GROUP	741503AS5=
CELGENE CORP	151020AE4=
GOLDMN SACHS	38141E5N5=
BANK OF AM	06048WBC3=
CAPITAL ONE FNCL	14040HAY1=
EL PASO	28336LBV0=
EMERSON ELECTRIC	291011BE3=
FEDEX	31428XAS5=
CITIGROUP FUNDNG	US049852797=
BLACKRCK	09247XAH4=
CONOCOPHILLIPS	20825CAQ7=
CATERPILLAR	149123BJ9=
METLIFE	59156RBF4=
NETFLIX	64110LAE6=
NVIDIA	67066GAD6=
MORGAN STANLEY	61747YCS2=

Source: Refinitiv®

As all of the entities described are high liquid frequently issued and traded corporate bonds with common or comparable bond structures (profiles), they seem to be an appropriate data base for the selected portfolios. Furthermore, the companies listed – especially the low volatility ones – possess rather strong balance sheets with high equity ratios and rather low debt profiles. [225] Common financial statement data like

[225] E.g. utilizing p. 9 of the manual, retrieved Mai 15, 2022, from https://www.scranton.edu/academics/ksom/alperin/Equity-Fundamental%20Analysis.pdf at a Bloomberg® terminal.

turnover/sales figures, cash flows and earnings (EBITDA) are strong, often further growing and indicate rather stable company choices over the next time (cf. Bloomberg® or Refinitiv® queries on the companies). Many of the companies, as commonly known, are holding a quasi-monopoly or oligopoly in their areas and entry barriers for new market participants are relatively high as CocaCola® in the area of soft drinks, McDonalds® in the fast-food industry or SAP® in business software (Blackstone & Darby, 2020; Esch et al., 2019).

Adding the broad range of branches and industry sectors involved in the bond portfolios, they appear rather representative and build a common financial industry choice for the desired requirements.

Furthermore, the spread – as highly liquid titles were selected – can be attributed to credit risk and merely to liquidity risk as also suggested by academic research and supported by the fact that a rather "normal" and timid macro-economic timeframe was chosen, excluding times of extreme market anxiousness, (liquidity) crisis as during the GFC or insecurity in light of the COVID-19 pandemic (Breckenfelder & Ivashina, 2021; Chaudhary et al., 2020; Covitz & Downing, 2007; Ericsson & Renault, 2000; Ericsson & Renault, 2006; Shirakawa, 1999).

Table 8 Rating agencies scales and the master scale for the empirical comparison.

	Moody's	**S & P**	**Fitch**	**Master scale (ordinary number)**
Top bonds	Aaa	AAA	AAA	19
Very good quality bonds	Aa1	AA+	AA+	18
	Aa2	AA	AA	17
	Aa3	AA-	AA-	16
Good quality bonds	A1	A+	A+	15
	A2	A	A	14
	A3	A-	A-	13

Middle quality	Baa1	BBB+	BBB+	12
	Baa2	BBB	BBB	11
	Baa3	BBB-	BBB-	10
Speculative grade/NIG	Ba1	BB+	BB+	9
	Ba2	BB	BB	8
	Ba3	BB-	BB-	7
	B1	B+	B+	6
	B2	B	B	5
	B3	B-	B-	4
Very bad (highly junk) bonds	Caa	CCC	CCC	3
	Ca	CC	CC	2
	C	C	C	1
Defaulted	-	D	D	0
Short-term ratings - "P"-terminology	P1			15
	P2			12
	P3			10
	F1+			18
	F1			15
	F2			14
	F3			13
	A-1+			18
	A-1			15
	A-2			14
	A-3			13
	A1+			18
	A1			15

A2			14
A3			13
P-1			15
P-2			12
P-3			10

Source: Own table, combined of short-term and long-term ratings of S & P ®, Moody's ® and Fitch®.

Disassembling of the components (with an average empirical ratio) by the author further yielded basically the same results showing no need for separation. The portfolios are then the finally formatted ones – with ratings according to the master scale, missing values coherently filled, synchronized time series and portfolios for the SVR (training and) testing. They are identified as "SVMTest" (=1),…, "SVMTest 4". In terms of ratings, the corporations selected were regularly rated by the presented major three accredited rating agencies in the world, Standard & Poor's ®, Fitch Ratings®, and Moody's®. The rating methodology and terminology of these agencies are rather similar, with details that might exceed the content already described in the thesis to be found on their websites and regular methodological publications. While the role of these rating agencies was discussed in the aftermath of the great financial crisis the trial to establish an independent, foundation-based European rating agency (then empowered by the European Commission and promoted, e.g., by the consulting company Roland Berger®) or other alternatives failed as mentioned before. Apart from required due diligence on their ratings the regulators still allow – and expect – the remaining "three big" agencies and their ratings to be used for regulatory and official purposes (European Commission, 2019; SEC, 2022c; Sinclair, 2005; Witzany, 2017). Besides certain agencies (like Hermes Euler for accreditives, etc.) being allowed for some specific credit types, the named ones are the only ones to be used overarchingly and across different areas of credit (SEC, 2022c; Witzany, 2017, p. 35). Their rating codes are – by their definition of an ordinal number encoding and categorizing the creditworthiness of a company and their description (economical meaning) of the single stages/grades – one-to-one transformable into each other and hence the result is a common ordinal scale utilizing rating master scale. This master scale for the thesis ranges from zero to nineteen (0-19) in ascending order, with zero meaning a company defaulted on its debt and nineteen the highest possible triple-A highest-solvency rating. Short-term ratings are generally coded by the P-/F-terminology as presented in the table above, however, can be transformed to an ordinal number in a similar fashion and combined – with the risk of minimal maturity mismatches – to the common (short- and long-term) master rating scale (Moody's, 2022).

Therefore, four portfolios with representative constituents and attached, rather widely distributed ratings are available.

An intuitive model in econometrics and in measuring credit risk would be a linear or multi-factor model. As indicated by the name it models the underlying (economic) risk factors in a linear way or a linear combination of basis risk factors (Witzany, 2017).

As linear models are rapidly approachable and commonly used for first comparisons, they are employed to show that a support vector machine AI approach is superior and hence principally usable for measuring credit portfolio risk.[226]

The statistical programming language R® is used (the code can be found in the annex/attached) to select an (optimal) linear model (LM) and to train the SVR with the SVM e1071 library.

Running the linear model LM and the SVR model in R on the same four described portfolios yielding a general volatility- and geography-overarching result, random training and test subsets of the portfolios for the SVR are selected (allowing cross-validation). The standard data partition with sizes of 80 % for training and 20 % for testing is utilized for the SVR approach. This is common practice in the data science field, guarantees proper training with a sufficient amount of data and would allow for cross-validation with five same-sized parts. Furthermore, the parameters epsilon for the steps with 0.1 equidistance from zero to one (0-1) and the cost function (ranging from 4 to 1024 $= 2^{10}$) are standard values. The step is fine enough to cover relevant changes yet not too small to generate performance issues. However, the author additionally experimented with various smaller steps in the grid, that indicated no significant deviation at all. Different kernel functions are applied with very similar results (except as expected for the linear kernel which is trivial) – hence using a frequently used radial kernel consequently made sense.

At first, a Kruskal-Wallis rank sum test for the predictions of LM and SVR concerning the four portfolios is performed, because (a priori) nonrelated nonparametric empirical distributions are considered ("distribution-free" approach). Therefore,

[226] Another standard comparison in credit risk is utilizing logistic regression in logit models. As econometric models are founded on these and the PD-barriers of CreditMetrics® implicitly as well (and even extended) this comparison, yielding similar results, is not further detailed here as the SVM approach is compared to CreditMetrics® in the following.

parametric tests as a *t*-test or even normality tests as an Anderson-Darling, Shapiro-Wilk, Doornik-Hansen, or Kolmogorov-Smirnov test are naturally not suitable, the same is true for related-sample tests as the nonparametric Wilcoxon signed-rank test. The first null hypothesis H0 states LM and SVR generated "similar predicted distributions" stemming from "the same group", formally a p-value > 0.05 for the Kruskal-Wallis (rank sum) test. The operationalized version of the research hypothesis into a formal null hypothesis writes:

H0: The predicted distributions of values from LM and SVR are equal (stemming from the same population). LM $\sim^{\text{Kruskal-Wallis}}$ SVR.

The results for all four portfolios and the corresponding out-of-sample predictions ("Test Sets") clearly indicate, that the underlying (predicted) distributions are significantly different and $p \ll 0.05$ for the Kruskal-Wallis Tests. Hence, the first null hypothesis is denied and the generated distribution results differ significantly.

Table 9 Test result of the Kruskal-Wallis rank sum test, comparing the out-of-sample predictions (test partition predictions) of LM and SVR for the four portfolios.

	Portfolio 1 (U.S. LV)	**Portfolio 2 (E.U. LV)**	**Portfolio 3 (E.U. HV)**	**Portfolio 4 (U.S. HV)**
Kruskal-Wallis rank sum Test on data: modelValY and modelValY2 from LM and SVR (*chi-squared*, *df* and *p*-value result)	Kruskal-Wallis *chi-squared* = 175, *df* = 58, *p*-value = 1.136e-13	Kruskal-Wallis *chi-squared* = 343, *df* = 114, *p*-value < 2.2e-16	Kruskal-Wallis *chi-squared* = 343, *df* = 65, *p*-value < 2.2e-16	Kruskal-Wallis *chi-squared* = 419, *df* = 88, *p*-value < 2.2e-16

Source: Own illustration from own R code results.

However, in the next step it is interesting to determine which one is "better" in regard to the real four portfolio distributions and "how much better".

As a comparison, a measure of the "distance to the real output", i.e., a sort of "goodness-of-fit" or "minimalizing of errors to the real data" is needed. A standard indicator to achieve this aim is the root mean square error (RMSE), which is therefore applied here (Dixon, 2020; Greene, 2003; Martin et al., 2014). The RMSE metric possesses useful mathematical properties (as differentiability, subadditivity etc.) and is naturally a nonparametric error measure, i.e., distribution-independent (Greene, 2003). It always yields larger or equal resulting values than, e.g., the corresponding mean absolute error values and is therefore prudent (Greene, 2003). Circumstances and data which would prefer another measure like that absolute mean error or weighted means are not applicable here, hence the RSME is the method of choice. The second null hypothesis can be formulated as:

H0: RMSE (LM, real portfolio data) ≤ RMSE (SVR, real portfolio data)

It is also denied. The RMSE results – applied already to the training set as well as to the out-of-sample test set for both approaches – in a much smaller RMSE (number) for the SVR case.

Results for the Test Subset for the four portfolios, where the columns denote the portfolios as described above and the rows constitute the model applied, are shown below:

Table 10 Results of the LM-SVR comparison on the test set.

Method/Portfolio	U.S. LV	E.U. LV	E.U. HV	U.S. HV
LM	21.44766	22.305105	20.484721	32.83905
SVR	16.16686	7.821679	7.101168	16.97299

Source: Own illustration from own R code results.

Results for the (less important) Training Subset for the four portfolios, where the columns denote the portfolios as described above and the rows constitute the model applied:

Table 11 Results of the LM-SVR comparison on the training set.

Method/Portfolio	U.S. LV	E.U. LV	E.U. HV	U.S. HV
LM	21.94749	21.426613	19.902117	32.36669
SVR	16.81485	6.764679	7.428752	15.08635

Source: Own illustration from own R code results.

The result tables show that the SVR model performs much better than the intuitive, feasible linear model LM with an RMSE number which is ~ 23 % to 68 % less. The null hypothesis H0 that RMSE(LM) ≤ RMSE(SVR) on average is denied – even not true at all, for all four portfolios. Especially for portfolios 2.-3. and higher volatility the method of SVR performs significantly better and clearly outperforms the linear model.

Furthermore, as accompanying information, an additional distribution-independent Kruskal-Wallis Test of the SVR results with the original portfolio data was indicating success for all portfolios (H0 that the SVR model output inherits the same distribution structure as the real portfolio was clearly not denied).

Hence, the support vector machine AI - model is superior to the linear model and principally useful and effective to measure credit portfolio risk. This is the first part to be proved. It is furthermore of high practical relevance. As AI-based models become more frequently used in banking also the regulators (supervisory bodies) set up standards regarding their use. The German BaFin published its guideline on BDAI (Big Data and Artificial Intelligence) this year, and the ECB and EBA published guidelines for the use and validation of AI models as in November 2021 (EBA, 2020; EBA, 2021f; ECB, 2021b).

Regulators therein acknowledge the frequent use and spread of AI-based models, define terminology, principles, and limitations to their use and make best-practice-type recommendations. In the coming years even more activity in this realm is to be expected with different task forces, working groups, and impact studies set up for this purpose by supervisory bodies (as well as within banks and insurance themselves).

Among other trends like decentralized finance (“defi”) based on blockchain type technologies of different layers, layer one is again a pure cryptocurrency like Bitcoin® (BTC) or Ethereum® (ETH), whereas layer two builds on that and sets up smart contracts or processes like staging crypto-credits upon it as seen, are platform-based approaches utilizing AI (Dixon, 2020). Furthermore, the areas of Robo advising and investing employ AI methods, and the sophisticated use of artificial-intelligence-based models in credit risk and also valuation and execution of deals might become a cutting-edge distinction criterion between competitors (Dixon, 2020).

5.4 Comparison of the SVR approach to Industrial Credit Portfolio Models

The main objective of the thesis is to show that the SVR model can perform better than traditional credit portfolio models. That means that the SVR-predicted VaR in such a case is closer to the real portfolio VaR in comparison to the ones predicted by classical models. As described before, that is a crucial point, since a more precise credit risk model improves credit risk management and risk-bearing capital allocation, hence optimizes risk/return metrics for portfolios and for the pricing of bonds and loans. Ultimately, it may lead to higher profits for a financial institute.

As shown in Chapter 4 two types of credit portfolio risk models are industry standards in the financial industry. The first ones are hazard rate or actuarial models modeling the default as a random exogenous stochastic process using a Poisson process as usual to count the number of (rare) default events and a Gamma function as a common risk (impact) factor resulting in a negative binomial distribution type function at the end. This risk model type with its widely used proponent CreditRisk+® is implemented and well-tested in an R package called CRP and hence used to compare it with the SVR method. The second type of credit portfolio risk models are structural or asset-value models introduced by Merton and with its main proponent being the extended migration model CreditMetrics®. For CreditMetrics® not being available as a common standard in R, an Excel-VBA-based standard version programmed by the author and previously used (license-free) and validated by clients in the banking industry is applied. The third type, econometric models, are principally convertible and reducible to structural settings with comparable results as shown in Chapter 4, hence they are not separately considered. As CreditMetrics® is superior to CreditRisk+® in most portfolio contexts (especially for rated, listed companies as dealt with in the four portfolios) the comparison of the SVR with the first one is the (far) most relevant in practice.

As CreditRisk+® models credit risk by an exogenous stochastic Poisson mixed process, the intensity parameter lambda of the Poisson process and the parameters (alpha, beta) of the Gamma function are normally empirically calibrated, and can be transformed to other default variables. In the case of the thesis standard parameters (in line with the CreditMetrics® settings) – finally even optimized by the R package in use – were chosen.

CreditMetrics® is founded on the asset price movements – derived or simplified from stock prices in practice – and underlying returns of a company as seen. As a reminder, they are assumed to move statistically in a normal distributed way and a company defaults once its debt D is higher than the assets V resulting in negative equity capital. The related subsequent quantiles of the normal distribution hence represent the rating grades. One has to keep in mind that the companies are not independent and isolated but connected and thereby bear a common (industrial or economic) risk factor and an individual idiosyncratic one. [227] Hence the returns and asset prices as well as the potential defaults are correlated via asset price correlation proxies, empirically calibrated. Furthermore, by applying the correct (periodical) discount factors the bond prices have to be re-calculated, especially discounted with the proper forward rates, after each period.

For CreditMetrics®, therefore, certain input parameters are needed and named in the corresponding tabs of the Excel® file for a well-calibrated model. These are the

- Transition matrices describe the probability of a bond migrating from one rating to another – where a bond time series consistent 2019 public version from S & P ® is selected
- The spot and forward rates (extracted from Refinitiv® for the same periods) for risk-adjusted discounting purposes and
- Plausibilized sector correlations (U.S. banks, U.S. tech, German banks as proxy for E.U. banks, 2019 from Refinitiv®, see also (Leske et al., 2004))

Furthermore – as computations and cross-correlations are too complex or not available otherwise in CreditMetrics® – an industrial standard is applied, where five random (by dice) constituents are taken from each of the portfolios instead of taking the whole portfolios themselves. The selected bonds are – as constraint, otherwise implying a redraw – distributed among the ratings real range and industries and have 20 initial stable ratings at the beginning of their time series as explained later. In the input sheet all bonds are equipped with unit 1 (here equal to one million) nominal amount, 7[228] years maturity and 5 %[229] coupon as in line with the Refinitiv® bonds and standard data and a standard recovery rate of 40 %[230] for senior unsecured bonds (hence a 60 % LGD – loss given default) is assumed.

Subsequently, further parameters are calculated directly standard-wise by the program as the product of the transition matrix, the discount rates (as reciprocal of the spot and forward rates), the thresholds (as the inverse of the normal distribution of the

[227] For a good overview of systematic and idiosyncratic risk factors see, e.g., (Moody's, 2013, p. 16).

[228] As medium-term notes (MTN) have a normal maturity of 5-7 years, 10 years is a standard longer-term bond. Therefore, 7 years is a suitable standard (and "average"/"middle ground"), see also (CFI, 2022e).

[229] Also the most common standard (5%, in some applications 10%) in law see, e.g., (Upcounsel, 2020) and practical textbook examples, as in https://www.investopedia.com/terms/c/couponbond.asp (retrieved Mai 14, 2022) or mainly in (Hull, 2015).

[230] Industry standard as in ISDA, cf. (ISDA, 2009) or collected by S & P ®: (S & P, 2021b).

different ratings' boundaries/PDs), the Cholesky-decomposition of the covariance matrix or the correlated random variables per Monte Carlo simulation (multiplied with the weighting of sectors plausibly chosen and correlations). The report and also the "CEO_report" tab then show the result as Value-at-Risk with a holding period of 250-days (i.e., one standard trading year) and confidence intervals of 95 % and 99 % (i.e., level of significance 5 % and 1 %). This result (95%) is then benchmarked with the SVR result for all four portfolios.

To apply SVR to the portfolios, for all four of them the five bonds (distributed over different branches and ratings, starting at the same PIT as mentioned), for each with 20 stable initial ratings (as one out of twenty corresponds to a VaR 95 % later) at the beginning of the time series are selected. This guarantees synchronous, coherent data and long data history.

After choosing T0…T19 – ordered from the beginning of the common time series period – the 20 matching changes in T-250 (trading days) hence T250…T269 are selected for the five bonds to calculate the real Value-at-Risk (VaR, 250 days, 95 % CI) and testing set base. These are also consistently used for CreditRisk®, its initial ratings and calibration, with results of the calculations in the four CreditMetrics®-Excel-Files.

The SVR model is furthermore trained with the whole residual portfolio (naturally except the T250-269 record sets) in the "SVMTest"-files. Therefore, enough data is available for a calibrated, well-fitting model. The parameters are standard ones as selected for the comparison with the linear model, hence again a radial base function kernel, with epsilon (preciseness) from zero to one in 0.1 steps, the cost function (balancing the width and penalty in SVR as regarded before) ranging from four to 1024 and tuned for the best optimized choice.

The trained SVR model is applied to the before-mentioned test set and the VaR results are to be found in VaRTest, Test file. Well-calibrating of CreditMetrics® yields a closer result than with CreditRisk+®, the latter one being analogously well-calibrated with LGD of 60 %, weights and PDs as in CreditMetrics® and 100 (%) EAD, hence, e.g., without any drawn credit lines as plain-vanilla bonds. The LGD of 60 % is an empirical industry standard for senior unsecured loans as shown before. It represents a solid choice throughout different branches, sectors, and company sizes while avoiding over-specification (or over-fitting) to a certain industry or company profile. Higher losses are especially common for subordinated loans, high-yield loans, or in terms of small companies or venture capital-backed start-ups. As these types of loans and companies are not within the scope of this thesis, the average of 60 % seems appropriate. On the other hand, also secured or collateralized bonds are not considered, and extremely well-handled recoveries of conveniently liquidated companies are rare – therefore the 60 % also seems to be well-chosen as an upper bound for the purpose of the thesis. The weights and PDs are – with PDs directly linked with the corresponding ratings – empirically (historically) chosen and are of typical height (and well replicable). One can therefore speak from an industry typical, empirically replicable setting for the test. The product of the unexpected distribution part of the default probability with (a normalization factor 12.5 and) the conservative loss given default again combines to the risk weight RW in regulatory settings:

$$RW = (VaR_{quantil}(PD) - PD) \cdot 12.5 \cdot LGD \tag{220}$$

To calculate the expected loss *EL*, as shown before, one has to multiply *PD* and *LGD* by the exposure at default, i.e., the amount which is relevant and "in danger of loss" once a default occurs:

$$EL = PD \cdot LGD \cdot EAD \tag{221}$$

Therefore, the *EAD* is chosen in this thesis as the standard unit amount of one (million) for each company and bond in the underlying currency unit as mentioned and is hence not impacting (or even give a reason for a biased approach) the results (*VaR*). The third null hypothesis states that the SVR model is not performing significantly better for the majority of chosen portfolios than classical credit portfolio models when predicting the Value-at-Risk (VaR) of the portfolios. It can be operationalized as:

H0: Min (CreditMetrics® VaR – real VaR; CreditRisk+® VaR - real VaR) ≤ (SVR VaR - real VaR) – for the majority of the portfolios. That null hypothesis is clearly denied.

CreditRisk+® generally leads to (far) worse results compared to the real VaR and SVR regression is superior in every case. For portfolios 1.-3. CreditRisk+® predicted VaRs of roughly 35-40 % less (2.4-2.6) than CreditMetrics® and for portfolio four it was better than CreditMetrics® yet still too low with absolute value 12, compared to a real VaR of 31. CreditMetrics® is underestimating the real VaR less and is superior (as expected), yet not as precise as the SVR for most of the portfolios. For portfolio 1 CreditMetrics® yields the best result, the SVR approach however is far more accurate for portfolios 2 and 4 and at least even for portfolio 3, both – also the SVR one – have underestimated the real VaR here for the European high volatility portfolio.

Table 12 Final result of comparing CreditMetrics® and the SVR approach.

Portfolio of Bonds | VaR, 1 year (250d), 95 %	CreditMetrics®	SVR	Real	Scores (+1,0,-1) for SVR
Portfolio 1	4	5.1	4.2	-1
Portfolio 2	4.1	18.4	15.8	+1
Portfolio 3	4.1	4.1	13.5	0
Portfolio 4	4.3	31.7	31.3	+1

Source: Own illustration own R code results and aggregation.

This results here in a score of 2:1 for the SVR model in comparison to CreditMetrics®, the superiority of the SVR method in terms of VaR calculation is implied for representative portfolios.

CONCLUSIONS

The research hypotheses of the thesis dealt with the applicability of support vector regression for measuring credit portfolio risk and the question of how precise it may measure it for standard bond portfolios compared to industrial credit portfolio models.

Therefore, the necessity for banks to quantify credit portfolio risk precisely was presented in the first chapters of the thesis, while having introduced the target objects assets (asset classes), portfolios, and funds with their underlying characteristics as well as measures for risk/return, optimal portfolio models, and models currently available to determine the specific risks in the banking industry. The inherent form of appearance of credit risk in various asset classes or project investments and its crucial role within a bank's risk management framework was discussed.

A comprehensive overview first treated single obligor credit risk and ratings by the inclusion of internal as well as external rating approaches rarely regarded in academic research so far, combined with current regulatory requirements and a review of the entire rating process. The preference for mathematical-statistical models was illustrated, and state-of-the-art validation techniques were summarized and presented. In the next step, multiple obligors and their correlated credit risk structure within a credit portfolio were regarded as the thesis aimed to compare SVR with thoroughly analyzed industrial credit portfolio models.

Therefore, the most comprehensive literature review in that area to date and (meta-)analysis of current credit portfolio models, their underlying assumptions, strengths, shortcomings as well as preferred portfolio type applications were given. The results show that apart from homogeneous retail portfolios where the hazard rate proponent CreditRisk+® is prone due to its feasible implementation, execution speed and analytical closed-form solution, CreditMetrics® (and equivalently the structural model variant KMV®) is currently the best performing credit portfolio model regarding VaR estimation overall and especially for listed companies and rated entities. This result is for instance in line with (Arora et al., 2005; Diaz & Gemmill, 2002; Gordy, 1998; Gordy, 2000; Kollár & Gondžárová, 2014; Kolman, 2010; RiskMetrics Group, 2007; Witzany, 2017). Structural models as illustrated thus outperform hazard rate or reduced form models in general; especially when utilized for out-of-sample predictions, as formerly already exemplified in (Duffie, 1999; Geyer et al., 2001; Kolman, 2010). Reduced form models further tend to underestimate common default rates, credit spreads, and thus VaRs whereas structural models' results seem to be more conservative – as also the thesis' results for the four test portfolios show, confirming (Arora et al., 2005; Crouhy et al., 2000; Diaz & Gemmill, 2002; Elizalde, 2006; Gordy, 1998b; J.P. Morgan, 1998; Kollár & Gondžárová, 2014; Kolman, 2010; Stein, 2002; Witzany, 2017). The similar econometrical models perform also considerably well in practice and are especially convenient when the VaR ought to be linked with and explained by macro-economic factors, as known from (Bluhm et al., 2003; Wilson, 1998). Such an approach, as implemented in CreditPortfolioView®, is frequently used by mutual and savings banks relying on pooled data available from common service units in the umbrella association or holding company. The thesis outlined in Chapter 4 that the three model types can be transformed into each other within a common theoretical framework utilizing mainly methods from (Bélanger et al., 2003; Hickman & Koyluoglu, 1998; Wong, 1998). Nevertheless, the literature review of the empirical research generally showed stronger performance metrics for structural models, referencing inter alia (Arora et al., 2003; Gordy, 1998b; Gordy, 2000; Grouhy et al., 2000; Kollár & Gondžárová, 2014; Kolman, 2010; Schwarz, 2006; Wahrenburg & Niethen, 2000; Witzany, 2017; Zhang et al., 2008). The model comparisons for the test portfolios in the thesis, employing E.U. and U.S. liquid bond portfolios and each of them within two different regimes of volatility, proved that aspect as well. Furthermore, CreditMetrics® as the primary structural model choice was shown to have several extensions and specialized applications, by using ideas of (Bielecki & Rutkowski, 2004; Black & Cox, 1976; Duffie & Lando, 2001; Giesecke & Goldstein, 2004; Goldstein et al., 2001; Ho & Singer, 1982; Ho & Singer, 1984; Jou & Lee, 2009; Leland, 1994; Leland & Toft, 1996; Lettau & Wachter, 2011; Longstaff & Schwartz, 1995; Madan & Unal, 2000; Vašíček, 1984). Model enhancements that even capture risk parameter dependencies as the PD-LGD nexus like a concept introduced by Emmer and Tasche, an empirically calibrated approach by Eckert et al., and an alternative variant from Witzany were presented in the thesis (Eckert et al., 2016; Emmer & Tasche, 2016; Witzany, 2011).

The most innovative one, however, was probably the CreditMetrics® extension ZPP that is utilizing copulas and is further achieving to outperforming the other CPMs, e.g., KVM® (de Giuli et al., 2007; Kamali et al., 2020).

Apart from that extension, there was less research regarding credit risk models and it brought few model innovations during the last ten years. While parts of credit risk research were however revived through AI-based methods and SVM has been already used for individual, single obligor scoring methodologies and showed superiority in comparison to some traditional methods in that area, it was not applied to assets in portfolios and credit portfolio risk yet (Baesens et al., 2003; de Laurentis et al., 2010; Lessmann et al., 2015; Roa et al., 2021).

The thesis introduced a novel approach for quantifying credit portfolio risk, namely support vector machines and support vector regression, and bridged the gap to a portfolio view. Various other artificial intelligence techniques as ANNs were presented in a detailed fashion, and applications of AI in the realm of finance were discussed, like in (De Spiegeleer et al., 2018; Dixon, 2020; Dixon & Halperlin, 2019; Dori et al., 2018; Feng et al., 2018; Hornik et al., 1989; Hutchinson et al., 1994; Roa et al., 2021). Certain superior SVR characteristics were outlined in the thesis which could be utilized for credit risk as well, building on works of (Halls-Moore, 2017; Schölkopf & Smola, 2001; Vapnik, 1998; Wilmott, 2007; Witzany, 2017).

The first research question considered the applicability of SVR to credit portfolio risk. To answer it, four realistic, representative portfolios in the liquid bond markets of the EU and USA with low and respectively high volatility for each geographical destination were selected. According to common data science standards, those were synchronized, ratings unified on a common ordinal master scale, it was conservatively dealt with missing values and non-ratings, and the files were adequately formatted. The statistical programming language R within RStudio®, besides Microsoft Excel®, was employed to implement the SVR model for the four standard portfolios.

The thesis proved the applicability by showing that SVR outperforms multi-factor linear models by far – in terms of a general Kruskal-Wallis test indicating a different generated distribution of outcomes, and concretely in terms of measuring the RMSE. This test is the standard statistical tool for comparing nonparametric independent sample distributions. By utilizing the Kruskal-Wallis test the null hypothesis of similar predicted distributions stemming from the same group in regard to the linear and support vector regression models' outcomes was clearly denied, with a p-value that was smaller or equal than $1.136 \cdot 10^{-13}$ for randomized samples and out-of-sample predictions for all four portfolios. The second null hypothesis stated that the RMSE of the multi-factor linear model (LM) is smaller than the one of the support vector regression (SVR) approach for the majority of selected portfolios. Since the RMSE measure has feasible mathematical properties, is generally applicable and nonparametric, and is more conservative (≥) than e.g. the mean absolute error (MAE) it was the method of choice to compare the LM and the SVR in the thesis. In terms of the root mean square error values, SVR outperformed the LM in each case, by more than 20 % in the worst case up to nearly 70 % in the best one, and on average by roughly 50 % across all four portfolios on training as well as out-of-sample test sets. Hence, the second null hypothesis could be denied, and the superiority of the SVR method compared to the LM method was proven.

After having shown the usefulness for measuring credit risk, the objective was to benchmark the SVR approach against the industrial models CreditRisk+® and especially CreditMetrics®.

Therefore, the third research hypothesis dealt with the comparison of SVR to industrial credit portfolio models, and the null hypothesis in the formalized, negated form of the corresponding research hypothesis stated that the latter are superior. To decide that research problem, the industrial proponents of structural models and hazard rate ones, CreditMetrics® and CreditRisk+®, as well as the novel applied SVR were calibrated to the same sub-portfolios of the four standard portfolios and equipped with the usual assumptions from academic literature. Afterward, the code implementations and VaR calculations followed.

As a result, CreditMetrics® performed well for a small real VaR and low volatility portfolios, however overall for the four portfolios it underperformed in terms of prediction accuracy compared to the SVR approach. The SVR's predicted VaRs were more precise and the error metric as distance to the real portfolio VaR was overall considerably smaller. Whereas SVR outperformed CreditRisk+® for all four portfolios and the latter one also only showed superior results compared to CreditMetrics® in the case of the last U.S. bond portfolio, SVR still equaled or even outperformed the former front-runner CreditMetrics® for three portfolios. Only in the case of the European high volatility portfolio, SVR was inferior, maybe due to the portfolio structure and the preselected kernel type of a radial base function. Especially for the E.U. and U.S. high volatility portfolios, SVR was able to capture the default and migration correlations of the bonds and the portfolio dynamics in a more precise fashion and yielded far superior results. The last research question was hence answered, and it was proven that SVR is not only applicable and useful for the area of credit portfolio risk in theory and practice but on average even shows superiority to currently employed industrial models for the selected portfolios.

Therefore, not only in general but especially for volatile portfolios, trend branches like tech, biomedicine, and parts of the finance sector, or for modeling portfolios in times of crises, an AI-based model like the support vector machine regression approach can be more accurate in terms of quantifying the adequate amount of credit risk-bearing capacity needed to cover the Value-at-Risk of a portfolio and might be the preferred method of choice. As profit margins are narrow, especially in a lower zero bound environment (LZB) with inherently thin interest margins and not foreseeable to rise extremely quickly as only moderate, time stretched[231] interest rate hikes by the Federal Reserve Bank in the United States took place and in case of the European Central Bank in the eurozone not before summer 2022 after ending the PEPP[232] and asset purchase program (APP), a superior risk model which calculates the Value-at-Risk and corresponding risk cover capital more precisely is, therefore, a competitive edge (ECB, 2022e; ECB, 2022f; ECB-Statement, 2022). Applying SVR for measuring credit portfolio risk is hence a potential tool that may help to decrease loss reserves and provisions and as a consequence increase profits for banks. The model is readily available and directly implementable since programming libraries and packages for all major statistical languages as SAS®, Python ®, or R exist.

An additional argument is that the credit portfolio is traditionally forming the major part of the risk for almost all banks, except a few investment banks or large trading book institutes, and constitutes the highest portion of risk-weighted assets (EBA, 2021b, p. 46). Post-pandemic economic recovery in the EU and its corresponding rise in the amount of loans reached out to corporations, and even already increased loan offerings, especially from state (backed) banks during the pandemic, makes a calculation of credit risk that is as exact as possible even more essential from a risk management perspective as well as from a macroprudential stability point of view (ECB, 2020, box 7; ECB, 2020b; KfW, 2020).

[231] Though considerably faster than assumed a year ago due to a rapidly rising inflation.
[232] The ECB council declared on December 16, 2021, that net purchases within the PEPP are ending by March 2022. Reinvestments of the principals due from PEPP holdings ought to be done by the end of 2024. However, in an emergency meeting on June 15, 2022 (the last day of the thesis' finalization), the European Central Bank declared that it will use proceeds from expiring bonds from the PEPP to reinvest them in a flexible manner and potentially asymmetrically in issuances from countries with high bond credit spreads (e.g., from Italy) to prevent a "fragmentation" of the market and ensure an ordered transmission of their monetary policy measures (ECB-Statement, 2022b).

Furthermore, regulatory reforms, e.g., the Volcker-rule and Dodd-Franck act in the US or the revised CRR and the introduction of the FRTB in Europe, as well as a scale-back of investment banking activities within most banks, underpin the predominant role of credit risk for banks' risk-weighted assets as laid out in the thesis. The trend is accompanied by the increasing need of companies and consumers to get credit access more rapidly, in a more flexible manner, more automatized and to compare loan conditions more intensively ("credit shopping", "real-time loan comparison") – resulting in an even higher necessity of precise risk control (Camba-Méndez & Mongelli, 2021; GDS, 2021; Jorda et al., 2015).[233] Often credit lines, however most frequently smaller ones, and loans are offered directly via an app on a smartphone or via platforms, sometimes even in the form of peer-to-peer (P2P) or crowd lending (GDS, 2021). That trend might even accelerate in the future.

An automatized, AI-based, flexible, and accurate calculation approach is, therefore, a valuable tool in that environment. As the regulatory focus changed to more dynamic and stress-test-based approaches which banks need to fulfill in the recent years – extended by the presented climate stress tests with physical and transitory scenarios introduced by the EBA and ECB in 2021– risk models that can adapt to swiftly changing environments and extreme or stress scenarios are of ever more importance (ECB, 2021b; Witzany, 2017). Generally, there is the precept of technical neutrality for regulators and methodological freedom of choice for a bank, as long as a model fulfills legal requirements and supervisory standards. From a regulator's perspective, it was indicated that authorities view the use of AI-based quantitative models in a more positive light than in the past, and standards were set under which the employment of such a method is allowed (EBA, 2020; EBA, 2021f; ECB, 2021b). Since in the field of credit scoring regulators have already allowed the use of for instance ANNs or SVMs under some restrictions, the same will probably hold true for artificial-intelligence-based credit portfolio models in the future as well. Furthermore, as banks have built up their AI-based model inventory in other pricing, hedging, and risk quantification areas during the last years, it might be conveniently extended by an SVR CPR component. It is therefore fitting in the enhanced model inventory which itself is integrated into the entire risk management framework of a bank, fulfilling the requirements of the European internal capital

[233] Closely monitored by the regulators such as the ECB and FED.

adequacy assessment process (SSM ICAAP) and the internal liquidity adequacy assessment process (SSM ILAAP) for significant institutes, respectively the risk-bearing capacity regulations by the national competent authorities.

SVM-based regression models are a suitable option for an artificial intelligence model in the area of credit portfolio risk as the test results showed its strong performance for assets with high volatility. The performance and flexibility of SVR in adapting to different data sets and distribution densities, represented by the kernel functions, to nonlinear circumstances and special situations as well as the ability to fine-tune its calculations by exact settings of the penalty parameters to avoid under- as well as overfitting are essential features. The SVR method should be employed in practice to measure credit portfolio risks more precisely, help to improve predictions, portfolio risk/return profiles, and risk management, and thereby potentially increase profits.

Support vector machines further have the advantage that they “learn” from patterns in data that are available and can be therefore calibrated well to historical settings. As neither underlying a priori distribution assumptions nor potentially non-realistic constraints regarding asset price movements nor common parametric default intensities are necessary for SVR, not even an economically feasible model framework, the support vector regression can be applied to the given real portfolio data in a remarkably flexible manner. The flexibility of various kernel functions captures the credit risk dynamics very well, is able to accurately illustrate the dependency structures, and is suitable for highly nonlinear and nonnormal distributed circumstances. As a method that is exploiting artificial intelligence and “big data”, the results further improve with more data sets and longer time series available, hence automatically “over time”. Common kernel functions like radial base kernels (RBF, radial base functions) or exponential smoothed kernels, Gaussian kernels, and polynomial kernels are accessible for that purpose as presented, as well as optimization methods in the corresponding packages, for instance in R, for their step-width adjusted grid and balancing parameter to guarantee a sufficient generality of the results. Therefore, increasing amounts of data and powerful kernel functions in line with state-of-the-art implementations will favor AI-based methods such as SVR even further in the future.

As a result, approaches exploiting AI and especially SVR, are not only useful for the pre-categorization of customers like automated scorings and ratings but also for the illustration of marginal risks and the underlying dependency risk structure in a whole standard portfolio and thus the quantification of its Value-at-Risk. AI-based methods can hence extend the existing structural models, hazard rate approaches, and econometrical models for standard corporate bond portfolios by a further model type employed in future credit risk research. Historically, as illustrated in Chapter 4, the structural models or Merton models are built on the assets and debts of a company and the movement of their values over time and are linking corporate finance and option pricing theory with credit risk. Hazard rate models stem from a different, actuarial background and utilize exogenous Poisson Gamma mixture processes from insurance mathematics to describe credit risk. Finally, econometrical models are employing a multi-factor approach with common macro-economic motivated indicators to derive credit risk values. Support vector regression, however, neither requires certain internal assumptions of the asset or debt structures of companies and their motions nor a pre-defined external stochastic process or macro parameters to value assets and the corresponding credit portfolio risk, it is more flexible and learns from previous patterns in bond and portfolio prices "as they are". The same holds true for dependencies between obligors.

The thesis showed that the research topic of dependency and credit correlation structures in a portfolio can be successfully treated and improved by utilizing SVR. Overall, SVR is a precise credit portfolio VaR estimator.

The SVR approach proved to be successful as an AI "pioneer" for credit risk management and is supposed to be among the most efficient artificial-intelligence-based methods when referring to illustrated comparisons with other methods for similar applications in the past. Apart from the RMSE measure also other advanced goodness-of-fit measures (AGOFs) might be used for such a benchmarking purpose. Additionally, combinations of different kernel functions could be investigated and setting-adapted kernels for special credit or funding structures and types, e.g., considering hybrid debt instruments or project finance investment, might be imagined well for future research.

Quantum SVR is a method that may be employed at some point as well, potentially accelerating calculations. These approaches utilize the phenomenon of quantum annealing in quantum physics to "parallelize" and accelerate the optimization problem in SVR as presented by Kadowaki (Kadowaki & Nishimori, 1998). Dalal et al. in their research connected with facial-landmark-detection, Li et al. in a Nature study dealing with a biological problem, and Willsch et al. by applying a standard D-wave quantum annealer to assist the "traditional" SVR showed the superior accuracy of quantum assisted methods compared to classical AI and SVR approaches (Dalal et al., 2021; Li et al., 2018; Willsch et al., 2020). Therefore, Quantum SVR seems to have further potential to improve SVR.

The thesis scope, due to liquidity considerations, was limited to developed credit markets. These, as shown in the thesis, reside – apart from South East Asia, partly China[234] and several other single countries like Canada, Australia or New Zealand – largely in the United States and in Europe, regarding the latter, especially within the Eurozone area (BIS, 2022). Access to the bond issuances is generally considered to be fairly transparent and open, with professional surveillance and regulation, unlike in many emerging market countries (European Commission, 2022b). However, with time, more widespread trading technologies, regulatory adoptions, and improving liquidity in emerging markets more research in terms of applying SVR to emerging market portfolios might be valuable as well.

Furthermore, the research of smaller, unlisted companies or consumer portfolios with higher default rates may lead to an indication of the (very probable) usefulness of AI-based risk management methods like the SVR method as well in these areas of credit. Since SVM was useful for default prediction accuracy and credit scoring for single obligors and as pointed out in the thesis SVR is effective for corporation portfolios, one assumes that support vector regression may work similarly well for small exposure size retail portfolios. As analyzed within the thesis, for banks a cost-effective approach and a high degree of automatization are especially crucial in the area of retail credit risk. As the size of loans is frequently rather small in that specific field, an individual, human resource intensive due diligence process is too expensive as well as a long expert-based calibration of standard models. Additionally, due to the fact that these retail portfolios are commonly treated within the advanced internal ratings-based approach by banks and are structure and size-wise often prone to "big data" algorithms the application of SVR seems to be a natural step.

For some companies in the field of listed corporations, apart from their bond credit spreads, also CDS spreads might be investigated utilizing SVR-based methods, thus incorporating even more "real-time" information. The principal method would be similar

[234] The Chinese bond emissions amount to the second highest worldwide, after the United States, and the third highest when regarding the EU cumulatively, as shown in the thesis (BIS, 2022). However, the access is not completely open for foreign investors and the Chinese yuan not freely convertible.

to the one used in the thesis, where bond prices and their spreads were included in the applied credit portfolio risk models like CreditMetrics® or SVR – only substituted with the corresponding single name credit default swap spread of the respective bond, at point of issuance represented in the premium leg contribution, and taking CDS correlations into account. As a consequence, the application could then also serve as a rapidly visible early warning indicator for individual default risks, sector-wise widening spread structures, or upcoming crises on a broader scale.

Additionally, other AI-based models like artificial neural networks, bagging methods, or maybe even reinforced machine learning models might be adequate for such scenarios – those might be interesting areas for future investigations. As mentioned before, one can apply bootstrapping methods within an SVR framework for widening the amount of useful data, and further utilize bagging methods by employing various different kernel functions for the support vector regression and averaging them to improve performance. The kernels might be chosen due to individual properties and pre-assumed distribution characteristics of the credit portfolio. In the case of support vector regression one could for instance exploit sigmoid kernels, radial base function kernels, and Gaussian kernels for the specified calculations and subsequently take the arithmetic mean of the corresponding predicted VaRs as averaged final VaR estimation.

Another common method in the field of machine learning was to combine distinct artificial intelligence models, as opposed to only different functions within the same model framework, and subsequently take some sort of average of them in a so-called ensemble. That method is increasingly used in AI-based research areas. In the case of credit portfolio risk, it would be an opportunity to combine support vector regression with for instance random forests, k-nearest-neighbors, neuro-fuzzy inference systems, or artificial deep neural networks for the task of calculating the Value-at-Risk of a portfolio. Since these distinct approaches have their own, unique techniques to model correlation structures, portfolio dynamics, and overall credit portfolio risk, one could analyze their individual strengths within various portfolio settings, weight them, and exploit them in the final aggregated ensemble. As a consequence, the method of ensemble predictors might be a promising topic for further artificial-intelligence-based credit portfolio risk research.

Until then the results as stated in the thesis are implying for certain standard portfolios the superiority of the support vector regression approach compared to industrial credit portfolio models like CreditRisk+® and CreditMetrics® – hence justify the primary future usage of SVR in measuring credit portfolio risk.

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ABBREVIATIONS

ABS – Asset Backed Security

ADC – Acquisition, Development and Construction (of Land)

ADD – Average Drawdown

ADR – American Depositary Receipt, or: ADR – Address Risk

AG – German: Aktiengesellschaft

AGOF – Advanced Goodness of Fit

AI – Artificial Intelligence, or: AI – Accrued Interest

AIC – Akaike Information Criterion

AIF – Alternative Investment Fund

AIFM – Alternative Investment Fund Manager

AIMA – Alternative Investment Management Association®

AIRBA – Advanced Internal Ratings-Based Approach

ALCO – Asset Liability Committee

ALM – Asset Liability Management

ALMM – Advanced Liquidity Monitoring Metrics

AMA – Advanced Measurement Approach

AML – Anti-Money Laundering

ANN – Artificial Neural Network

AOCI – Accumulated Other Comprehensive Income

APEC – Asia-Pacific Economic Cooperation

APP – Asset Purchase Programme

APT – Asset Pricing Theory

AR – Accuracy Ratio

ARCH – Autoregressive Conditional Heteroscedasticity

ARDL – Autoregressive Distributed Leg

ARIMA – Autoregressive Integrated Moving Average

ARR – Average Rate of Return

ASRF – Asymptotic Single Risk Factor (Model)

ATF – Anti-Terror Financing

AT 1 – Additional Tier 1

AUC – Area Under the Curve

AUROC – Area Under Receiver Operating Characteristic

BAPT – Behavioral Asset Pricing Theory

BB – Banking Book

BCA – Base(line) Credit Assessment

BCBS – Basel Committee on Banking Supervision

BCM – Blocher, Cooper, Molybaga Model

BDAI – Big Data and Artificial Intelligence

BI – Business Intelligence

BIA – Basic Indicator Approach

BIC – Bayes Information Criterion

BICRA – Banking Industry Country Risk Assessment®

BIS – Bank for International Settlements

BLUE – Best Linear Unbiased Estimator

BM – Brownian Motion

BMS – Black-Merton-Scholes (Model)

BOE – Bank of England

BOFA – Bank of America®

BOJ – Bank of Japan

BRICS – Brasilia, Russia, Indonesia, China, South Africa

BRRD – Bank Recovery and Resolution Directive

BS – Black-Scholes

BSF – Binary Step Function

BTC – Bitcoin®

B2B – Business to Business

B2C – Business to Consumer

BV – Book Value

BVBP – Base Value of a Basis Point

CA – Carrying Amount

CAIA – Chartered Alternative Investment Analyst Association®

CAL – Capital Allocation Line

CAP – Cumulative Accuracy Profile

CAPEX – Capital Expenditure

CAPM – Capital Asset Pricing Model

CAR – Capital Adequacy Ratio

CART – Classification and Regression Tree

CBDC – Central Bank Digital Currency

CBOE – Chicago Board Options Exchange®

CBOT – Chicago Board of Trade®

CBR – Combined Buffer Requirement

CCAF – Cambridge Center for Alternative Finance®

CCAR – Continuously (Continued) Compounded Annual Rate, or:

CCAR – Comprehensive Capital Analysis and Review

CCB – Capital Conservation Buffer

CCF – Credit Conversion Factor

CCP – Core and CorePlus

CCR – Counterparty Credit Risk

CCyB – Counter-Cyclical Buffer

CDD – Conditional Drawdown

CDF – Cumulative Distribution Function

CDR – Conditional Default Rate

CDO – Collateralized Debt Obligation

CDS – Credit Default Swap

CED – Conditional Expected Drawdown

CEO – Chief Executive Officer

CET 1 – Common Equity Tier 1

CF. – Confer/Conferatur

CFA – Chartered Financial Analyst®

CFI – Corporate Finance Institute®

CFO – Chief Finance Officer

CI – Carried Interest

CID – Conditionally Independent Default

CIO – Chief Investment Officer

CIR – Cox-Ingersoll-Ross (Model)

CIRR – Common interest Rate Risk

CLGD – Conditional Loss Given Default

CLN – Credit Linked Note

CLO – Collateralized Loan Obligation

CM – CreditMetrics®

CME – Chicago Mercantile Exchange®

CMO – Collateralized Mortgage Obligation

CNAV – Constant Net Asset Value

CoCo – Contingent Convertible Bond

COD – Certificate of Deposit

COREP – Common Reporting (Framework)

CO2 – Carbon Dioxide (Chemical Element C02)

COV – Covariance

COVID – Coronavirus Disease

CP – Conditional Probability, or: CP – CorePlus, or: CP – Commercial Paper

CPD – Conditional Probability of Default, or:

CPD – Cumulative Probability Distribution

CPI – Consumer Price Index

CPM – Credit Portfolio Model

CPPI – Constant Proportion Portfolio Insurance

CP(R)M – Credit Portfolio (Risk) Model

CPR – Credit Portfolio Risk

CPV – CreditPortfolioView ®

CR – Credit Risk

CR+ – CreditRisk+®

CRA – Credit Rating Agency

CRAN – Comprehensive R Archive Network®

CRCU – Credit Risk Control Unit

CRE – Commercial Real Estate

CRM – Credit Risk Management, or: CRM – Customer Relationship Management, or: CRM – Credit Risk Model

CRO – Chief Risk Officer

CRR – Capital Requirements Regulation

CRD – Capital Requirements Directive

CS – Credit Spread

CSA – Credit Standard Approach (German: KSA)

CSFB – Credit Suisse First Boston®

CSM – Credit Spread Model

CSPM – Credit Suisse Portfolio Management/Manager®

CVA – Credit Value Adjustment

CVAR, CVaR – Credit Value-at-Risk (=UL in Credit Risk) or:

CVaR – Conditional Value-at-Risk (=ES)

CZK – Koruna Česká

D – Default (Rating Grade)

DACH – Deutschland (D), Austria (AT), Switzerland (CH)

DAX – Deutscher Aktien-Index, DAX®

DBRS – Dominion Bond Rating Service®

DBS – Defined Benefit Scheme

DCF – Discounted Cash Flow

DCS – Defined Contribution Scheme

DD – Drawdown, or DD: Distance to Default

DF – Default Frequency

DFAST – Dodd-Frank Act Stress Test

DLT – Distributed Ledger Technology

DNN – Deep Neural Network

DPT – Default Point

DR – Default Rate

DSGE – Dynamic Stochastic General Equilibrium Model

DSGV – Deutscher Sparkassen- und Giroverband®, Association of German S & Ls

DVA – Debt Value Adjustment

EAD – Exposure at Default

EBA – European Banking Authority

EBIT – Earnings Before Interest and Taxes

EBIDTA – Earnings Before Interest and Taxes, Depreciation and Amortization

EC – European Commission

ECAI – External Credit Assessment Institution

ECB – European Central Bank

ECL – Expected Credit Loss

ECMU – European Capital Market Union

ECO-CIR – Economic Cox-Ingersoll-Ross (Model)

ECRA – External Credit Risk Assessment Approach, or:

ECRA – External Credit Rating Agency

EDF – Expected Default Frequency

EDGAR – Electronic Data Gathering, Analysis, and Retrieval

EDHEC - École Des Hautes Études Commerciales Du Nord

EEG – German: Erneuerbare Energien Gesetz

E.G. – Exempli Gratia

EIOPA – European Insurance and Occupational Pensions Authority

EL – Expected Loss

ELL – Expected Lifetime Loss (Expected Loss Over Lifetime)

ELU – Exponential Linear Unit

ELBE – Expected Loss Bet Estimate

EM – Euler-Maruyama or Milstein

EMA – Exponential Moving Average

EMEA – Europe and Middle or East Asia

EMH – Efficient Market Hypothesis

EMIR – European Markets Infrastructure Regulation

EONIA – Euro Overnight Index Average

ES – Expected Shortfall

ESG – Environmental, Social and Governance

ESMA – European Securities and Markets Authority

ESPEL – Environment Social Political Economic Law®

€STR (or ESTER) – Euro Short-Term Rate

ETC – Exchange-Traded Commodity

ETC. – Et Cetera

ETF – Exchange-Traded Fund

ETH – Ethereum®, or: ETH – Eidgenössische Technische Hochschule, in Zürich

ETN – Exchange-Traded Note

E.U. – European Union (as adjective use, for the political union itself: the EU)

EURIBOR – Euro Interbank Offered Rate

EV – Enterprise Value

EVE – Economic Value of Equity

EVT – Extreme Value Theory

EWMA – Exponentially Weighted Moving Average

EY – Ernst & Young ®

EZK – German: Eigenmittelzielkennziffer

FA – Factor Analysis

FCR – Foreign Currency Rating

FD – Frequency Distribution

FDIC – Federal Deposit Insurance Company®

FED – Federal Reserve Bank

FF – Plural of “folio” (as pp. of p.)

5G – Fifth Generation (Mobile Phone Technology)

FFM – French-Fama Model

FICO – Fair Isaac Cooperation®

FINREP – Financial Reporting (Framework)

FIRBA – Foundational Internal Ratings-Based Approach

FLTF – Failing or Likely to Fail

FOF – Fund of Funds

FOMC – Federal Reserve Open Market Committee

FOREX – Foreign Exchange

FPR – False-Positive-Rate

FR – Forward Rate

FRA – Forward Rate Agreement

FRTB – Fundamental Review of the Trading Book

FS – Fung Hsieh

FT – Financial Times®

FTP – Funds Transfer Pricing

FV – Future Value

FWD – Forward

FX – Foreign Exchange, or: FOREX

GAAP – Generally Accepted Accounting Principles

GARCH – Generalized Autoregressive Conditional Heteroscedasticity

GARP – Global Association for Risk Management Professionals®

GB – Great Britain

GBM – Geometric Brownian Motion

GBP – (Great) British Pound (Sterling)

GD – Gradient Descent

GDP – Gross Domestic Product

GDR – Global Depositary Receipt

GFC – Great Financial Crisis

GHOS – Group of Governors and Heads of Supervision

GL – Guideline

GOF(I) – Goodness of Fit (Index)

GP – General Partner

GPU – Graphics Processing Unit

G-SIB – Global Systemically Important Banks

GUI – Graphical User Interface

GRU – Graphics Regression Unit

HGB – Handelsgesetzbuch

HHI – Herfindahl-Hirschman Index

HJM – Heath-Jarrow-Morton

H0 – Null-Hypothesis

HPP – Homogeneous Poisson (Point) Process

HQLA – High-Quality Liquid Assets

HS – Historical Simulation

HV – High Volatility

I.A. – Inter Alia

IACPM – International Association of Credit Portfolio Managers®

IASB – International Accounting Standards Board

IBM – International Business Machines® Corporation

IBO – Investors Buyout

IBOR – Interbank Offered Rate

ICAAP – Internal Capital Adequacy Assessment Process

ICE – InterContinental Exchange®

ICLAAP – Internal Capital and Liquidity Adequacy Assessment Process

ICLMAAP – Internal Capital, Liquidity and Management Adequacy Assessment

Process

ICO – Initial Coin Offering

ICR – Issuer Credit Rating

ICRA – Investment Information and Credit Rating Agency® (of India)

ICRE – International Commercial Real Estate

ICS – Internal Control System

I.E. – Id Est

IFIP – International Federation For Information Processing

IFRS – International Financial Reporting Standards

I.I.D – Independently Identically Distributed

ILAAP – Internal Liquidity Adequacy Assessment Process

ILN – Insurance Linked Note

IMA – Internal Models Approach

IMAAP – Internal Management Adequacy Assessment Process

IMF – International Monetary Fund

IMM – Internal Models Method

IO – Initial Offering

IPO – Initial Public Offering, or: IPO – Input-Processing-Output

IR – Interest Rate

IRB(A) – Internal Ratings-Based Approach

IRB(A)-F/A or F/A-IRB(A) – Foundational /Advanced – Internal Ratings-Based Approach

IRD – Interest Rate Derivative

IRR – Internal Rate of Return, or: Interest Rate Risk

IRRBB – Interest Rate Risk in the Banking Book

ISIN – International Securities Identification Number

IT – Information Technology

JPM, J. P. Morgan® – John Pierpont Morgan

JST – Joint Supervisory Team

KAG – German: Kapitalanlagegesellschaft

KAGB – German: Kapitalanlagegesetzbuch

KG – German: Kommanditgesellschaft

KKT – Karush-Kuhn-Tucker

KMV – Kealhofer, McQuown, Vašíček – original founders of the company KMV® (bought by Moody's)

KNN – K-Nearest Neighbors

KPI – Key Performance Indicator

KRI – Key Risk Indicator

KRM – Key Risk Metric(s)

KSA – Kreditstandardansatz

KVG – German: Kapitalverwaltungsgesellschaft

KWG – German: Kreditwesengesetz

KYC – Know Your Customer

LAF – Linear Activation Function

LAR – Liquidity-at-Risk

LBO – Leveraged Buyout

LCR – Liquidity Coverage Ratio, or: LCR – Local Currency Rating

LE – Large Exposure

LEL – Lifetime Expected Loss

LGD – Loss Given Default

LGF – Loss Given Failure

LIBOR – London Interbank Offered Rate

LIS – Low Interest Survey

LLC – Limited Liability Company

LLP – Limited Liability Partnership

LM – Linear Model, or: LM – Lagrange Multiplier

LME – London Metals Exchange®

LMC – Large Margin Classifier

LMM – LIBOR Market Model

LN – Logarithmus Normalis (Logarithm to the Base of the Euler Number e)

LP – Limited Partner

LR – Logistic Regression, or (Rarely): Linear Regression, or: Leverage Ratio

LPM – Lower Partial Moments

LS – Least Squares, or: L/S – Long-/Short

LSE – London Stock Exchange®

LSI – Less Significant Institute(s)

LSTM – Long Short-Term Memory

LTA – Long-Term Average

LTD – Long-Term Debt

LTRO – Long-Term Refinancing Operation

LV – Low Volatility

LVAR – Liquidity Value-at-Risk

LVMH – Louis Vuitton Moet Hennessy®

LZB – Lower Zero Bound

M – Maturity

MA – Moving Average

M & A – Mergers and Acquisitions

MaRisk – Mindestanforderungen an das Risikomanagement von Banken, German guidance accompanying the CRD

MATLAB – Matrix Laboratories®

MBI – Management Buy-In

MBO – Management Buyout

MBS – Mortgage Backed Security

MC – Monte Carlo

MCMC – Markov Chain Monte Carlo

MDA – Maximum Distributable Amount, or: MDA – Multivariate Discriminatory Analysis

MDD – Maximum Drawdown (also MaxDD)

MICA – Markets in Crypto-Assets

MIP – Monthly Income Plan

ML – Machine Learning

MLE – Maximum Likelihood Estimator

MLP – Multi Layer Perceptron

MMF – Money Market Fund(s)

MMI – Money Market Instrument

MOC – Margin of Conservatism

MPR – Market Price Risk

MPT – Modern Portfolio Theory

MRB – Model Risk Buffer

MR – Market Risk

MREL – Minimum Requirement on Own Funds and Eligible Liabilities

MSCI – Morgan Stanley Capital International®

MSE – Mean Square Error

MTN – Medium-Term Note

MWRR – Money-Weighted Rate of Return

NA – No-Arbitrage, or: NA – Not Available

NAREIT – North American Real Estate Investment Trust®

NASDAQ – National Association of Securities Dealers Automated Quotations®

NAV – Net Asset Value

NCA – National Competent Authority(-ies)

NCWO – No Creditor Worse Off

NFT – Non-Fungible Token

NGAAP – National GAAP

NII – Net Interest Income

NMA – National Mining Association®

NMR – Nonmodelable Risks (British English: Non-Modellable Risks)

NMRF – Nonmodelable Risk Factors (British English: Non-Modellable Risk Factors)

NNA – National Numbering Agency

NPE – Non-Performing Exposure

NPL – Non-Performing Loan

NRSRO – Recognized Statistical Ratings Organization

NSFR – Net Stable Funding Ratio

NYSE – New York Stock Exchange®

NZU – Niedrigzinsumfrage (=LSI Stresstest in Germany)

OA – Outstanding Amount

OBO – Owners Buyout

OCR – Overall Capital Requirement, or: OCR – Office of Credit Ratings

OECD – Organisation for Economic Co-operation and Development

OGAW – German: Organismen für Gemeinsame Anlagen in Wertpapiere

OIS – Overnight Indexed Swap

OPR – Op(erational) Risk

OS – Operating System

O-SIB – Other Systemically Important Bank

OTC – Over-the-Counter

P.A. – Per Annum

P & L – Profit and Loss

PCA – Principal Component Analysis

PCRI – Private Credit Risk Insurance

PD – Probability of Default, or: PD – Private Debt

PDF – Probability Distribution Function

PE – Private Equity, or: PE – Price Earnings

PEPP – Pandemic Emergency Purchase Programme

PIIGS – Portugal, Ireland, Italy, Greece and Spain

P1 – Pillar 1, Pillar One

PIT – Point-in-Time

PL – See P & L

PLC – Public Limited Company

PLN – Polish Złoty

PM – Probability of Migration, or: Portfolio Management

PMPT – Post-Modern Portfolio Theory

PNL – Promissory Note Loan

POT – Peak Over Threshold

PP – Paginae (pages)

PPP – Public Private Partnership

PR – Public Relations

P2 – Pillar 2, Pillar Two

P2G – Pillar 2 Guidance

P2R – Pillar 2 Requirement

PPP – Public Private Partnership

P2P – Peer-to-Peer

PU – Partial Use

PV – Present Value

Q – Quarter

QML – Quantum Machine Learning

QIS – Quantitative Impact Study

QP – Quadratic Programming

QRRC – Qualified Retail Revolving Credit

R – Correlation, or: (Statistical) Programing Language and Toolkit “R”, or:

R – Recovery Rate, cf. RR

RA – Risk Appetite, or: RA – Requirement Analysis

RAF – Risk Appetite Framework

RAR – Risk-Adjusted Return

RAROC – Risk-Adjusted Return on Capital

RARORAC – Risk-Adjusted Return on Risk-Adjusted Capital

RBC – Risk-Bearing Capacity

RBF – Radial Base Function

RC(U) – Risk Control (Unit)

RCP – Risk Classification Procedure

REIT – Real Estate Investment Trust

RELU – Rectified Linear Unit

REPO – Repurchase Agreement

RF – Risk Factor, or: RF – Risk Framework

RFR – Risk-Free Rate

RICS – Royal Institution of Chartered Surveyors

RM – Risk Measure, or: RM – Risk Metrics, or: RM – Risk Model

RMBS – Residential Mortgage Backed Security

RMSE – Root Mean Square Error

RNIV – Risks Not in VaR

RNN – Recurrent Neural Network

ROC – Receiver Operating Characteristic, or: ROC – Return On Capital

ROE – Return on Equity

ROI – Return on Invest

RORAC – Return on Risk-Adjusted Capital

ROW – Rest of The World

RR – Recovery Rate

RSU – Rating Service Unit®

RT – Random Tree

RTF – German: Risikotragfähigkeit (=RBC)

RV – Random Variable

RW – Risk Weight

RWA – Risk-Weighted Assets

SA – Standardized Approach (e.g., for Credit Risk)

SAA – Strategic Asset Allocation

SABR LMM –Stochastic Alpha Beta Rho Libor Market Mode

S & L – Savings and Loans

S & P – Standard & Poor‘s ®

SAP – Systemanalyse Programmentwicklung®

SAR – Stand-Alone Rating

SAS – Statistical Analysis Systems, or: SAS – Software as a Service®

SB – Systemic Buffer

SBP – Supervisory Benchmarking Portfolio

SCC – Standard Cost of Credit

SCHUFA – Schutzgemeinschaft für Allgemeine Kreditsicherung®

SCL – Security Characteristic Line

SCP – Stand-Alone Credit Profile

SCRA – Standardized Credit Risk Assessment Approach

SD – Standard Deviation

SDR – Special Drawing Right

SE – Société Européen

SEC – Securities and Exchange Commission

SEM – Structured Equational Model

SGD – Stochastic Gradient Descent

SGM – Stochastic Gradient Method, or: SGM – Sub-Gradient Method

SI – Significant Institute(s)

SIB – Systemically Important Banks

SIC – Schwarz Information Criterion

SICAF –Société d'Investissement à Capital Fixe

SICAV – Société d'Investissement à Capital Variable

SIRR – Standard Interest Rate Risk

S&P – Standard & Poor’s ®

SLP – Single Layer Perceptron

SMA – Standardized Measurement Approach

SME – Small and Medium (Sized) Enterprises

SMO – Sequential Minimal Optimization

SNA – System of National Accounts

SND – Standard Normal Distribution

SOFR – Secured Financing Offered Rate

SONIA – Sterling Overnight Index Average

SPC – Special Purpose Company

SPV – Special Purpose Vehicle

SQL – Structured Query Language

SRB – Single Resolution Board

SREP – Supervisory Review and Evaluation Process

SRMR – Single Resolution Mechanism Regulation

SRM – Single Resolution Mechanism

SRT – Securitization for Risk Mitigation

SSA – Supranationals, Sub-Sovereigns and Agencies

SSD – Schuldscheindarlehen, engl.: PNL – Promissory Note Loan

SSD-IO – SSD Initial Offering

SSM – Single Supervisory Mechanism

ST – Stress Test

S.T. – Subject to

STA – Standardized Approach for Credit Risk

STD – Short-Term Debt

SVaR – Stressed Value-at-Risk

SVD – Singular Value Decomposition

SVM – Support Vector Machine

SVR – Support Vector (Machine) Regression

SWOT – Strength and Weaknesses, Opportunities and Threats

T 1 – Tier 1 (capital)

T 2 – Tier 2 (capital)

TAA – Tactical Asset Allocation

TB – Trading Book

TBTF – Too Big to Fail

TC – Total Capital

TCE – Tail Conditional Expectation

TCO –Total Cost of Ownership

TER – Total Expense Ratio

TFP – Total Factor Productivity

TITF – Too Interconnected to Fail

TLAC – Total Loss Absorbing Capacity

TPR – True-Positive-Rate

TREA – Total Risk Exposure Amount (=RWA. TREA Term is Mainly Used by the EBA)

TRIM – Targeted Review of Internal Models

TRS – Total Return Swap

TSCR – Total SREP Capital Requirement

TT – Turing Test

TTC – Through-the-Cycle

TTM – Time to Maturity

TVAR, TVaR – Tail Value-at-Risk (=ES)

TMWRR – Time- and Money-Weighted Rate of Return

TWRR – Time-Weighted Rate of Return

UCITS – Undertakings for Collective Investments in Transferable Securities

UCL – Undrawn Credit Line

UDA – Univariate Discriminatory Analysis

UDR – Unexpected Default Rate

UL – Unexpected Loss

UPM – Upper Partial Moments

U.S. – United States (as adjective use, for the country itself: the US)

U.S.A. – United States of America (as adjective use, for the country itself: the USA)

UTP – Unlikeliness to Pay

VA – Value Added

VAR, VaR –Value-at-Risk, or: VAR – Vector Autoregressive

VARIMA – Vector Autoregressive Integrated Moving Average

VC – Vapnik-Chervonenkis, or: VC – Venture Capital

VCM – Variance-Covariance Model

VDAX – Volatilitäts DAX®

VEC(M) – Vector Error Corrected (Model)

VIX – Volatility Index

VK – Vašíček Kealhofer, See KMV®

VNAV – Variable NAV

WDCC – Write-Down and Capital Conversion

WEF – World Economic Forum

W.L.O.G – Without Loss of Generality

WTI – West Texas Intermediate

XVA – Otherwise/X Value Adjusted

YTD – Year-to-Date

ZPP – Zero-Price Probability

ANNEX A: R CODE FOR THE COMPARISON LINEAR MODEL VS. SVM MODEL

```
library(e1071)#SVM working library

# Load the data from the csv file

dataDirectory <- "C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-
R/LinearerVergleich/"

#define rmse function which calculates the root of the mean square error (with vector
“error”)

rmse <- function(error)

{

  sqrt(mean(error^2))

}


#package to read in excel-file

library(readxl)

# the tidyverse standard package to perform the necessary data manipulation

# and visualization

library(tidyverse)


# package to compute the utilized

# cross - validation methods  (for cross-validation techniques cf. the source
geeksforgeeks)

library(caret)

library(xlsx)

#i as a counter variable for the loop

i=0

#setting up the  2*4 output matrices for the test set and training set results

resultsTest=c(0,0,0,0,0,0,0,0)

dim(resultsTest)=c(2,4)

resultsTrain=c(0,0,0,0,0,0,0,0)

dim(resultsTrain)=c(2,4)


dataset<-0

#writing the Excel result file

write.xlsx(dataset, "C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-
R/LinearerVergleich/TestVert.xlsx", sheetName = "Sheet1",

           col.names = TRUE, row.names = TRUE, append = FALSE)

#following code part used before, not needed in the final version
```

```
#library(openxlsx)

#workbk <-loadWorkbook("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-
R/LinearerVergleich/TestVert.xlsx")

#addWorksheet(workbk,"Sheet2")

#addWorksheet(workbk,"Sheet3")

#addWorksheet(workbk,"Sheet4")


#loop for reading in the four portfolios (0-3)

for (i in 0:3)

  {


  if (i == 0) #first portfolio

    {SVMTest <- read_excel("SVMTest.xlsx")}

  else #second to forth portfolio

  {


      SVMTest <-read_excel(paste(dataDirectory,"SVMTest",i+1, ".xlsx",sep=""))

      }


# reproducible random sampling

set.seed(123)


# creating partition with training data as 80% of the dataset  (20% testing data set)

random_sample <- createDataPartition(SVMTest$Wert,

                                     p = 0.8, list = FALSE)


# generating training dataset

# from the random_sample

training_dataset  <- SVMTest[random_sample, ]

training_dataset

# generating testing dataset

# from rows which are not

# included in random_sample

testing_dataset <- SVMTest[-random_sample, ]


#Creating the SVM model and Linear Model and assign them to the correspondingly named
variables

#Created with the training data, “trained”

modelsvm <- svm(Wert ~. , training_dataset )

modellin <- lm(Wert ~ ., training_dataset )
```

```
#predictedY with SVM model on testing set (directly)

predictedY <- predict(modelsvm, testing_dataset)

#fine tuning the SVM model on the training data by using a radial base function as
kernel, an #epsilon increasing by standard 0.1 steps from zero to one and a cost
parameter(from 4 to 1024)

tuneResult <- tune(svm, Wert ~ .,  data = training_dataset, kernel="radial",

                   ranges = list(epsilon = seq(0,1,0.1), cost = 2^2:10)

)

#the result of the tuning, i.e., the best model is assigned to the tunedModel variable

tunedModel <- tuneResult$best.model

#output ("print") of the tuning

tuneResult

#the residuals of the tuned model are assigned to the error variable

error <- tunedModel$residuals

#counter i and variables for the Root Mean Square Error of the models' predictions on
the #training set (where RMSE2 refers to the linear model RMSE to SVM)

i

predictionRMSE2 <-rmse(modellin$residuals)

predictionRMSE2

predictionRMSE <- rmse(error)

predictionRMSE

#results of RMSE on training set is assigned to the results variables

resultsTrain[1,i+1]=predictionRMSE2

resultsTrain[2,i+1]=predictionRMSE


#test dataset out-of-sample with the testing data for the linear model

#modelValY

modelValY <-predict(modellin, testing_dataset)

rmse(modelValY-testing_dataset$Wert)


#and for the SVM model in the same way

#modelValY2

modelValY2 <-predict(tunedModel, testing_dataset)

rmse(modelValY2-testing_dataset$Wert)

#Kruskal-Wallis Test between output of LM and SVM, hence modelValY and modelValY2

Kruskal.test(modelValY, modelValY2)

#safe the results of the RMSE on the testing data set in the result variables

resultsTest[1,i+1]=rmse(modelValY-testing_dataset$Wert)

resultsTest[2,i+1]=rmse(modelValY2-testing_dataset$Wert)
```

```
#create a combined dataset

dataset <-data.frame(testing_dataset$Wert,modelValY2)


#load Excel workbook for comparison

wb <- loadWorkbook("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/LinearerVergleich/TestVert.xlsx")

sheets <- getSheets(wb)

sheet <- sheets[[1]]  # or another

# data to put into A1,C1,E1,G1

s=1+2*i

# modify contents

addDataFrame(dataset, sheet, col.names = FALSE, row.names = FALSE,

             startRow = 1, startColumn = s)

# save to disk

saveWorkbook(wb,"C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/LinearerVergleich/TestVert.xlsx")



}

#execute a Kuskal-Wallis test for all portfolios/values of real output compared with SVM

kruskal.test(W2~W1,read_excel("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/LinearerVergleich/TestVert.xlsx"))

kruskal.test(W4~W3,read_excel("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/LinearerVergleich/TestVert.xlsx"))

kruskal.test(W6~W5,read_excel("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/LinearerVergleich/TestVert.xlsx"))

kruskal.test(W8~W7,read_excel("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/LinearerVergleich/TestVert.xlsx"))


#print the RMSEs of Linear vs. the SVM model on the training and testing set

resultsTest

resultsTrain
```

ANNEX B: R CODE FOR THE COMPARISON CREDITRISK+® (CREDITMETRICS® IN EXCEL ATTACHMENT) VS. SVM MODEL

```
library(e1071)#SVM working library

# Load the data from the csv file

dataDirectory <- "C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-
R/VergleichCreditMetrics/"

#RMSE function to calculate the Root Mean Square Error of an error vector

rmse <- function(error)

{

  sqrt(mean(error^2))

}


#package to read in excel-file

library(readxl)

# tidyverse is a package to perform the necessary data manipulation

# and also visualization

library(tidyverse)


# package to compute the

# cross - validation methods

library(caret)

i=0

#setting up the  2*4 output matrices for the test set and training set results (filled
by zeros)

resultsTest=c(rep(0,8))

dim(resultsTest)=c(2,4)

resultsTrain=c(rep(0,8))

dim(resultsTrain)=c(2,4)


#again going through all four portfolios by a for-loop


for (i in 0:3)

{


  if (i == 0)
```

```
  { #read in the test for SVM and VaR calculation (comparison) for portfolio 1
    VaRTest <- read_excel(paste(dataDirectory,"VaRTest.xlsx",sep=""))
    SVMTest <- read_excel(paste(dataDirectory,"SVMTestV.xlsx",sep=""))
    }
  else #read in the test for SVM and VaR calculation (comparison) for portfolio 2-4
  {
    VaRTest <-read_excel(paste(dataDirectory, "VaRTest", i+1, ".xlsx",sep=""))
    SVMTest <-read_excel(paste(dataDirectory,"SVMTest",i+1, "V.xlsx",sep=""))
  }


  #Create an SVM model on the specified portfolio selection (SVMTest is training data!)
  modelsvm <- svm(W6 ~. , SVMTest)

  #predictedY
  predictedY <- predict(modelsvm, SVMTest)
  #Tune the SVM model with a radial base function used (RBF), with epsilon in 0.1 steps
from zero
  #to one and cost parameter ranging from 4 to 1024
  tuneResult <- tune(svm, W6 ~ .,  data = SVMTest, kernel="radial",
                     ranges = list(epsilon = seq(0,1,0.1), cost = 2^2:10)
  )
  #safe the best tuned model in the tunedModel variable
  tunedModel <- tuneResult$best.model
  #print the result of the tuning
  tuneResult
  #assign the residuals of the tuned model to the error variable
  error <- tunedModel$residuals
  #counter i
  i
  #calculate the RMSE of the error vector and print it
  predictionRMSE <- rmse(error)
  predictionRMSE
 #this result is also stored in the results variable of the training set
 resultsTrain[1,i+1]=predictionRMSE


  #test dataset out-of-sample (VaRTest) and prediction on it
  modelValY2 <-predict(tunedModel, VaRTest)
  modelValY2
```

```
  #rmse(modelValY2-Test$W6) #for internal control purposes, not used

  #modelValY2


  #store the result of the SVM on the testing data set in the variable resultsTest

  resultsTest[,i+1]=modelValY2


}


#include library tidyverse again

library(tidyverse)


#install packages for credit risk (CreditRisk+®)

install.packages("GCPM")

library(GCPM)

help(GCPM)


sec.var <- c(0.02, 0.02, 0.02) #sector variances calibrated as in CreditMetrics®

names(sec.var) <- c("A","B","C") #"USBANKS", "USTECH", "GBANKS"

#setting the model type (CreditRisk+®) loss unit, etc.

Testmodell <- init(model.type = "CRP", loss.unit = 1, alpha.max = 0.9999,

                    sec.var = sec.var)


#read in the data for the equivalent portfolio

portfolioX=data.frame(read_excel("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/VergleichCreditMetrics/PortfolioCRP1.xlsx"))

#print it

portfolioX

#calculate the 95 % CI VaR

CRP.classic <- analyze(Testmodell, portfolioX, 0.95)

#repeat that for portfolio 2

portfolioX=data.frame(read_excel("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/VergleichCreditMetrics/PortfolioCRP2.xlsx"))

portfolioX

CRP.classic <- analyze(Testmodell, portfolioX, 0.95)


#repeat it for portfolio 3

portfolioX=data.frame(read_excel("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/VergleichCreditMetrics/PortfolioCRP3.xlsx"))

portfolioX
```

```
CRP.classic <- analyze(Testmodell, portfolioX, 0.95)


#repeat it for portfolio 4

portfolioX=data.frame(read_excel("C:/Users/Raphael Reinwald/Documents/SVM-Test/SVM-in-R/VergleichCreditMetrics/PortfolioCRP4.xlsx"))

portfolioX

CRP.classic <- analyze(Testmodell, portfolioX, 0.95)


#alpha <- c(0.95, 0.99) #levels for tail measures, not used here

#VaR(CRP.classic)

#print the tuned model and the results on the training and test sets (comp. with the CR+ above)

tuneResult

resultsTrain

resultsTest
```

ANNEX C: STOCHASTIC FOUNDATIONS

Filtration of a probability space (cf. Björk, 2009, pp. 43, 462, 491): Let (Ω, A, P) be a probability space and let I be an index set[235] (totally ordered by $\leq$, i.e., for each two elements x, y in I one can clearly say if $x \leq y$ is true (or false) and $y \leq x$ is true (or false)) and F_i a sub-σ-algebra of A for all $i \in I$ (i.e., a subset of A and σ-algebra in itself). If $F_i \subset F_j \subset A$ für alle $i, j \in I$ then one says $(F_i)_i := \mathscr{F}$ is a filtration and (Ω, A, $\mathscr{F}$, P) a filtered probability space (Björk, 2009, p. 491).

In finance filtrations with σ-algebras are commonly used for the amount of information (or prices e.g.) available at a time t, cf. (Björk, 2009, pp. 487, 489). Utilizing filtrations, over the time the information increases and old information does not get lost. The thesis makes use of filtrations in connection with sets of random variables. These as well as further stochastic terms are defined in the following paragraphs.

Random variable (Björk, 2009, p. 484): Let (Ω, A, P) be a probability space with σ-Algebra A and probability measure P. $M_{\mathscr{B}}(\Omega, \mathbb{R}^k)$ is defined as the space of $\mathbb{R}^k$-valuable (k-dimensional vector in the field of real numbers $\mathbb{R}$) random variables

$$X: \Omega \rightarrow \mathbb{R}^k, \omega \mapsto X(\omega)$$

A-measurable mapping (Björk, 2009, pp. 461-463): Hence, referring to A and the Borel σ-algebra, measurable mappings are also called A-measurable mappings. Vice versa a random variable is defined as the above mapping which is A-measurable.

Stochastic independence cf. (Bingham & Kiesel, 2004; Björk, 2009, p. 492): Two random variables X_1, X_2 are called (stochastically) independent if

$$P(\{\omega \in \Omega: X_1(\omega) \in B_1, X_2(\omega) \in B_2\}) = P(\{\omega \in \Omega: X_1(\omega) \in B_1\})\, P(\{\omega \in \Omega: X_2(\omega) \in B_2\})$$

for all $B_1 \in A_1$, $B_2 \in A_2$, where A_1, A_2 are the corresponding σ-algebras.

[235] An index set is one whose members index (“label”, ”name”) the members of another set

Random/Stochastic Process (Bingham & Kiesel, 2004; Björk, 2009, p. 486): A stochastic (or random) process is hence a collection of random variables indexed by a (possibly infinite or even continuous) set of numbers, normally representing the time-axis, e.g., $\{A_t, t$ in $[0,1], A_t$ a random variable for all $t\}$.[236]

State space (Bielecki & Rutkowski, 2004): Let (Ω, A, P) be a probability space with σ-Algebra A and probability measure P. Further let I be an index set (ordered by $\leq$), $\mathscr{F}$ filtration of A, ($\mathbb{R}^k$, $\mathscr{B}$), $k \in \mathbb{N}$, a measurable space called "state space" and $X: I \times \Omega \rightarrow \mathbb{R}^k, (t, \omega) \mapsto X(t, \omega)$ a stochastic process.

Adapted process (Björk, 2009, p. 43): The process is called an adapted process (or non-anticipative) process (to the filtration $\mathscr{F}$) if the random variable $X_t: \Omega \rightarrow \mathbb{R}^k$ is (F_t, $\mathscr{B}$)-measurable for each $t \in I$. Hence, for every realization and every t, X_t is known exactly at time t – it is not possible to anticipate (look into) the "future".

Martingale Process (Björk, 2009; p. 504): A (discrete) time stochastic process Y_1, Y_2.... is said to be a (discrete) martingale with respect to the stochastic process X_1, X_2... (possibly $X = Y$) if for all t

1. $\mathrm{E}[|Y_t|] < \infty$ (i.e., finite)
2. $\mathrm{E}[Y_{t+1}|X_1, X_2 \dots X_t] = Y_t$

A probability measure Q is called a martingale measure (risk-neutral probability measure, equilibrium measure) if the stochastic (price) process of any tradable security/claim (which for now pays no coupons or dividends) in the state space becomes an F-martingale under Q, when discounted by the savings account (also risk-free account or numeraire) B, given by

$$B_t = e^{\int_0^r r_u du}$$

as in (Bielecki & Rutkowski, 2004; Björk, 2009).

[236] Sometimes random function is used to coin a random process as a stochastic process is also interpretable as a (random) element in a function space.

Risk measures: Coherent risk measure (cf. Embrechts & Wang, 2015, p. 2; Hull, 2015, p. 299): A risk measure ρ is called coherent if it upholds the following properties:

- $\rho(X + \alpha) = \rho(X) + \alpha$, for all $X \in M(\Omega, \mathbb{R})$ and all α (translation invariance)
- $\rho(\alpha X) = \alpha\rho(X)$, for all $X \in M(\Omega, \mathbb{R})$ and all positive α (positive homogenous)
- $X_1 \geq X_2$ almost everywhere $\Rightarrow \rho(X_1) \geq \rho(X_2)$ for all X_1, $X_2 \in M(\Omega, \mathbb{R})$ (monotone)
- $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$ for all X_1, $X_2 \in M(\Omega, \mathbb{R})$ (sub-additivity)

The VaR measure upholds the first three properties but not sub-additivity.

Convexity (Embrechts & Wang, 2015; Hull, 2015, pp. 299f., 702; McNeil et al., 2015): Another property is called convexity. A risk measure is convex if for all $\alpha \in [0,1]$ and for all $X_1, X_2 \in M(\Omega, \mathbb{R})$ the following inequality upholds:

$$\rho(\alpha X_1 + (1-\alpha)X_2) \leq \alpha\,\rho(X_1) + (1-\alpha)\,\rho(X_2)$$

The spectral measure (Raskin, 2006, p. 3): Let (Ω, A, μ) denote a probability space with σ-Algebra A and probability measure μ. Then an integrable function $\varphi: A \rightarrow \mathbb{R}$ is called weighting function, if φ fulfills:

- $\varphi(\alpha) \geq 0$ for almost all $\alpha \in A$,
- $\int_A \varphi(\alpha) d\mu(\alpha) = 1$

So given a weighting function $\varphi \in L^1([0,1])$[237]. Then the risk measure

$$M\varphi\,(X) = \int_0^1 VaR_p(X)\,\varphi(p)\,dp$$

is the spectral (risk) measure of φ – the term referring to the spectrum of an operator and its functional analysis background. It generalizes the VaR measure (Adam

[237] $L^1([0,1])$ denotes the class of all (Lebesgue-)measurable functions in the real interval [0,1].

et al., 2008; Raskin, 2006). Similarly, a stressed VaR (SVaR) can be extended to a spectral type measure in the analogous integral formula.