Autoreferat

Dr. Ana Belén Sainz

Summary of Professional Accomplishments

February 12, 2021

Contents

1 NAME AND SURNAME

Ana Belén Sainz.

2 DIPLOMAS, DEGREES

- **PhD in Physics** – February, 2014.
  Thesis: "Characterizing and witnessing multipartite correlations: from nonlocality to contextuality".
  Advisor: Prof. Antonio Acín
  Class: 1st, Award: Cum Laude
  Quantum Information Theory group
  ICFO–Institut de Ciències Fotòniques, Castelldefels (Barcelona), Spain.

- **MSc in Photonics** – July, 2010.
  Thesis: "Entanglement and non-locality of pure quantum states".
  Advisor: Prof. Antonio Acín
  Quantum Information Theory group
  UPC, UB, UAB and ICFO, Barcelona, Spain.

- **MSci in Physics** – March, 2009.
  Thesis: "Study of quasi-equilibrium states and spin quantum dynamics, in liquid crystals, by NMR".
  Advisor: Prof. Ricardo Zamar.
  Grade Average: 9.85 (over 10)
  Facultad de Matemática, Astronomía y Física - Universidad Nacional de Córdoba, Córdoba, Argentina.

3 INFORMATION ON PREVIOUS EMPLOYMENT IN SCIENTIFIC INSTITUTIONS

- **Group leader, Adiunkt** – June, 2019 - present.
  Foundational Underpinnings of Quantum Technologies group
  International Centre for Theory of Quantum Technologies, University of Gdańsk, Poland.

- **Postdoctoral Fellow – Part-time visitor** – June, 2019 - August, 2019.
  Quantum Foundations
  Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada.

- **Postdoctoral Fellow** – September, 2016 - May, 2019.
  Quantum Foundations
  Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada.

- **Postdoctoral Research Assistant** – July, 2014 - August, 2016.
  Quantum Information Theory group
  Group Leader: Prof. Sandu Popescu
  HH Wills Physics Laboratory, Tyndall Avenue, University Of Bristol, Bristol, UK.

- **Postdoctoral Researcher** – March, 2014 - June, 2014.
  Quantum Information Theory group
  Group Leader: Dr. Antonio Acín
  ICFO–Institut de Ciències Fotòniques, Castelldefels (Barcelona), Spain.

4 DESCRIPTION OF THE ACHIEVEMENTS ACCORDING TO ART. 219 PARA 1 POINT 2 OF THE ACT

4.1 Title of the achievement

Single-themed series of publications, titled *Theoretical foundations of the possibilities and limitations of nonclassical phenomena for quantum information processing.*

4.2 List of Selected Publications

The list of publications related thematically:

1. *Bipartite post-quantum steering in generalised scenarios*
   Ana Belén Sainz, Matty J. Hoban, Paul Skrzypczyk, Leandro Aolita.
   Physical Review Letters 125, 050404 (2020) – total pages: 21

2. *Quantifying Bell: The Resource Theory of Nonclassicality of Common-Cause Boxes*
   Elie Wolfe, David Schmid, Ana Belén Sainz, Ravi Kunjwal, Robert W. Spekkens
   Quantum 4, 280 (2020) – total pages: 53

3. *Quantum violations in the Instrumental scenario and their relations to the Bell scenario*
   T. Van Himbeeck, J. Bohr Brask, S. Pironio, R. Ramanathan, A. B. Sainz, E. Wolfe
   Quantum 3, 186 (2019) – total pages: 12

4. *Almost Quantum Correlations are Inconsistent with Specker's Principle*
   T. Gonda, R. Kunjwal, D. Schmid, E. Wolfe, A. B. Sainz
   Quantum 2, 87 (2018) – total pages: 24

5. *A formalism for steering with local quantum measurements*
   A. B. Sainz, L. Aolita, M. Piani, M. J. Hoban, P. Skrzypczyk
   New Journal of Physics 20, 083040 (2018) – total pages: 27

6. *Almost quantum correlations violate the no-restriction hypothesis*
   Ana Belén Sainz, Yelena Guryanova, Antonio Acín, Miguel Navascués
   Physical Review Letters 120, 200402 (2018) – total pages: 7

7. *A channel-based framework for steering, non-locality and beyond*
   Matty J. Hoban and Ana Belén Sainz
   New Journal of Physics 20, 053048 (2018) – total pages: 55

8. *Almost quantum correlations and their refinements in a tripartite Bell scenario*
   James Vallins, Ana Belén Sainz and Yeong-Cherng Liang
   Physical Review A 95, 022111 (2017) – total pages: 10

9. *Adjusting inequalities for detection-loophole-free steering experiments*
   Ana Belén Sainz, Yelena Guryanova, Will McCutcheon, Paul Skrzypczyk
   Physical Review A 94, 032122 (2016) – total pages: 12

10. *Classical communication cost of quantum steering*
Ana Belén Sainz, Leandro Aolita, Nicolas Brunner, Rodrigo Gallego, Paul Skrzypczyk
Physical Review A 94, 012308 (2016) – total pages: 7

11. *Postquantum steering*
Ana Belén Sainz, Nicolas Brunner, Daniel Cavalcanti, Paul Skrzypczyk, Tamás Vértesi
Physical Review Letters 115, 190403 (2015) – total pages: 9

12. *Macroscopic non-contextuality as a principle for Almost Quantum Correlations*
Joe Henson and Ana Belén Sainz
Physical Review A 91, 042114 (2015) – total pages: 21

5 PRESENTATION OF SIGNIFICANT SCIENTIFIC ACTIVITY

The scientific achievement is a part of collective publications. My contribution is described in point I2 of the annex "List of scientific or artistic achievements which present a major contribution to the development of a specific discipline". The contribution statements provided by the coauthors are presented in the attached documents.

Hereon, the publications that belong to the series will be cited with roman letters, e.g. [A], publications by the applicant that do not correspond to the series will be cited with numbers, e.g. [1], and other references will be cited following author-year convention, e.g. [Bel64,KS67].

5.1 Introduction

The 20th century witnessed a revolution in physics where we had to let go of our classical intuitions to explain the world. Quantum theory played a key role in this, by providing bizarre yet remarkably accurate explanations for our observational data. For instance, Nonlocality [Bel64], Contextuality [KS67] and Steering [Sch36, WJD07] are manifestations of powerful correlations which arise naturally within quantum mechanics, but, that cannot be explained in the classical world. Beyond their foundational implications on the underpinnings of physics, these nonclassical phenomena prove to be a resource for information processing. This realisation triggered a change in scientific activity, where the focus shifted from solely trying to understand the peculiarities of quantum theory, to also exploiting them for our benefit.

There are currently two fundamental properties of quantum theory that we know how to exploit. Firstly, entanglement: quantum systems can be highly correlated, and may remain so after being separated. Secondly, measurement incompatibility: there are properties of systems whose values we cannot know simultaneously with certainty. As a consequence of these properties, quantum theory displays nonclassical features that are harnessed as a resource for information-theoretic tasks. For instance, entanglement allows for Bell nonlocality [Bel64] and steering [Sch36, WJD07], which serve as resources for quantum cryptography [BCP+14b, CS16]. In turn, measurement incompatibility makes quantum theory display contextuality [KS67], which plays a crucial role in quantum computing [HWVE14].

Bell experiments can be recast in the language of causal inference, which sets them up as a particular case among a family of more general causal structures [WS15]. The broader problem of causal inference (of which certification of nonclassical Bell correlations is only a special case) is to determine if a given observed statistical data is compatible with some causal structure. In other words, it asks which hypotheses about the causal mechanism can explain the given statistics. Nowadays, partial solutions to this problem are used in the fields of medicine, bio-informatics, image processing, and experiments on quantum foundations. The incompatibility of quantum statistics with classical causal structures is a well established fact for Bell experiments [Bel64], however very little is known about general scenarios [HLP14]. There is still out there a diverse plethora of lessons that we must learn about the very foundations of Nature and how to harness its nonclassical power.

5.2 Motivation and Scientific goals

It is already extraordinary that Nature is Bell-nonlocal and contextual. Quantum theory, however, does not fully exploit these phenomena – there are limitations on how nonclassical Quantum theory can be, and therefore on how powerful its information processing capabilities may be. For instance, one can write down examples of statistics on measurement outcomes obtained by parties in distant laboratories, with the following properties: these correlations do not allow the parties to communicate faster than the speed of light, but cannot be generated experimentally with quantum systems [PR94, Cir80, Tsi93]. That is, these correlations are logically consistent with the operational constraints of the thought-experiment, yet quantum mechanics cannot realise them. This observation is not just a mathematical curiosity, but also has a practical implication: if one was able to produce such 'not quantumly allowed' (i.e., *post-quantum*) correlations in the lab, our power to process information would be much stronger – e.g., communication complexity problems would be rendered trivial [VD13]. This fact provides one motivation to explore the possible correlations one could observe beyond quantum theory and try to understand why Nature would forbid them – this line of research is usually referred to as "the characterisation of quantum correlations", or colloquially as "understanding quantum correlations from the outside". One of the goals in this research line is therefore to develop an intuitive understanding of these quantum correlations, tailored at their role in information processing.

In terms of information processing applications, for instance, quantum correlations in Bell scenarios have proven to power quantum cryptographic protocols [BCP+14b], some of which are already being commercialised – the company ID Quantique is indeed already providing quantum cryptographic equipment with which SK Telecom's 5G smartphones are equipped. Contextuality, on the other hand, has been linked to providing quantum advantage in computation [HWVE14]. Developing an intuition behind the strength of quantum correlations would help us to understand how these and other information theoretical applications are enabled by quantum theory.

Another motivation to engage into such research line come from the question of whether Quantum theory is the final correct theory to describe the physics of the world. Indeed, quantum theory is currently our most accurate description of Nature, at least in the microscopic scale, but whether quantum theory is the ultimate theory of Nature remains unclear. One of the main reasons for this is the tension it displays with the theory of General Relativity – both theories differ in crucial fundamental aspects, such as the role of "time". Traditional approaches to resolving this tension, such as String Theory, have preserved the structure of quantum theory whilst modifying General Relativity [Pol98]. However, recent insight has suggested the need to radically modify both theories [Har07]. Hence, part of the scientific community believes there is value in exploring post-quantum correlations beyond the goal of developing the physical intuition behind quantum correlations – for such part of the community, post-quantum correlations could indeed be a potential candidate for the observable statistics in the real world.

The standard approach to characterising quantum correlations consists of postulating simple and intuitive physical principles that constrain the set of correlations that may arise in Bell-type and contextuality experiments. Examples of these are *No Signalling principle* [PR94], *non-trivial communication complexity* [BBL$^+$06, VD13], *information causality principle* [PPK$^+$09], *macroscopic locality* [NW10], *local orthogonality principle* [1], and *consistent exclusivity principle* [2]. This problem is far from being solved, since all the principles proposed so far (with the possible exception of Information Causality) are satisfied by some correlations called "almost-quantum" [NGHA15][2] which are not entirely allowed by quantum theory.

A Steering experiment has a similar setup to that of a Bell experiment, but moves beyond the minimalistic defining feature of the latter (who focuses merely on outcome statistics). A steering experiment interprets the setup as an experimenter who – by performing measurements on the shared system – remotely influences (i.e., 'steers') the state of the subsystem held by a distant party (Bob). Steering hence allows us to investigate nonclassical correlations under a different set of assumptions than those of Bell, since it also considers the information available on the particular quantum state of Bob's system. The conceptual conclusions one may draw from this setup, as well as the types of information processing that it enables, are therefore substantially different to those of Bell. For instance, consider a cryptographic scenario where one powerful party – such as a bank – aims to securely communicate with others who may have more limited technology – such as their clients. These situations are best conceptualised as steering scenarios (rather than Bell scenarios), and have more accessible standards for security certification [BCW$^+$12, BES$^+$12]. The power and scope of steering as a resource or information processing is still yet unknown, and half a decade ago it was still impossible to study quantum steering "from the outside" [Gis89, HJW93, Pus13]. In addition, a systematic approach to certifying steerablity of experimentally collected data was then not yet fully developed.

From a more general perspective, the ubiquitous Bell scenario is just a particular case of what is known as a generalised Bayesian network [HLP14]. Traditional Bayesian networks have been an active area of research for computer scientists and statisticians in the last few decades, pioneered in particular by Pearl [Pea09]. Such networks, which focus on classical experiments, are studied in a broad range of disciplines to address the problem of causal inference – e.g., in a medical trial, what relations should our statistical data display to allow us to conclude that a drug cures a disease? A generalised Baesian network extends the traditional framework by allowing the so-called latent (unobserved) variables to be systems of arbitrary nature (such as quantum systems), instead of restricting them to be classical random variables. Each such network then encodes the particular causal structure assumed for the experimental setup, which includes the hidden variables (i.e., the latent ones) which may also mediate causal influences. The recent work of Ref. [HLP14] classified a variety of causal structures, and identified a set which have the potential to display a gap between classically allowed and quantumly allowed statistics. However, it was left as an open question whether there exists a simpler scenario than that of Bell which indeed does display such a gap, and what the consequences of it would be.

Finally, there is the question of exploiting these nonclassical correlations (observed in generalised Bayesian networks, contextuality or steering scenarios) to power information processing protocols and other technologies. The progress made in answering this question is quite diverse, since some scenarios (such as Bell) have been more deeply studied than others. However, the study of any such correlations as a resource is underpinned by the recently formalised concept of a Resource Theory [CFS16]. A resource theory explores how resourceful some physical system is with regards to a particular task in mind – for example, in the context of thermodynamics, one can seek to understand resources of thermal nonequilibrium in terms of their ability to do useful work. The particular way in which a Resource Theory approaches this question is by studying what resources can be converted into which ones by processes that are considered "free" of cost. Resource theories are nowadays ubiquitous across fields, with frameworks being developed for the resources of, e.g., athermality [BHO$^+$13], asymmetry [GS08], entanglement [HHHH09], non-Gaussianity [BESP03], non-Markovianity [RHP14], knowledge [DRKR15], coherence [BCP14a, SL19], contextuality [GHH+14, VMGE14], and Bell nonlocality [DV14, GA17]. in order to develop a complete understanding and quantification of the utility of the nonclassical correlations that we have discussed so far, one then needs to formally take these resource theoretic considerations into account. For instance, only when a resource theory for Bell nonlocality is developed, can one then make complete statements regarding which correlations are more classical than others, and quantify such nonclassicality. This ordering of resources, and how to quantify their cost, could help us identify which are the actual nonclassical features of Nature that do provide the power for information processing and related tasks. Even though some steps have been taken towards the development of resource theories for the phenomena investigated in this Habilitation thesis, a general approach and fundamental understanding of quantum correlations as a resource is still an open question.

Our research presented in this Habilitation thesis aims at making crucial step towards addressing the open problems presented here. We aim at

- developing the theoretical tools necessary to study "quantum from the outside" in experiments beyond the traditional Bell and contextuality ones,
- making progress on understanding Steering as a resource, taking into account practical considerations,
- developing an intuition behind almost-quantum correlations, and principles that would challenge the plausability of their existence,
- exploring correlations in more general causal structures, and developing the theoretical tools to enable their study as a resource for information processing.

5.3 Summary

The first phenomenon that I explored was Steering, which by then had not been widely studied. This allowed me to explore various different aspects of it. First, I studied Quantum steering: we quantified quantum steering by how much classical information would Alice and Bob need to exchange to simulate a quantum assemblage with classical resources [A]. We found that certain cases would require an infinite amount of communication, unlike the similar situation for a Bell scenario (which needs at most 2 bits of communication). We also found closed expressions for the lower bounds for the communication cost, including the situation where the simulation may tolerate errors. The next aspect of quantum steering that I explored was how to certify it in non-idealised setups [B]. That is, we asked how to devise a method to tell that an assemblage is or is not steerable, when we take into account experimental errors and losses. The technique we developed takes into account the overall setup efficiencies η, and constructs steering inequalities which can certify steerable assemblages while closing the so-called detection loophole. Our method is general: it can admit a different efficiency for each measurement setup, and even handle correlated losses between Alice and Bob's setups. This allows us to explore a so-far not studied range of models for experimental losses, not only in steering experiments, but also in Bell tests.

In addition, I studied post-quantum steering: the possibility of steering beyond the realm of quantum theory was discovered in Ref. [C], which is an achievement of this Habilitation thesis. We found multi-partite [C, D, E] and generalised bipartite steering scenarios [F] where steering compatible with the No Signalling principle exists but that may not be produced with quantum resources. Since we were the first ones to discover post-quantum steering, the next step was to develop a mathematical formalism to explore it in a unified manner with quantum steering. Indeed, we developed one formalism by connecting steering with Hermitian operators [D], and another formalism by connecting steering with casual quantum channels [E].

These formalisms even allowed us take a step forward from steering, and define the concepts of post-quantum Buscemi nonlocality, and post-quantum nonclassical teleportation, which are two other nonclassical phenomena [E]. Importantly, we also showed that post-quantum steering is a genuinely new phenomenon and not merely another expression of post-quantum Bell or Instrumental nonclassicality. That is, we found post-quantum steering which can never give rise to post-quantum correlations in Bell or Instrumental scenarios [C, D, E, F]. Finally, we developed numerical techniques to test post-quantumness of steering [C, F].

The second topic I studied pertains to the statistical predictions that are observable in Bell tests and contextuality experiments. Motivated by the question of how to characterise correlations from simple principles, I focused on understanding the set of almost-quantum correlations [G, H] and how to distinguish it from the quantum one [I, J]. First, we found a physical principle that singles out precisely the set of almost-quantum correlations in Bell and contextuality scenearios, and called it Macroscopic Non-contextuality [G]. Second, we explored the structure of almost-quantum correlations, especially in multi-partite Bell tests [H]. We found that the relationship between the almost-quantum set and the set of correlations defined by the Navascués-Pironio-Acín hierarchy [NPA07, NPA08, PNA10] is more complex than what is hinted by their behavior in bipartite setups, hence insight to understand the former cannot be drawn solely from the latter. Then we moved on to try to identify a physical property that quantum correlations satisfy but almost-quantum do not. On the one hand, we found that any physical theory whose statistical predictions are given by almost-quantum correlations violates the No Restriction Hypothesis [I]. On the other hand, we showed that almost-quantum correlations violate the statistical implications of Specker's principle, and hence any almost-quantum physical theory would violate Specker's principle at the level of sharp measurements [J]. These results bring progress to a research program that was thought to have fundamentally stalled.

The final topic I studied moved on from the traditional phenomena explored in quantum foundations, into the realm of causal inference. First, we showed how the simplest interesting causal structure, called the Instrumental scenario, may display similar features to those in Bell nonclassicality, steering, and contextuality: the scenario may logically allow for observed statistics that are not compatible with classical theory, and also even some which are not compatible with quantum predictions [K]. The crucial insight here was to mathematically connect the bipartite Bell scenario with the Instrumental scenario, which allowed us to import into the latter all the knowledge and techniques from the well studied Bell scenarios. Ref. [K], hence, shows how the study of Bell nonclassicality may have an impact on the study of more general causal mechanisms. Ref. [L], on the other hand, may be posed as a manifestation of the converse: how the insight driven from taking a causal perspective may push forward the development of our understanding of Bell nonclassicality. In Ref. [L] we developed a resource theory for correlations in a Bell scenario. The crucial point here was to move on from the traditional approach to Bell's theorem (which confronts the notions of locality and realism), and instead take a causal perspective to it. This allowed us to justify the choice of Local Operations and Shared Randomness (LOSR) as the appropriate one for the set of free operations, and triggered the discovery of various properties of the order of resources in Bell scenarios. For example, checking resource interconvertibility can be done by a numerically efficient algorithm, and even in the simplest Bell scenario one would need at least eight distinct resource monotones to quantify nonclassicality – indeed, the monotones defined so far in the literature are far from forming a complete set. Crucially, we found that there are infinite sets of incomparable resources, even when restricted to quantum correlations. And, moreover, there is an infinite set of quantum correlations at the top of the order of quantum resources – that is, there is no analog of a maximally entangled state for quantum correlations. Finally, a key fundamental aspect in Ref. [L] is that our technique to define the resource theory readily applies to causal structures beyond Bell scenarios. We gave an example of this by setting up the resource theory for the triangle-with-settings causal structure. Our results hence have the potential to enable the systematic study of resources beyond Bell scenarios.

5.4 Einstein-Podolsky-Rosen steering in quantum theory

The concept of steering was first introduced by E. Schrödinger in 1935 [Sch36, Sch35] in response to the Einstein, Podolsky and Rosen paradox [EPR35]. It refers to the phenomenon where one party, Alice, by performing measurements on one part of a shared system, seemingly remotely 'steers' the state of the system held by a distant party, Bob, in a way which has no explanation in terms of classical causal influences. Steering has only recently been formally defined in a quantum information-theoretic setting [WJD07], as a way of certifying the entanglement of quantum systems without the need to trust one of the parties, or when one of the parties is using uncharacterised devices. In this setting, the uncharacterised party convinces the other party that they shared entanglement by demonstrating steering. Furthermore, if all parties are uncharacterised (or untrusted) then one recovers the device-independent setting of a standard Bell test. Steering thus may be seen as one in a family of nonclassical phenomena, closely related to entanglement and Bell non-locality [Bel64]. Indeed, Bell non-locality implies steering, and steering implies entanglement, however all three concepts are inequivalent [WJD07, QVC+15].

Formally, the simplest steering test is defined as follows: there are two parties, Alice and Bob, in distant laboratories, each of which playing a different role in the experiment (see Fig. 1). Alice (a.k.a. the 'steering' party) is thought of as having access to a "black box", and her actions are then restricted to choosing the classical input x to be put into the box, and recording the classical output a that is produced. In a quantum mechanical realisation of the experiment, x would denote a choice of measurement to be performed on her share of a quantum system, and a the obtained outcome, but in this black-box picture no information on the particular working of measurement device is relied on – only the values of (a, x) are considered. The situation at Bob's lab (who is known as the 'steered' party), in turn, is fully described by means of quantum mechanics: he has access to a system whose marginal state, given by ρ_R, corresponds to that of a quantum system. Each round in the experiment consists of Alice choosing an input x and obtaining an outcome a, with probability $p(a|x)$, and Bob obtaining the conditional marginal state $\rho_{a|x}$ into which his system has been steered. Let $\mathbb{X}$ denote the set of possible values for x, and $\mathbb{A}$ the set of possible outcomes[1] a that Alice can obtain. It is convenient to work with the unnormalised steered states $\sigma_{a|x} := p(a|x)\, \rho_{a|x}$, which contains information about both the probabilities of the steering party ($p(a|x) = \text{tr}\left\{\sigma_{a|x}\right\}$) and the conditional marginal states of the steered party ($\rho_{a|x} = \frac{\sigma_{a|x}}{\text{tr}\left\{\sigma_{a|x}\right\}}$). The collection of such unnormalised conditional states is referred to as *assemblage* [Pus13], and here we will denote is as $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}} := \left\{\sigma_{a|x}\right\}_{a\in\mathbb{A}\,,\,x\in\mathbb{X}}$.

Intuitively, if Alice and Bob share an entangled quantum state, one could expect Alice to be able to steer Bob into certain assemblages which could not be achievable if they were to only share a separable state – and indeed, this is the case. The assemblages that Alice can steer Bob into without the need of entanglement are usually referred to as *unsteerable* or *LHS* assemblages, where LHS stands for "local hidden state". An experiment is then said to have demonstrated steering if an assemblage that is not LHS is prepared.

In this Habilitation thesis we took initial steps towards understanding quantum assemblages as a resource, by showing how much classical communication Alice and Bob would need to exchange in order to simulate with shared classical resources those steerable quantum assemblages

[1] In principle, different measurements can have different outcome sets. However, one can take all those outcome sets to be the same, without loss of generality, and we will follow that approach in this work.

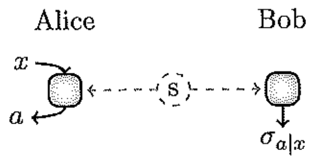


Figure 1: **Traditional bipartite steering scenario:** a source S distributes a system between Alice and Bob. Alice steers Bob by performing a measurement labelled by $x \in \mathbb{X}$ on her share of the system, and obtaining outcome $a \in \mathbb{A}$. Bob's share of the system is steered into the marginal conditional state $\sigma_{a|x}$ of a quantum system.

[A]. Moreover, we further explored steering as a resource in more realistic setups, by revisiting the question of 'is an assemblage steerable' – we developed tools to certify steering beyond the idealised setup, considering experimental errors and losses [B]. These two contributions are elaborated on below.

5.4.1 Communication cost of quantum steering

In a traditional bipartite steering scenario (see Fig. 1), an assemblage is deemed unsteerable if it admits a *local hidden state model.* In an LHS model the source S distributes a classical random variable λ to Alice and Bob, Alice produces outcome a with probability given by the response function $p(a|x\lambda)$, and Bob prepares a normalised quantum state ρ_λ. The assemblage elements that can be produced through such a model are:

$$\sigma_{a|x} = \int d\lambda\, p(\lambda)\, p(a|x\lambda)\, \rho_\lambda\,,$$

where $p(\lambda)$ is the probability distribution of the source over the possible values of λ.

It is known that some quantum assemblages (i.e., assemblages produced be measuring shared quantum states) do not admit an LHS model [WJD07]. Here we ask how these steerable assemblages could be simulated using classical communication between Alice and Bob instead of entanglement [A]. Let $\mathbf{m}$ be a classical message of t bits that Alice sends to Bob after choosing the input x. In this setup, hence, Bob is allowed to prepare a normalised quantum state $\rho_{\mathbf{m},\lambda}$ using the partial (or complete) knowledge on x that he learns through $\mathbf{m}$. The assemblage elements that Alice and Bob can simulate through this protocol are:

$$\sigma^{\text{sim}}_{a|x} = \sum_{\mathbf{m}} \int d\lambda\, p(\lambda)\, q(\mathbf{m}|x,\lambda)\, p(a|x\lambda)\, \rho_{\mathbf{m},\lambda}\,,$$

where $q(\mathbf{m}|x,\lambda)$ is the probability that Alice sends message $\mathbf{m}$ given that she's received variable λ and chosen input x.

First, we show that if Alice and Bob aim at simulating an arbitrary assemblage (in the limit $|\mathbb{X}| \to \infty$ for the number of classical inputs x) produced from a bipartite pure entangled state (even with local dimension 2), then the length t of the required message must be infinite [A, Result 1]. We also show that this behaviour prevails for some bipartite mixed entangled states [A, Result 2]. These results show that the communication cost of arbitrary steering is infinite, but more remarkably, they also show that steering scenarios display contrasting features compared to their counterpart in Bell tests: the communication costs of quantum steering and of quantum nonlocality are totally different. While the communication cost of steering is infinite for any pure entangled two-qubit state, few bits of communication of enough in the context of nonlocality. Specifically, the statistics of local projective measurements on a maximally entangled state can be reproduced with a single bit of communication [TB03], while two bits are enough for partially entangled states [TB03]. For higher dimensional states, it was shown that two bits of communication suffice to reproduce the correlations of dichotomic measurements on any bipartite entangled states [RT09]. Nevertheless, it is known that the statistics of general measurements on $d \times d$ maximally entangled states, require an amount of communication that increases (at least) as $O(d)$ [BCT99].

In this setup, we further explored the communication cost of approximately simulating an arbitrary assemblage produced from a bipartite pure entangled state of two qubits. We found that the the length t of the message is lower bounded by [A, Result 4]

$$t \geq t_{\text{bound}} = \log_2 \left(\frac{2}{1-\sqrt{1-4\epsilon^2}} \right) ,$$

where $\epsilon > 0$ is the tolerated error, i.e.:

$$\mathrm{d}(\rho^{\text{sim}}_{a|x}, \rho_{a|x}) \leq \epsilon , \quad \forall a, x \text{ , such that } p(a|x) \neq 0 ,$$

where $\mathrm{d}(\rho, \rho') = \frac{1}{2}||\rho - \rho'||$ is the trace distance, and $\rho^{\text{sim}}_{a|x}$ (resp. $\rho_{a|x}$) is the normalised state corresponding to $\sigma^{\text{sim}}_{a|x}$ (resp. $\sigma_{a|x}$). We see then that $\epsilon \to 0$ implies $t \to \infty$, which is consistent with Result 1 of [A].

Finally, for steering scenarios with an arbitrary (but finite) number of measurements $|\mathbb{X}|$, we provided a method for placing a lower bound on the communication cost for simulating an arbitrary assemblage. More precisely, we showed that if Alice and Bob want to simulate the assemblage $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}}$, then the length t of the message is lower bounded by [A, Result 3]

$$t \geq \log_2 [\nu(\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}}) + 1] ,$$

where $\nu(\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}})$ is a quantifier of steering termed the *LHS robustness* [PW15].

5.4.2 Certifying steerability in experimental setups: analytical methods

Steerable quantum assemblages are a resource for certain quantum information tasks [BCW+12, CS16]. Therefore, a natural question is how to certify that a certain assemblage is steerable. Similarly to the case of Bell nonlocality, the usual approach is to find an inequality that steerable assemblages may violate, while LHS assemblages cannot. If a given assemblage then violates one such inequality, this acts as a certificate that the assemblage is steerable. Formally, in the traditional bipartite scenario, a linear steering inequality is given by a set of operators $\{F_{ax}\}_{a\in\mathbb{A},\, x\in\mathbb{X}}$ through the linear functional:

$$\beta := \mathrm{tr} \left\{ \sum_{a\in\mathbb{A}, x\in\mathbb{X}} F_{ax}\, \sigma_{a|x} \right\} . \tag{1}$$

Denote by β^{LHS} the maximum value that β can take when evaluated over LHS assemblages – computing such a bound corresponds to solving one instance of a semidefinite program [CS16]. Then, the steering inequality defined by the functional of Eq. (1) is:

$$\mathrm{tr} \left\{ \sum_{a\in\mathbb{A}, x\in\mathbb{X}} F_{ax}\, \sigma_{a|x} \right\} \leq \beta^{\text{LHS}} . \tag{2}$$

An assemblage that yields $\beta > \beta^{\mathrm{LHS}}$ is hence steerable.

In any real experimental demonstration of steering, though, there will necessarily be experimental imperfections that mean that the discussed idealised treatment will not be strictly applicable. Here we develop a method to modify the steering inequalities such that they can faithfully certify steerable assemblage under certain experimental imperfections [B]. The imperfections that we address are the ones that give rise to the so-called *detection loophole*, such as particle losses and imperfect detectors. The detection loophole refers to the fact that if one makes the fair sampling assumption for Alice – that the conclusive events (where no particle is lost) constitute a faithful representative of the complete experimental data – and apply the above idealised treatment to it, then one may erroneously conclude that an assemblage is steerable, even though the underlying shared state was separable. To prevent this from happening, we developed a way to modify the steering inequalities, to take into account the detection efficiency[2] η_x of each measurement setting $x \in \mathbb{X}$. We denote $\boldsymbol{\eta} := \{\eta_x\}_{x\in\mathbb{X}}$.

The main idea is to consider the steering scenario (hereon denoted *a priori* scenario) where the measurements have $|\mathbb{A}| + 1$ outcomes each, and understand the initial steering experiment through the lens of this larger scenario [B]. This extra outcome (hereon, outcome 0) in the a priori scenario represents the situation where, due to the above mentioned experimental imperfections, no event was registered at Alice's device. An assemblage $\boldsymbol{\Sigma}_{\mathbb{A}|\mathbb{X}}$ in the steering experiment is then understood as a post-selection of an assemblage in the a priori scenario, where only the successful detection rounds are considered. Formally, let $\{\sigma^0_{a|x}\}$ be an assemblage in the a priori scenario. Its post-selection then corresponds to the assemblage $\boldsymbol{\Sigma}_{\mathbb{A}|\mathbb{X}}$ with elements:

$$\sigma_{a|x} = \frac{1}{\eta_x}\,\sigma^0_{a|x}\,.$$

A steering test safe of the above mentioned "false positives" is then one that tells that an assemblage $\boldsymbol{\Sigma}_{\mathbb{A}|\mathbb{X}}$ is steerable only when $\{\sigma^0_{a|x}\}$ is not LHS. Given a steering functional as in Eq. (1), we formally impose this condition by computing a new bound $\beta^{\mathrm{LHS}}(\boldsymbol{\eta})$ whose violation certifies steerability [B, Eq. (11)].

Our method for adapting the steering inequalities not only allows us to certify steerability free of the detection loophole, but also allows us to explore which steering inequalities and which range of detection efficiencies would permit the construction of a steering test – indeed, if the efficiencies are too low, or the inequality is not chosen optimally, then no value of the steering functional could ever certify that an assemblage is steerable.

We then explored a family of steering inequalities, in particular their robustness and usefulness to serve as steering certificates. We considered the case where the operators F_{ax} are proportional to rank-1 projectors. We showed that these inequalities are robust under losses, in the sense that they tolerate the lowest allowed efficiencies. More precisely, we showed that these inequalities can be violated by some steerable assemblage as long as $\langle\eta\rangle > \frac{1}{|\mathbb{X}|}$, where $\langle\eta\rangle$ is the average detection efficiency. This bound that we found is moreover tight, since it has been shown that steering can never be demonstrated when $\langle\eta\rangle \leq \frac{1}{|\mathbb{X}|}$ [SGDA+12, SNC14]. In addition, we found the requirements for the minimum detection efficiencies for steering certification when the assemblages are generated from pure quantum states subject to white noise, in the form of precise relations between the average detection efficiencies and the amount of white noise [B, Eq. (26)].

Then we showed how our method can be applied to multi-partite steering scenarios [HR13, CSA+15], and focused on tests that certify that an assemblage is not full-separable. We explicitly stated the new adjusted bound for a steering scenario with two untrusted parties [B,

[2]Here, "detection efficiency" refers to the overall probability that the particle reaches the detector and that the detector does click, for the particular experimental arrangement of the corresponding measurement setting.

Eq. (38)]. To give a particular example of our technique, we further discussed two steering functionals, and explored their efficiency range for certifying steerability when the underlying shared quantum states is one of the two archetypal tripartite entangled states – the GHZ and the W states –, and for isotropic and uncorrelated loss.

Finally, we also discussed how our technique can be applied to Bell scenarios, to modify the local bounds of Bell inequalities so they can certify nonlocality free of the detection loophole. We applied our method to a family of tilted CHSH inequalities [AMP12] and recovered two known results: Eberhard's result that a detection-loophole-free test is not possible whenever $\eta \leq \frac{2}{3}$ [Ebe93], and the asymmetric case (where Alice has perfect detectors) where a detection-loophole-free test is not possible whenever $\eta \leq \frac{1}{2}$ [Gar10]. Our method, however, allows us to go beyond the approach to detection efficiencies given in Ref. [Bra11], which assume that the probability of no-click events in Alice's and Bob's labs are uncorrelated. This is a very reasonable assumption, but does not need to hold in full generality. To give an example of the generality of our approach, we hence considered the case where the no-clicks in Alice lab are perfectly correlated with those in Bob's, i.e., if in one round one gets a no-click then the other party gets one as well. Here we found that the modified bound $\beta^{\text{local}}(\eta)$ always coincides with the ideal LHV bound, and therefore provides a detection-loophole-free test for any value of η. Our analysis hence shows that in fact such perfectly correlated loss is, from the viewpoint of a malicious adversary (or source), the worst possible type of loss they could use to try and open the detection loophole, since in this instance no loophole is in fact opened.

5.5 Einstein-Podolsky-Rosen steering: beyond quantum theory

The exploration of post-quantum Bell nonclassicality is ubiquitous in quantum foundations. However, little was known about steering beyond quantum setups. One of the reasons for this is the intrinsic quantum nature of Bob's subsystem: how can we generalise the steering scenario to arbitrary physical theories while keeping Bob's system quantum? Another obstacle to the study of post-quantum steering is a celebrated theorem by Gisin [Gis89] and Hughston, Josza and Wootters [HJW93] (GHJW), which states that the traditional steering scenario of Fig. 1 may never admit post-quantum steering (see below for a formal statement). In this Habilitation thesis, we showed how the concept of steering beyond quantum theory may be formalised [C, D, E, F], and showed that there exist situations where the GHJW theorem does not apply, i.e., post-quantum steering consistent with the No Signalling principle may exist [C, F].

Let me begin by presenting the concept of post-quantum steering. Consider an assemblage $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}}$ in a traditional bipartite steering scenario (see Fig. 1). The assemblage is said to satisfy the *No Signalling* principle if and only if

$$\sum_{a\in\mathbb{A}} \sigma_{a|x} = \sum_{a\in\mathbb{A}} \sigma_{a|x'}\,, \quad \forall\, x\,,\, x' \in \mathbb{X}\,, \tag{3}$$

$$\operatorname{tr}\left\{\sum_{a\in\mathbb{A}} \sigma_{a|x}\right\} = 1\,, \quad \forall x \in \mathbb{X}\,. \tag{4}$$

Assemblages that satisfy the No Signalling principle are referred to as *No Signalling assemblages*.

Moreover, an assemblage $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}}$ is said to have a *quantum realisation* if and only if there exists a Hilbert space $\mathcal{H}_A$ for Alice, a normalised quantum state $\rho \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$, and a (complete)

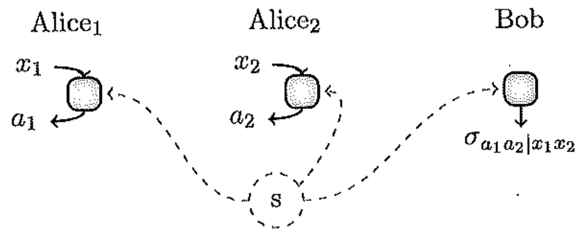


Figure 2: Multi-partite steering scenario: example where two steering parties (Alice$_1$ and Alice$_2$) steer the quantum state of Bob's system.

projective measurement[3] $\{\Pi_{a|x}\}_{a\in\mathbb{A}}$ for each $x \in \mathbb{X}$, such that

$$\sigma_{a|x} = \mathrm{tr}_A \left\{ \left(\Pi_{a|x} \otimes \mathbb{I}_{\mathcal{H}_B}\right) \rho \right\} \,, \quad \forall a \in \mathbb{A} \,, x \in \mathbb{X} \,. \tag{5}$$

The question then that we are interested in is the following:

Given an arbitrary No Signalling assemblage $\boldsymbol{\Sigma}_{\mathbb{A}|\mathbb{X}}$,
can we find a quantum realisation for it?

For the case of the traditional bipartite steering scenario discussed so far, the answer is yes. Namely, everything that is compatible with the No Signalling principle may be reproduced in an experiment with quantum states and measurements. This is a consequence of the GHJW theorem, and a review of it in contemporary notation may be found in Ref. [F, Appendix 1].

The lesson that we learn hence is that, for post-quantum steering to be theoretically possible, one needs to go beyond the traditional bipartite steering steering scenario. In this habilitation thesis, as presented below, we show that there exist generalised scenarios where some No Signalling assemblages do not admit a quantum realisation. These generalised scenarios correspond to considering a multi-partite steering setup with many steering parties [C, D, E], the bipartite Bob-with-Input scenario [F], and the Instrumental steering scenario [F].

5.5.1 Multi-partite steering scenarios

Here we consider a multi-partite steering scenario, with one steered party and many steering parties (see Fig. 2). For simplicity I present here the case of two steering parties (Alice$_1$ and Alice$_2$), since this is sufficient for post-quantum steering to arise. In this case, an assemblage is given by $\boldsymbol{\Sigma}_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2} = \left\{\sigma_{a_1 a_2|x_1 x_2}\right\}_{a_j\in\mathbb{A}_j,\, x_j\in\mathbb{X}_j,\, j\in\{1,2\}}$, where $\mathbb{X}_j$ is the set of measurement choices of Alice$_j$, and $\mathbb{A}_j$ the set of possible outcomes of the measurements in $\mathbb{X}_j$.

The first thing to note is that here the No Signalling principle imposes constraints beyond those of Eq. (3). Indeed, an assemblage $\boldsymbol{\Sigma}_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ is No Signalling iff

$$\sum_{a_1\in\mathbb{A}_1} \sigma_{a_1 a_2|x_1 x_2} = \sum_{a_1\in\mathbb{A}_1} \sigma_{a_1 a_2|x_1' x_2} \,, \quad \forall x_1 \,, x_1' \in \mathbb{X}_1 \,, a_2 \in \mathbb{A}_2 \,, x_2 \in \mathbb{X}_2 \,, \tag{6}$$

$$\sum_{a_2\in\mathbb{A}_2} \sigma_{a_1 a_2|x_1 x_2} = \sum_{a_2\in\mathbb{A}_2} \sigma_{a_1 a_2|x_1 x_2'} \,, \quad \forall x_2 \,, x_2' \in \mathbb{X}_2 \,, a_1 \in \mathbb{A}_1 \,, x_1 \in \mathbb{X}_1 \,, \tag{7}$$

$$\mathrm{tr}\left\{ \sum_{a_1\in\mathbb{A}_1 \,, a_2\in\mathbb{A}_2} \sigma_{a_1 a_2|x_1 x_2} \right\} = 1 \,, \quad x_1 \in \mathbb{X}_1 \,, x_2 \in \mathbb{X}_2 \,. \tag{8}$$

[3]Because of the Stinespring dilation theorem, since we do not have restrictions on the dimension of Alice's Hilbert space, one can consider projective measurements without loss of generality.

A quantum realisation of an assemblage $\Sigma_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ then requires the existence projective measurements $\{\Pi^{(j)}_{a_j|x_j}\}_{a_j\in\mathbb{A}_j}$, for each $x_j \in \mathbb{X}_j$, and for each party Alice$_j$, as well as a tripartite quantum state $\rho \in \mathcal{B}(\mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \mathcal{H}_B)$, such that

$$\sigma_{a_1a_2|x_1x_2} = \mathrm{tr}_{A_1A_2}\left\{(\Pi^{(1)}_{a_1|x_1} \otimes \Pi^{(2)}_{a_2|x_2} \otimes \mathbb{I}_{\mathcal{H}_B})\,\rho\right\}\,, \quad \forall a_j \in \mathbb{A}_j\,,\, x_j \in \mathbb{X}_j\,,\, j \in \{1,2\}\,. \tag{9}$$

What we show is that not all No Signalling assemblages $\Sigma_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ have a quantum realisation [C]. Indeed, consider the simplest scenario where $\mathbb{A}_1 = \mathbb{A}_2 = \{0,1\}$, $\mathbb{X}_1 = \mathbb{X}_2 = \{0,1\}$, and the dimension $d = 2$. Here define the assemblage $\Sigma^*_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ with elements:

$$\sigma^*_{a_1a_2|x_1x_2} = \tfrac{1}{4}\,\delta_{a_1\oplus a_2 = x_1x_2}\,\mathbb{I}_2\,, \tag{10}$$

where $\oplus$ denotes sum mod 2, and $\mathbb{I}_2$ is a two-dimensional identity operator. One can straightforwardly check that $\Sigma^*_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ is a No Signalling assemblage by evaluating Eqs. (6) to (8). However, $\Sigma^*_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ does not have a quantum realisation as we will now show. To do this, we will focus on the correlations $p(a_1a_2|x_1x_2) = \mathrm{tr}\left\{\sigma^*_{a_1a_2|x_1x_2}\right\}$ between the steering parties. These correlations are

$$p(a_1a_2|x_1x_2) = \mathrm{tr}\left\{\sigma^*_{a_1a_2|x_1x_2}\right\} = \tfrac{1}{2}\,\delta_{a_1\oplus a_2 = x_1x_2}\,,$$

which correspond to the PR-box correlations [PR94]. Since PR-box correlations cannot be produced in quantum theory, this implies that the assemblage $\Sigma^*_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ cannot admit a quantum realisation to begin with.

This example shows that steering beyond what quantum theory allows may exist in multipartite scenarios while complying with the No Signalling principle. This example, however, leverages the phenomenon of post-quantum Bell nonclassicality to prove that the assemblage is not quantum – this may give the impression that post-quantum steering is just a consequence of the well known post-quantum Bell nonclassicality. Nonetheless, we show that post-quantum steering exists which does not follow from post-quantum Bell nonclassicality, which proves that post-quantum steering is a genuinely new phenomenon [C]. These results will be presented in Section 5.5.5.

In Ref. [C] we further developed a method that can certify when an assemblage is post-quantum, for some No Signalling assemblages, which does not rely on the correlations produced in Bell tests. The idea is to define a relaxation of the set of quantum assemblages, which we call 'almost-quantum', which has a much simpler structure and contains within the set of quantum assemblages. The name comes from its close relation to the definition of set of almost-quantum correlations [NGHA15]. Whether or not an assemblage is inside the almost-quantum set can be checked efficiently using a semidefinite program (SDP), and hence if an assemblage is found to be outside this set, then it is also certified to be post-quantum. In Section 5.5.5 I will comment on applications of this numerical technique.

5.5.2 Mathematical framework for multi-partite steering: Hermitian operators

The discovery of post-quantum steering [C] raised the question of how to best understand the phenomenon, including its possibilities and its limitations (which could ultimately lead to an information-theoretic reason why post-quantum steering does not appear in nature). Indeed, the usual approach to post-quantum Bell nonclassicality is not appropriate for steering, given that the steered party cannot be viewed as a "black box". Hence, at the moment when Ref. [C] was published, there was no mathematical framework within which to study quantum as well as post-quantum steering in a unified manner.

Here we developed a unified approach to classical, quantum and post-quantum steering [D]. The framework is based on the steering parties performing quantum measurements on their share of a (possibly unphysical) quantum state $\tilde{\rho}$, and its starting point is the characterisation of No Signalling assemblages via non-positive local hidden-state models. In Ref. [D] we work within a generic multi-partite scenario, with many steering parties, as well as many steered parties. Here I will focus on the case of two steering parties, such as in the previous section, for clarity in the presentation.

First we show that, given an arbitrary No Signalling assemblage $\Sigma_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$, there exists a Hilbert space $\mathcal{H}_{A_j}$ for each Alice$_j$, a (complete) projective measurement $\{\Pi^{(j)}_{a_j|x_j}\}_{a_j\in\mathbb{A}_j}$, for each $x_j \in \mathbb{X}_j$ and $j \in \{1,2\}$, as well as a tripartite normalised Hermitian operator $\tilde{\rho}$ (not necessarily a quantum state) in $\mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \mathcal{H}_B$, such that

$$\sigma_{a_1a_2|x_1x_2} = \mathrm{tr}_{A_1A_2}\left\{(\Pi^{(1)}_{a_1|x_1} \otimes \Pi^{(2)}_{a_2|x_2} \otimes \mathbb{I}_{\mathcal{H}_B})\,\tilde{\rho}\right\}\,, \quad \forall\, a_j \in \mathbb{A}_j\,,\, x_j \in \mathbb{X}_j\,,\, j \in \{1,2\}\,. \qquad (11)$$

From here, it follows straightforwardly that the assemblage $\Sigma_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ is quantum if there exists a realisation per Eq. (11) where $\tilde{\rho}$ is a positive semidefinite operator, i.e., a quantum state. Moreover, the assemblage $\Sigma_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ is LHS if there exists a realisation where $\tilde{\rho}$ is a separable quantum state. Hence, LHS, quantum, and post-quantum assemblages can be studied as different cases within the same mathematical expression.

An advantage of this formalism is that it allows us to define moreover other families of assemblages. For example, we defined the concept of a *Gleason assemblage*, which corresponds to a No Signalling assemblage $\Sigma_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ where the operator $\tilde{\rho}$ in Eq. (11) is a normalised entanglement witness. This opened the door to a plethora of possibilities for constructing examples of post-quantum steering. As an example when $d = |\mathbb{A}|$, following Ref. [3], take an arbitrary Local Orthogonality inequality in a tripartite Bell scenario with $|\mathbb{X}|$ measurements of $|\mathbb{A}|$ outcomes per party. From this inequality one can construct an unextendible product basis (UPB) [BDM+99] or a weak UPB [ABB+10] (for scenarios with nondichotomic measurements) for the Hilbert space $\mathcal{H}_B^{\otimes 3}$. This UPB then defines a normalised entanglement witness W and measurement operators $\Pi^{(j)}_{a_j|x_j}$ which give rise to an assemblage $\Sigma^{\mathrm{UPB}}_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ via Eq. (11) taking $\tilde{\rho} = W$. This assemblage is provably post-quantum. To see this, it suffices to study the correlations $p(a_1a_2b|x_1x_2y) = \mathrm{tr}\left\{\Pi^B_{b|y}\,\sigma^{\mathrm{UPB}}_{a_1a_2|x_1x_2}\right\}$, where $y \in \mathbb{X}_1$, $b \in \mathbb{A}_1$, and $\Pi^B_{b|y} = \Pi^{(1)}_{b|y}$. Indeed, these correlations violate the original Local Orthogonality inequality beyond its maximal quantum value, and hence the assemblage cannot have had a quantum realisation to begin with.

Another family of assemblages defined in Ref. [D] is the now referred to as *PTP assemblages*, which is a subset of the Gleason assemblages. PTP stands for Positive and Trace Preserving maps, and the idea is as follows. Take a quantum assemblage $\Sigma^{\mathrm{Q}}_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ and a PTP map Λ_B in $\mathcal{H}_B$. On the one hand, the assemblage $\Sigma^{*}_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ with elements given by

$$\sigma^{*}_{a_1a_2|x_1x_2} = \Lambda_B[\sigma^{\mathrm{Q}}_{a_1a_2|x_1x_2}]$$

is by definition No Signalling. On the other hand, the assemblage $\Sigma^{*}_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ will admit a decomposition as per Eq. (11) with

$$\tilde{\rho} = \mathcal{I}_1 \otimes \mathcal{I}_2 \otimes \Lambda_B[\rho]\,,$$

where $\mathcal{I}_j$ is the identity channel, and ρ a normalised quantum state. Since Λ_B is not completely positive, then the operator $\tilde{\rho}$ may not be a quantum state. Hence, this opens the possibility for the assemblage $\Sigma^{*}_{\mathbb{A}_1,\mathbb{A}_2|\mathbb{X}_1,\mathbb{X}_2}$ to not admit a quantum realisation. It is not necessary that every PTP map Λ_B gives rise to a post-quantum assemblage, but we found examples of maps that do [D]. These examples are presented in Section 5.5.5 below. The added value of these constructions is that, unlike the family of UPB assemblages from above, PTP assemblages do not exhibit post-quantum Bell nonclassicality. Indeed, for any set of measurements $\{\Pi^B_{b|y}\}$ that Bob can perform on his system, the correlations

$$\begin{aligned} p(a_1a_2b|x_1x_2y) &= \mathrm{tr}\left\{\Pi^B_{b|y}\,\sigma^*_{a_1a_2|x_1x_2}\right\} = \mathrm{tr}\left\{(\Pi^{(1)}_{a_1|x_1}\otimes\Pi^{(2)}_{a_2|x_2}\otimes\Pi^B_{b|y})\,\mathcal{I}_1\otimes\mathcal{I}_2\otimes\Lambda_B[\rho]\right\} \\ &= \mathrm{tr}\left\{(\Pi^{(1)}_{a_1|x_1}\otimes\Pi^{(2)}_{a_2|x_2}\otimes\Lambda^\dagger_B[\Pi^B_{b|y}])\,\rho\right\} \end{aligned}$$

are always quantum, since $\Lambda^\dagger_B$ is unital. To certify that a PTP assemblage is post-quantum, hence, we used the numerical techniques of Ref. [C] mentioned above.

Finally, in Ref. [D] we defined a quantifier of post-quantum steering. We defined the *steering negativity* as the minimum value of the absolute value of the sum of the negative eigenvalues of $\tilde{\rho}$, over all possible decompositions of the assemblage as per Eq. (11). The steering negativity will always be 0 for quantum and LHS assemblages, but will be strictly positive for post-quantum assemblages. The free operations here are one-way Local Operations and Classical Communication, supplemented with arbitrary shared entanglement, with the extra constraint that a steering party only interacts classically with the assemblage. Under these operations, we show that the steering negativity is a convex steering quantifier.

5.5.3 Mathematical framework for multi-partite steering: 'causal' quantum channels

We also developed [E] an alternative formalism to that of Ref. [D], which is based on the study of quantum channels. We show that types of steering, whether quantum or post-quantum, are directly related to particular families of quantum channels that have been previously introduced by Beckman, Gottesman, Nielsen, and Preskill [BGNP01]. Utilising this connection we also demonstrate new analytical examples of post-quantum steering, give a quantum channel interpretation of almost-quantum non-locality and steering, easily recover and generalise the celebrated Gisin-Hughston-Jozsa-Wootters theorem, and initiate the study of post-quantum Buscemi non-locality and nonclassical teleportation. In the following I'll present a summary of these results.

The framework is based on quantum channels on multi-partite systems that satisfy a form of the No Signalling principle, introduced first by Beckman, Gottesman, Nielsen, and Preskill [BGNP01]. These channels are there referred to as *'causal' channels*, but to avoid confusion with causal networks I will here refer to them as *nonsignalling channels*. The idea is then to think of the elements of an assemblage as the (unnormalised) states of a quantum system that is output from a quantum channel. In the case of a traditional bipartite steering scenario, the situation is conceptualised as depicted in Fig. 3. For multi-partite setups the generalisation is straightforward, using a multi-partite quantum channel where the only parties with free output wires are the steered ones.

First, we note how different sets of assemblages may arise from different classes of channels: an assemblage is No Signalling if it may arise from a nonsignalling channel [E, Prop. 11], quantum if it may arise from a localisable channel [E, Prop. 13], and LHS if it may arise from a local channel [E, Prop. 10]. Moreover, this connection between channels and assemblages allows us to find an alternative characterisation of the almost-quantum assemblages defined in Ref. [C]. This is done by first defining a new class of quantum channels, which we call *almost-localisable* channels, and then showing that almost-quantum assemblages as those which arise from them [E, Thm. 15]. We also show that almost-quantum correlations in Bell scenarios [NGHA15] are

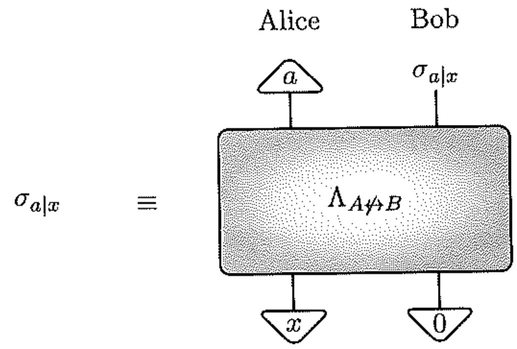


Figure 3: An assemblage $\Sigma_{\mathbb{A}|\mathbb{X}} = \{\sigma_{a|x}\}$ seen as generated by a nonsignalling quantum channel $\Lambda_{A\not\leftrightarrow B}$. Here, $\{|x\rangle\}_{x\in\mathbb{X}}$ is an orthonormal basis for the input Hilbert space of Alice, $\{|a\rangle\}_{a\in\mathbb{A}}$ an orthonormal basis for her output Hilbert space, and $|0\rangle$ a fixed state for Bob's input system.

also fully characterised by almost-localisable channels, hence bringing a unified approach to almost-quantum steering and Bell nonclassicality.

The connection we developed between quantum channels and steering also opened the door to constructing new examples of post-quantum steering. For instance, by extending the results of Ref. [BGNP01] we constructed a non-localisable tripartite quantum channel that defined a post-quantum assemblage in a tripartite steering scenario with two steering parties. Also, by defining the notion of a *local-limited* quantum channel, we initiated the systematic study of post-quantum assemblages that may only feature local correlations in Bell scenarios (namely, an analogue of the before mentioned PTP assemblages, but where the correlations are local instead of quantum).

In Ref. [E] we also initiated the study of post-quantum nonclassical teleportation, and post-quantum Buscemi non-locality. nonclassical teleportation [CSicvac17] and Buscemi non-locality [Bus12] (or semi-quantum non-locality) have been introduced very recently within the quantum information community as generalisations of steering and Bell non-locality respectively. The pioneering work by Buscemi consisted in defining a semi-quantum non-local game and arguing that any entangled state is more useful than a separable one for winning at it [Bus12]. In a nutshell, these semi-quantum games are in spirit Bell-type test, but where the inputs are quantum states instead of random classical variables. In our framework we then reinterpret these semi-quantum games from the perspective of quantum channels, and discover that the existence of nonsignalling yet not-localisable channels can be used to define post-quantum nonclassicality in these semi-quantum games. Similarly, we studied the phenomenon of nonclassical teleportation [CSicvac17]. In a nonclassical teleportation scenario, one party aims to "teleport" quantum information to the other, even when their resources are noisy. In this scenario, the state to teleport is chosen from a known (to all parties) finite set of quantum states, unlike conventional teleportation where there is a single unknown state that is to be teleported. Through our framework, hence, we reinterpret the nonclassical teleportation experiment, even for a multi-partite setup, by relating it to quantum channels. We then leverage the existence of nonsignalling yet not-localisable channels to define the concept of post-quantum nonclassical teleportation. Finally, the almost-localisable channels that we defined allows us to introduce the new concepts of almost-quantum Buscemi non-locality, and almost-quantum nonclassical teleportation. The relationship chart between forms of post-quantum nonclassicality found in Ref. [E] is presented in Fig. 4.

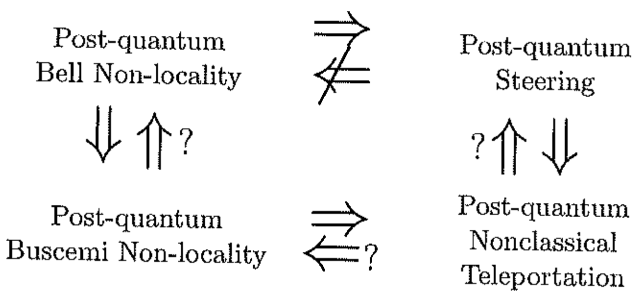


Figure 4: Implication relations among the different forms of post-quantum non-locality. Where there is a question mark next to an implication, this means that it is open whether there is an implication. One can also infer from the diagram that Post-quantum Bell Non-locality infers Post-quantum nonclassical Teleportation, but the reverse implication definitely does not hold.

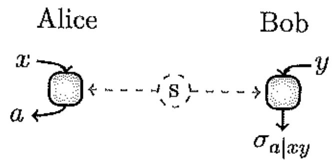


Figure 5: The Bob-with-input (BWI) steering scenario: Bob now also has an input y which influences the preparation of a state in his laboratory.

5.5.4 Generalised bipartite steering scenarios

Another alternative to open the door to post-quantum assemblages is to keep the bipartite structure of the steering scenario but generalise the setup in some other way. This avenue was explored on Ref. [F], where we discovered two relaxations of the traditional setup where bipartite steering incompatible with quantum theory is possible: (i) one where Bob also has an input and operates on his subsystem, which we call *Bob-with-Input* scenario, and (ii) the 'instrumental steering' scenario.

In the Bob-with-Input scenario, depicted in Fig. 5, Bob's device also accepts an input before producing a quantum state. Intuitively, we can think that this input may determine the preparation of some quantum system, which could come about from a transformation on a system inside Bob's device. Let y denote the choice of Bob's input, and let $\mathbb{Y}$ be the set of all possible such choices. An assemblage in the Bob-with-Input scenario will then be given by $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}\mathbb{Y}} = \{\sigma_{a|xy}\}_{a\in\mathbb{A}\,,\,x\in\mathbb{X}\,,\,y\in\mathbb{Y}}$.

An assemblage $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}\mathbb{Y}}$ is said to have a quantum realisation if and only if there exists a Hilbert space $\mathcal{H}_A$ for Alice, a Hilbert space $\mathcal{H}_{B'}$ for Bob, a normalised quantum state $\rho \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_{B'})$, and a (complete) projective measurement $\{\Pi_{a|x}\}_{a\in\mathbb{A}}$ for each $x \in \mathbb{X}$, and a completely positive and trace preserving map $\mathcal{E}_y : \mathcal{H}_{B'} \to \mathcal{H}_B$ for each $y \in \mathbb{Y}$ such that

$$\sigma_{a|xy} = \mathcal{E}_y \left[\mathrm{tr}_A \left\{(\Pi_{a|x} \otimes \mathbb{I}_{\mathcal{H}_{B'}})\,\rho\right\}\right] \,, \quad \forall\, a \in \mathbb{A}\,,\, x \in \mathbb{X}\,,\, y \in \mathbb{Y}\,. \tag{12}$$

In turn, an assemblage $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}\mathbb{Y}}$ is No Signalling if the following are fulfilled:

$$\sum_{a\in\mathbb{A}} \sigma_{a|xy} = \sum_{a\in\mathbb{A}} \sigma_{a|x'y} \,, \quad \forall\, x\,,\, x' \in \mathbb{X}\,,\, y\in\mathbb{Y}\,, \tag{13}$$

$$\mathrm{tr}\left\{ \sum_{a\in\mathbb{A}} \sigma_{a|xy} \right\} = 1\,, \quad \forall x\in\mathbb{X}\,,\, y\in\mathbb{Y}\,, \tag{14}$$

$$\mathrm{tr}\left\{\sigma_{a|xy}\right\} = p(a|x)\,, \quad \forall\, a\in\mathbb{A}\,,\, x\in\mathbb{X}\,,\, y\in\mathbb{Y}\,. \tag{15}$$

The question then that we are interested in is:

Given an arbitrary No Signalling assemblage $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X}\mathbb{Y}}$,
can we find a quantum realisation for it?

In Ref. [F] we answer this in the negative. A simple example we found is given by the assemblage $\mathbf{\Sigma}^*_{\mathbb{A}|\mathbb{X}\mathbb{Y}}$ with elements:

$$\sigma^*_{a|xy} := \frac{1}{2} \left(|a\rangle\langle a|\; \delta_{xy=0} + |a\oplus 1\rangle\langle a\oplus 1|\, \delta_{xy=1} \right)\,, \tag{16}$$

with $\mathbb{A} = \mathbb{X} = \mathbb{Y} = \{0,1\}$. One can readily check that this assemblage satisfies the No Signalling conditions of Eqs. (13) to (15). To show that $\mathbf{\Sigma}^*_{\mathbb{A}|\mathbb{X}\mathbb{Y}}$ does not have a quantum realisation we focus on the correlations $p(ab|xy) = \mathrm{tr}\left\{\Pi_b\, \sigma_{a|xy}\right\}$ that arise when Bob performs the dichotomic projective measurement $\{\Pi_b = |b\rangle\langle b|\}_{b\in\{0,1\}}$. These correlations turn out to be $p(ab|xy) = \frac{1}{2}\,\delta_{a\oplus b=xy}$, which correspond to the PR-box correlations [PR94]. Since PR-box correlations cannot be produced in quantum theory, this implies that the assemblage $\mathbf{\Sigma}^*_{\mathbb{A}|\mathbb{X},\mathbb{Y}}$ cannot admit a quantum realisation to begin with.

Similarly to the case of Ref. [C], the simplicity of this proof relies on post-quantum Bell nonclassicality. Hence, this may give the impression that post-quantum steering in the Bob-with-Input scenario is just a consequence of the well known post-quantum Bell nonclassicality. Nonetheless, just like it happened in Ref. [C], we show that post-quantum steering exists which does not follow from post-quantum Bell nonclassicality, which proves that post-quantum steering is a genuinely new phenomenon [F]. These results will be presented in Section 5.5.5. A key part in their construction is the definition of a new family of assemblages in the Bob-with-Input scenario, inspired by Ref. [D]: we call them Bob-with-Input PTP assemblages [F, Def. 19]. Similarly to Ref. [D], an assemblage $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X},\mathbb{Y}}$ is PTP if there exists a PTP map Λ_y for each $y\in Y$ and a quantum assemblage $\mathbf{\Sigma}^{\mathrm{Q}}_{\mathbb{A}|\mathbb{X}}$ in the traditional bipartite steering scenario, such that $\sigma_{a|xy} = \Lambda_y[\sigma^{\mathrm{Q}}_{a|x}]$. Similarly to the multi-partite case, the correlations in a Bell setup that may arise via $p(ab|xy) = \mathrm{tr}\left\{N_b \sigma_{a|xy}\right\}$ will always have a quantum realisation [F] for PTP assemblages for any (complete) measurement $\{N_b\}$ of any number of outcomes.

In Ref. [F] we also expanded the techniques of Ref. [C] to devise a method to certify that certain assemblages are post-quantum, independently of the correlations that they generate in a Bell setup. A first step in this method is to define a relaxation of the set of quantum assemblages in the Bob-with-Input scenario, which we call $\tilde{\mathcal{Q}}$ [F, Def. 17], which contains all quantum assemblages and some post-quantum ones. The advantage of this is that checking whether an assemblage $\mathbf{\Sigma}_{\mathbb{A}|\mathbb{X},\mathbb{Y}}$ belongs to $\tilde{\mathcal{Q}}$ or not is a single instance of a semidefinite program. If an assemblage does not belong to $\tilde{\mathcal{Q}}$ then it is certified to be post-quantum. This method will be applied to show post-quantumness in the examples of Section 5.5.5.

Finally, there is the steering scenario referred to as *Instrumental steering* [NTCA18], depicted in Fig. 6 (for a causal perspective on the Instrumental steering scenario, see Fig. 11 in

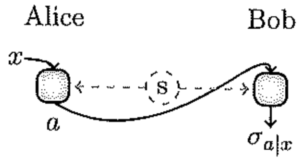


Figure 6: The instrumental steering scenario: is similar to Bob-with-Input except that Bob's input now depends on Alice's outcome.

Section 5.7.1). Here, the state preparation of Bob's system may moreover be influenced by the information on Alice's classical outcome a, unlike in the traditional bipartite steering scenario. An assemblage in the Instrumental steering scenario is hence given by $\mathbf{\Sigma}^{I}_{\mathbb{A}|\mathbb{X}} = \{\sigma^{I}_{a|x}\}_{a\in\mathbb{A}\,,x\in\mathbb{X}}$, where the states $\sigma^{I}_{a|x}$ may arise through different mechanisms as those $\sigma_{a|x}$ in the traditional bipartite scenario.

An assemblage $\mathbf{\Sigma}^{I}_{\mathbb{A}|\mathbb{X}}$ is said to have a quantum realisation [NTCA18] if and only if there exists a Hilbert space $\mathcal{H}_A$ for Alice, a Hilbert space $\mathcal{H}_{B'}$ for Bob, a normalised quantum state $\rho \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_{B'})$, and a (complete) projective measurement $\{\Pi_{a|x}\}_{a\in\mathbb{A}}$ for each $x \in \mathbb{X}$, and a completely positive and trace preserving map $\mathcal{E}_a : \mathcal{H}_{B'} \to \mathcal{H}_B$ for each $a \in \mathbb{A}$ such that

$$\sigma^{I}_{a|x} = \mathcal{E}_a \left[\mathrm{tr}_A \left\{(\Pi_{a|x} \otimes \mathbb{I}_{\mathcal{H}_{B'}})\,\rho\right\}\right]\,, \quad \forall\, a \in \mathbb{A}\,,\, x \in \mathbb{X}\,. \tag{17}$$

The instrumental steering scenario, however, has no straightforward no signalling constraints. Hence, we defined 'no signalling' assemblages [F] by adopting the relation found in Ref. [K] between nonsignalling Bell correlations and generic instrumental correlations in the black-box scenario. Namely, an assemblage $\mathbf{\Sigma}^{I}_{\mathbb{A}|\mathbb{X}}$ is 'no signalling'[4] if there exists a no signalling assemblage $\{w_{a|xy}\}$ in a Bob-with-Input scenario with $\mathbb{Y} = \mathbb{A}$, such that $\sigma^{I}_{a|x} = w_{a|xa}$. That is, we view an assemblage in the instrumental steering scenario as coming from a Bob-with-Input scenario after applying the post-selection $y = a$.

The question then that we are interested in is:

Given an arbitrary 'No Signalling' assemblage $\mathbf{\Sigma}^{I}_{\mathbb{A}|\mathbb{X}}$,
can we find a quantum realisation for it?

Here we answer this question in the negative. Indeed, we show that post-quantum steering is admissible in the Instrumental steering scenario. The proof consist of presenting an example of a 'no signalling' assemblage and showing it does not have a quantum realisation. The proof, presented in the next section, works by contradiction: we show that if the assemblage in the example was to have a quantum realisation, then one could use those states, measurements, and maps to give a quantum realisation in the Bob-with-Input scenario for a provably post-quantum assemblage, hence the contradiction. With this, Ref. [F] initiates the study of post-quantum steering in the instrumental steering scenario.

5.5.5 Post-quantum steering: a genuinely new phenomenon

In the previous sections I explained how post-quantum steering may emerge in steering scenarios when we go beyond the traditional bipartite setup. Here I will discuss how post-quantum

[4]Here, by a 'no signalling' assemblage we mean one that is operationally admissible in the scenario. In Ref. [F] we refer to them as *general assemblages*, but for a consistent narrative in this Habilitation thesis I will refer to them as 'no signalling'.

steering is a phenomenon independent of post-quantum Bell nonclassicality, i.e., that post-quantum assemblages that do not exhibit post-quantum correlations may exist.

This was first discovered in Ref. [C]. There, we found an example of a tripartite assemblage $\mathbf{\Sigma}^{(1)}_{\mathbb{A}_1\mathbb{A}_2|\mathbb{X}_1\mathbb{X}_2}$ of subnormalised states for a qutrit (i.e., the dimension of $\mathcal{H}_B$ is 3), with $\mathbb{A}_1 = \mathbb{A}_2 = \mathbb{X}_1 = \mathbb{X}_2 = \{0,1\}$, and with the following property: the correlations $p(a_1 a_2 b|x_1 x_2 y) = \text{tr}\left\{N_{b|y}\,\sigma^{(1)}_{a_1 a_2|x_1 x_2}\right\}$ admit a local hidden variable model for the tripartite Bell scenario Alice$_1$-Alice$_2$-Bob, for any set of generalised measurements (POVM[5]) $\{N_{b|y}\}$ on qutrits, and for any cardinality of the classical variables b and y. This example was found by conducting a complex numerical optimisation [C], and the assemblage elements are hence given numerically.

This example shows how remarkably distinct phenomena steering and Bell nonclassicality are: there are situations where the former does not admit a quantum explanation while the latter can arise within classical mechanics. However, the numerical nature of the example prevented us from starting a systematic study of that form of post-quantum steering. This obstacle was overcome in Refs. [D, F], where quantum and post-quantum steering was studied through a unified mathematical formalism.

The second example of post-quantum steering without post-quantum correlations was given in Ref. [D]. There, we found an example of a tripartite assemblage $\mathbf{\Sigma}^{(2)}_{\mathbb{A}_1\mathbb{A}_2|\mathbb{X}_1\mathbb{X}_2}$ of subnormalised states for a ququart (i.e., the dimension of $\mathcal{H}_B$ is 4), with $\mathbb{A}_1 = \mathbb{A}_2 = \mathbb{X}_1 = \mathbb{X}_2 = \{0,1\}$, and with the property that it is a PTP assembage, hence the correlations $p(a_1 a_2 b|x_1 x_2 y) = \text{tr}\left\{N_{b|y}\,\sigma^{(2)}_{a_1 a_2|x_1 x_2}\right\}$ admit a quantum realisation for the tripartite Bell scenario Alice$_1$-Alice$_2$-Bob, for any set of generalised measurements $\{N_{b|y}\}$ on ququarts, and for any cardinality of the classical variables b and y. The elements of $\mathbf{\Sigma}^{(2)}_{\mathbb{A}_1\mathbb{A}_2|\mathbb{X}_1\mathbb{X}_2}$ are mathematically defined as $\sigma^{(2)}_{a_1 a_2|x_1 x_2} = \Lambda[\sigma^{(2)\text{Q}}_{a_1 a_2|x_1 x_2}]$, since it's a PTP assemblage, and the particular Λ and $\sigma^{(2)\text{Q}}_{a_1 a_2|x_1 x_2}$ are defined as follows. First, let us denote by X, Y, and Z the Pauli-X, Pauli-Y, and Pauli-Z operators, respectively. The PTP map is defined as $\Lambda = \frac{1}{2}\left(\text{tr}\,\{\rho\}\,\mathbb{I} - \rho - U\,\rho^T\,U^\dagger\right)$, where $U = X \otimes Y$ is an antisymmetric unitary. To obtain the elements $\{\sigma^{(2)\text{Q}}_{a_1 a_2|x_1 x_2}\}$ we think of a quantum steering experiment where the dimension of the Hilbert spaces of Alice$_1$ and Alice$_2$ are each of value 2. Define as the local measurements for the Alices the following operators: $M^{(1)}_{a_1|0} = \frac{\mathbb{I}+(-1)^{a_1} X}{2}$, $M^{(1)}_{a_1|1} = \frac{\mathbb{I}+(-1)^{a_1} Z}{2}$, $M^{(2)}_{a_2|0} = \frac{\mathbb{I}+\frac{(-1)^{a_2}}{\sqrt{2}}(X+Z)}{2}$, and $M^{(2)}_{a_2|1} = \frac{\mathbb{I}+\frac{(-1)^{a_2}}{\sqrt{2}}(-X+Z)}{2}$. Finally, let Alice$_1$-Alice$_2$-Bob share the quantum state of two qubits and a ququart given by $|\Psi\rangle = \frac{|\Psi_1\rangle + i\,|\Psi_2\rangle - |\Psi_3\rangle}{\sqrt{14}}$, where (for $k \in \{1,2,3\}$):

$$|\Psi_k\rangle = \sum_{\substack{a_1,a_2,b,b' \in \{0,1\},\\ a_1+a_2+b+b'=k}} |a_1\rangle_{A_1}\,|a_2\rangle_{A_2}\,|b\,b'\rangle_B\,.$$

The elements $\{\sigma^{(2)\text{Q}}_{a_1 a_2|x_1 x_2}\}$ are computed via $\sigma^{(2)\text{Q}}_{a_1 a_2|x_1 x_2} = \text{tr}_{A_1 A_2}\left\{(M^{(1)}_{a_1|x_1} \otimes M^{(2)}_{a_2|x_2} \otimes \mathbb{I})\,|\Psi\rangle\,\langle\Psi|\right\}$.

This example shows how the PTP technique (i.e., the construction of PTP assemblages) can give rise to post-quantum assemblages. With this, we justified and initiated the use of the PTP technique to explore in a systematic fashion post-quantum steering without post-quantum correlations.

Another example of this kind that I would like to explicitly mention was given in Ref. [F]. There, we found an example of a PTP post-quantum bipartite assemblage $\mathbf{\Sigma}^{(3)}_{\mathbb{A}|\mathbb{X}\mathbb{Y}}$ in the Bob-with-Input scenario, with $\mathbb{A} = \mathbb{Y} = \{0,1\}$ and $\mathbb{X} = \{1,2,3\}$, and where the local dimension of

[5] A generalised measurement is mathematically expressed in terms of a positive-operator valued measure (POVM).

Bob is 2. The elements of this assemblage are $\sigma^{(3)}_{a|xy} = \frac{1}{4}(\mathbb{1} + (-1)^{a+\delta_{x,2}\delta_{y,1}}\nu_x)$, where $x \in \{1,2,3\}$ and $(\nu_1, \nu_2, \nu_3) = (X, Y, Z)$ are the Pauli operators. This assemblage is PTP since it may be mathematically expressed as one where Alice and Bob share a maximally entangled state of two qubits, Alice chooses between the three Pauli measurements to perform on her qubit, and Bob applies on his qubit the identity channel when $y = 0$ and the transpose when $y = 1$.

This example shows how post-quantum steering is also a genuinely new phenomenon independent of post-quantum Bell nonclassicality, even in bipartite scenarios. Moreover, it justifies and initiates the study of post-quantum steering in generalised bipartite scenarios via the PTP technique.

In the three examples that I mentioned in this section, a similar method was used to certify that the assemblages are post-quantum. This can be summarised by three steps:

- First, find a steering inequality in the corresponding steering scenario. That is, operators $\{F_{a_1a_2x_1x_2}\}_{a_j\in\mathbb{A}_j,\, x_j\in\mathbb{X}_j,\, j\in\{1,2\}}$ for the multi-partite case, and $\{F_{axy}\}_{a\in\mathbb{A},\, x\in\mathbb{X},\, y\in\mathbb{Y}}$ for the Bob-with-Input case.
- Second, upper-bound the maximum quantum violation of that inequality by the almost-quantum assemblages (in the multi-partite case) or $\tilde{\mathcal{Q}}$ assemblages (in the Bob-with-Input case). Computing this is a single instance of a semidefinite program.
- Evaluate the inequality with the corresponding assemblage from the example: the value the assemblage gives is larger than the maximum quantum value, and hence it does not admit a quantum realisation.

Finally, let me briefly comment the case of the Instrumental steering scenario. Here, we constructed the assemblage $\Sigma^{(4)I}_{\mathbb{A}|\mathbb{X}}$ by defining $\sigma^{(4)I}_{a|x} = \sigma^{(3)}_{a|xa}$ [F]. The fact that $\Sigma^{(3)}_{\mathbb{A}|\mathbb{XY}}$ is a PTP assemblage in the Bob-with-Input scenario implies that the correlations $p(ab|x) = \mathrm{tr}\left\{N_b\sigma^{(4)I}_{a|x}\right\}$ in the traditional Instrumental scenario will admit a quantum realisation for any (complete) measurement $\{N_b\}$ of any number of outcomes. Moreover, we showed that any quantum realisation of $\Sigma^{(4)I}_{\mathbb{A}|\mathbb{X}}$ would translate into a quantum realisation of $\Sigma^{(3)}_{\mathbb{A}|\mathbb{XY}}$, hence $\Sigma^{(4)I}_{\mathbb{A}|\mathbb{X}}$ has to be post-quantum.

This example shows that post-quantum steering is a form of nonclassicality that cannot be identified by merely studying the correlations $\{p(ab|x)\}$ in the Instrumental causal network. In addition, it also highlights how the close connection between the Instrumental and Bell scenarios found in Ref [K] (see Section 5.7.1) also plays a fundamental role in the study of steering.

5.6 Quantum vs. "Almost-quantum" correlations in Bell and contextuality scenarios

How to characterise the set of quantum correlations in Bell and contextuality scenarios is an important open question in the foundations of quantum physics, with the potential to impact the field of quantum information which uses such correlations as a resource for information processing. Until now, all the principles that have been postulated for characterising the quantum set fail to do so[6], since they are satisfied by the set of so-called almost-quantum correlations [NGHA15].

In this Habilitation thesis we focus on developing a better understanding of what the set of almost-quantum correlations is: how to fully characterise it in terms of physical principles [G], how it relates to the other sets of NPA-hierarchy correlations (where almost-quantum correlations were first identified) [H], and which physical principles satisfied by quantum theory do almost-quantum correlations violate [I, J].

[6] More precisely, there is only one principle, called Information Causality [PPK+09], for which the question of whether almost-quantum correlations violate it is still open.

5.6.1 Macroscopic non-contextuality: a principle to characterise correlations

The first question we address is whether the full set of almost-quantum correlations – and only that – can be totally characterised by a single physical principle. We answer this is the positive in Ref. [G], where we define the principle of *Macroscopic non-contextuality*.

The basic idea behind the Macroscopic non-contextuality principle is that a macroscopic version of the experiment should not lead to any observations of nonclassical effects. Intuitively, if we move from a "single photon source" to a "very strong light source" in a certain way, the experimental data should look classical. In Ref. [G] we properly define all these words: what this transition to the macroscopic realm should be, and when data would admit a classical explanation. Here I will briefly mention these concepts and results.

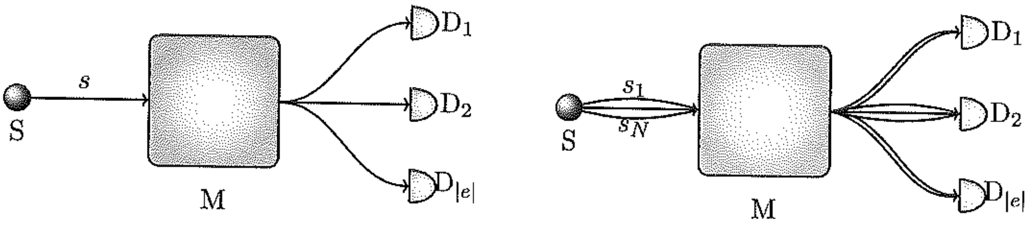
First, we will work within a contextuality scenario as defined in Ref. [2]. There, we use tools from hypergraph theory to define and manipulate contextuality scenarios. The advantage of this approach, in contrast to that of Ref. [NW10], is that Bell nonclassicality and Kochen-Specker contextuality can be studied simultaneously through a unified mathematical formalism. Hence, even though it may at first glance appear the the results apply only to single-system experiments, bipartite and multi-partite Bell scenarios are also being represented through the figures and formulae, when the corresponding Bell hypergraphs are used. Throughout this section I will hence use the notation defined in Ref. [2], and denote by v a measurement event (i.e., an outcome from a possible measurement), by V the set of all possible measurement outcomes of all measurement choices, by e a measurement choice, and by E the set of measurements.

Consider a physical system s and a set of measurements E, from which we choose one to perform on s. In the hypergraph approach to contextuality such a scenario is represented by a hypergraph $H = (V, E)$; the (normalised) probability $p(v)$ of obtaining an outcome $v \in V$ given that a measurement $e \ni v$ is performed, for all outcomes, defines a *probabilistic model*[7] on H. An experiment of this type is depicted in Fig. 7(a), and we refer to it as *microscopic experiment.* The macroscopic version of such an experiment, which we call its "macroscopic extension", is defined as follows. Suppose now that the source produces N independent copies of this system s, and that these N systems reach the measurement device (see Fig. 7(b)). Now we assume that we are no longer able to distinguish individual outcomes, but only the fraction of instances (or "intensity") of each outcome v given a measurement e. The experimental results for a particular measurement e in the macroscopic experiment are thus described by a probability distribution $\mathcal{P}_e(\{I^v\}_{v\in e})$ where I^v denotes the intensity for outcome v. This can be described as a joint measurement scenario in which the constituent experiments are the measurements of the intensity I^v for each v.

The probabilities $\{\mathcal{P}_e(\{I^v\}_{v\in e})\}_{e\in E}$ for the macroscopic extension setup are determined by the probabilistic model $\{p(v)\}_{v\in V}$ of the underlying microscopic setup. Imposing that $\{\mathcal{P}_e(\{I^v\}_{v\in e})\}_{e\in E}$ has a classical realisation will hence impose constraints on the underlying microscopic mechanism $\{p(v)\}$. The formal connection between $\{p(v)\}$ and $\{\mathcal{P}_e(\{I^v\}_{v\in e})\}_{e\in E}$ is derived in Ref. [G] by connecting the microscopic probabilistic model to both (i) the deviation of the intensity I^v from its mean value, and (ii) the probability distribution over the intensity fluctuations for each experiment e, which, according to the central limit theorem [Tij07], converges to a multivariate Gaussian distribution $\gamma^e_{u,v}$ in the limit $N \to \infty$.

The probabilities $\{\mathcal{P}_e(\{I^v\}_{v\in e})\}_{e\in E}$ are said to have a classical realisation if, in the limit $N \to \infty$, there exists a probability distribution $\mathcal{P}_{\mathrm{NC}}$ over a set of intensities $\{I^v\}_{v\in V(H)}$, such

[7] In this Habilitation thesis, I use the terms *probabilistic model, correlations*, and *conditional probability distribution* interchangeably, to encompass the notation used in the papers included in the series. Each term is slightly more appropriate to each nonclassical phenomena, but I will not make those refined distinctions here.



(a) Microscopic experiment

(b) Macroscopic experiment

Figure 7: **(a) Microscopic experiment.** A source S prepares a system s, which is sent to the measurement device M. There, an interaction between the measurement apparatus and the system sends the system towards one of a set of detectors, where its presence can be observed as a "detector click". The clicking of detector D_k corresponds to obtaining outcome k. **(b) Macroscopic experiment.** A source S prepares N independent copies of a system s, which are sent to the measurement device M. There, for each system (and independently for each system), an interaction between the measurement apparatus and the system sends the system towards one of a set of detectors, However, in this case, rather than a single click, there is a distribution of 'clicks' over the detectors, given by the arrival of various systems to each of them, according to the probabilities for each outcome in the microscopic experiment. The 'output' of this macroscopic experiment is the collection of intensities I_e^v registered at the detectors.

that the experimental probabilities $\{\mathcal{P}_e(\{I^v\}_{v\in e})\}_{e\in E}$, can be obtained as marginals from $\mathcal{P}_{\rm NC}$:

$$\mathcal{P}_e(\{I^v\}_{v\in e}) = \int \Big(\prod_{v\in V(H)\setminus e} dI^v \Big)\, \mathcal{P}_{\rm NC}(\{I^v\}_{v\in V(H)}), \tag{18}$$

where $\setminus$ is set difference. This condition translates into the existence of a covariance matrix $\gamma_{u,v}$ for this probability distribution, which reduces to $\gamma^e_{u,v}$ when the events u and v are restricted to e.

Finally, Ref. [G] shows that the existence of this matrix $\gamma_{u,v}$, given the connection between $\{p(v)\}$ and $\{\mathcal{P}_e(\{I^v\}_{v\in e})\}_{e\in E}$, implies the existence of a moment matrix (see [2, Def. 6.1.2]) for the probabilistic model $\{p(v)\}$ that certifies that it is almost-quantum. In addition, Ref. [G] also shows that if $\{p(v)\}$ is an almost-quantum probabilistic model, its associated moment matrix implies the existence of the covariance matrix $\gamma_{u,v}$ that certifies that $\{\mathcal{P}_e(\{I^v\}_{v\in e})\}_{e\in E}$ admits a classical realisation.

With this, Ref. [G] provides the first full characterisation of almost-quantum correlations in terms of a physical principle, for both Bell and contextuality scenarios. This tells us that requiring a classical macroscopic limit may be compatible with physical theories other than quantum theory. A natural next step in the characterisation of quantum correlations is to try and find physical principles that almost-quantum correlations violate.

5.6.2 Almost-quantum correlations in multi-partite scenarios

Almost-quantum correlations were first identified in bipartite Bell scenarios as the set of correlations that satisfy the 1+AB level of the Navascués-Pironio-Acín (NPA) hierarchy of SDP tests [NPA07, NPA08, PNA10]. There, by definition, the almost-quantum set is included within the set of correlations that satisfy the first level of NPA, and in turn includes the set of correlations that satisfy the second level of NPA. Almost-quantum correlations were later given their proper name, and extended to multi-partite Bell scenarios [NGHA15].

In Ref. [H] we asked the questions of how the set of almost-quantum correlations in multi-partite Bell scenarios relates to the quantum set and to the levels of the NPA hierarchy. The motivation here is to learn about the structure of such set of correlations. The only set of correlations that satisfies all levels of the NPA tests is that given by quantum theory, and multi-partite Bell scenarios are known to provide a much richer structure for the set of correlations, especially when exploring principles to characterise conditional probability distributions obtained in Bell tests [1, 3]. Hence, understanding how almost-quantum relates to NPA in these scenarios may shed light on how to differentiate quantum correlations from almost-quantum ones at a fundamental level.

In Ref. [H] we focused on the simplest multi-partite Bell scenario: one with three parties (Alice, Bob, and Charlie), each of which having access to two dichotomic measurements to choose from to perform on their respective systems. The set of classical correlations in this scenario is fully characterised by 46 classes of facet Bell inequalities [Śli03]. Among these are the Mermin [Mer90] and the tripartite guess your neighbor's input (GYNI) [ABB+10] inequalities, which display curious properties. On the one hand, the maximum value of Mermin's inequality for no-signalling (NS) correlations coincides with that for quantum (and hence almost-quantum) correlations, while classical ones achieve a lower value. On the other hand, for the GYNI inequalities, the classical maximum coincides with that for the almost-quantum (and hence the quantum) correlations [1], while the NS ones achieve a larger value. We see hence how the range of possibilities that this tripartite scenario displays is much richer than its bipartite counterpart.

The first question we addressed in Ref. [H] is whether we can tell apart the sets of quantum and almost-quantum correlations by only looking at their violation of facet Bell inequalities. We found that, out of the 45 nontrivial inequalities, 43 display the same phenomenon as in the bipartite Clauser-Horne-Shimony-Holt (CHSH) scenario: the maximum violation attainable by the almost-quantum correlations ([H, column four, Table I]) can also be achieved by the quantum ones ([H, column three, Table I]). However, two inequalities (n° 23 and 41) display a gap, that is, they demonstrate that quantum correlations cannot be as nonclassical as the almost-quantum ones. This is the first time that such a behavior is observed for scenarios with two dichotomic measurements per party using facet Bell inequalities.

Next, we studied the relations between the set of almost-quantum correlations and those sets of post-quantum correlations defined by the NPA hierarchy. We showed that in this tripartite, and therefore in a general n-partite scenario (where $n \geq 3$), neither the $(n-1)$-th level of the NPA hierarchy is contained within the almost-quantum set, nor otherwise [H]. To see this we showed that, in the tripartite scenario, for some of these 45 non-trivial tripartite Bell inequalities, there exist correlations in the second level of NPA that are more nonclassical (in the sense of giving a stronger Bell violation) than those restricted to the almost-quantum set. Hence, the set of correlations that satisfy the second level of NPA is not included within the almost-quantum set. Conversely, we constructed a specific Bell inequality in the tripartite setup with the following property: the maximal violation of that inequality by almost-quantum correlations is larger than the violation achieved by the correlations in the second level of NPA. This implies that there exist tripartite almost-quantum correlations that do not satisfy the constraints of the second level of NPA.

5.6.3 Almost-quantum correlations violate the No restriction Hypothesis

The sets of quantum and almost-quantum correlations are so close to each other, that researchers have so far failed to identify a single foundational principle that quantum correlations satisfy but almost-quantum don't [NGHA15]. This is a crucial question in the research program on deriving the correlations produced by quantum mechanics solely from physical principles, i.e., without making reference to the underlying mathematical structure of Hilbert spaces, vectors, self-adjoint operators, and so forth. This research program, initiated by Popescu and Rohrlich [PR94], aims at developing an understanding of quantum correlations alone in terms of physical principles, which would facilitate the study of their power as a resource for information processing. The discovery that almost-quantum correlations satisfy all the principles proposed by 2014[8] [NGHA15] opened the question of whether there is a fundamental limitation to this research program, or if it is the case that almost-quantum correlations could really be the correlations allowed by a alternative theory to quantum mechanics.

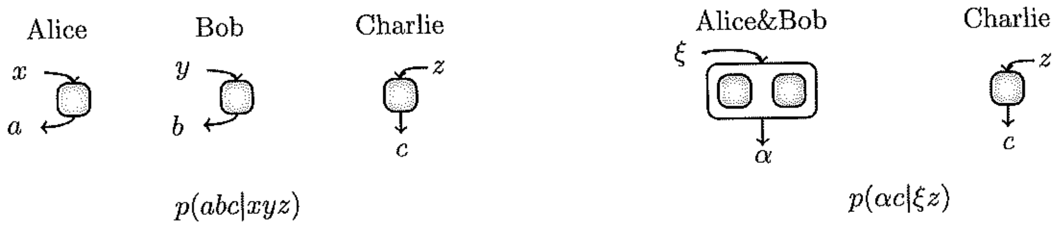
In this Habilitation thesis we study the properties that a physical theory should have in order to predict almost-quantum correlations. We found two principles that quantum theory satisfies and that no theory that realises almost-quantum correlations could: on the one hand, the so-called No-Restriction Hypothesis (NRH) [I], and on the other hand, Specker's principle [J]. Our results hence bring crucial progress to this research program that had been thought stalled.

In this section I will present the results of Ref. [I], namely, that almost-quantum correlations violate the no-restriction hypothesis [CDP10]. In Ref. [I], we consider a (hypothetical) physical theory that satisfies the following property: the correlations it generates in a Bell experiment are indeed almost-quantum correlations. Hereon I will refer to such a theory as an almost-quantum theory. We frame the study of this theory in the framework of Generalised Probabilistic Theories (GPT) [Bar07], which allows us to explore its statistical properties in a simple way. Here, we prove that any almost-quantum theory must violate NRH, and the proof works by contradiction: we assume that NRH is satisfied, and show this implies that the almost-quantum theory can produce correlations in Bell experiments that lie outside the almost-quantum set, hence reaching the contradiction [I]. Let me explain next the details of our proof.

The no-restriction hypothesis [CDP10], in summary, imposes that every mathematically conceivable measurement should be physically realisable. More precisely, let me denote by $\mathcal{S}$ the set of states that systems in a physical theory can be prepared in. The no-restriction hypothesis, hence, imposes that any linear map $\mathcal{L}$ that evaluates to a real number within $[0,1]$ when acting on any normalised state in $\mathcal{S}$ should be a physically realisable operation within the theory. In the language of Generalised Probabilistic theories, the no-restriction hypothesis imposes that the set of effects is equal (rather than strictly included within) the dual set of the set of states. Quantum theory satisfies NRH. As an example of a theory that does not satisfy NRH, one can consider Spekkens' toy model [Spe07, JL13].

In the case of an almost-quantum GPT, the no-restriction hypothesis implies that certain linear functionals, which we identified in Ref. [I] and denoted by *normalised Bell functionals* (NBFs), must be allowed joint operations in composite systems of two constituents. Hence, we used these NBFs to define measurements in a Bell experiment, as used them as follows: consider a tripartite Bell scenario, where Alice, Bob, and Charlie act locally on their systems. By definition, the tripartite correlations that they can observe, $p(abc|xyz)$, are within the almost-quantum set, where a (resp. b and c) denotes Alice's (resp. Bob's and Charlie's) outcome, and x (resp. y and z) denotes Alice's (resp. Bob's and Charlie's) choice of measurement. Now, allow Alice and Bob to come together: this defines a new bipartite Bell scenario, where Alice&Bob are one party, and Charlie is the second one. Denote by ξ the measurement choice of Alice&Bob, and by α the outcome they obtain when measuring their composite system. If the underlying almost-quantum theory is defined in a consistent way, then, one would expect that the bipartite correlations $p(\alpha c|\xi z)$ that they produce be almost-quantum. However, we found that if Alice and Bob measure their composite system using these NBFs, these bipartite correlations $p(\alpha c|\xi z)$ they observe with Charlie may lie outside the almost-quantum set. This shows that these

[8]With the possible exception of Information Causality [PPK$^+$09].



(a) Tripartite scenario

(b) Emerging Bipartite scenario

Figure 8: **(a) Tripartite scenario**: a Bell scenario where three distant parties perform local measurements on their share of a system. The outcome statistics that is studied is given by $p(abc|xyz)$. **(b) Emergent bipartite scenario**: The bipartite scenario that arises when Alice and Bob are brought to the same location and allowed to perform joint measurements on their share of the system. These joint measurements, within our framework, are mathematically given by Normalised Bell Functionals. The outcome statistics that is studied is given by $p(\alpha c|\xi z)$. In Ref. [I] we found that if all normalised Bell functionals are allowed to give rise to physically implementable measurements, the corresponding almost-quantum theory would feature outcome statistics beyond those in the set of almost-quantum correlations.

NBFs cannot be considered as allowed physical operations in the almost-quantum theory, and therefore, that the almost-quantum theory violates the no-restriction hypothesis.

5.6.4 Almost-quantum correlations violate Specker's principle

Another principle that we proved almost-quantum correlations violate [J] is Specker's principle. Ernst Specker originally said that [Cab12]

> *"If you have several questions and you can answer any two of them, then you can also answer all of them."*

Formally, this means that if in a set of measurements every pair is compatible, then all the measurements are compatible. One statistical consequence of this principle is that, if in a set of measurements every pair is compatible, then – for every preparation of the system – the statistics generated by these measurements are marginals of some joint probability distribution, i.e., are non-contextual. This formalization of Specker's principle holds true in quantum theory for sharp (i.e., projective) measurements.

In Ref. [J] we showed that almost-quantum correlations violate the statistical consequence of Specker's principle, and hence any almost-quantum theory violates Specker's principle. To prove this, we work within the hypergraph framework of Ref. [2], and focus on contextuality scenarios where the measurements to be performed are pairwise compatible. Within the hypergraph framework, these scenarios have the following property: the set of quantum probabilistic models that they feature always admit a non-contextual (i.e., classical) explanation. In particular, we consider the scenario with four pairwise compatible dichotomic measurements. In this scenario, each facet of the classical polytope[9] is either a so-called consistent exclusivity inequality or a pentagonal inequality [4]. Namely, the set of classical probabilistic models is precisely defined

[9] The set of classical probabilistic models in a contextuality scenario is a polytope, i.e., a convex set with a finite number of extreme points. Each of these extreme points corresponds to a deterministic assignment of an outcome to each measurement. This convex body can alternatively be characterised in terms of a finite number of hyperplanes called facets.

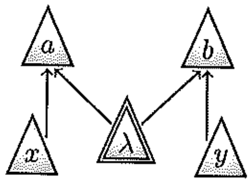


Figure 9: The causal structure corresponding to a bipartite Bell scenario: the observed classical variables x and y correspond to Alice's and Bob's choices of measurement, while the observed classical variables a and b correspond to the obtained outcomes. The common cause (latent variable) λ is a classical random variable. The observed statistics are given by $\{p(ab|xy)\}$. Bell's theorem says that quantum correlations may be incompatible with this causal structure. A new structure where the unobserved classical variable λ is replaced by an unobserved quantum variable ρ (i.e., a shared quantum state) is, however, compatible with quantum statistics.

by these inequalities. Since here by construction quantum models are always non-contextual, it follows that these inequalities also fully define the set of quantum probabilistic models in this scenario. Since almost-quantum correlations satisfy the Consistent Exclusivity principle [2], they do satisfy the consistent exclusivity inequalities. However, what we show in Ref. [J] is that almost-quantum correlations violate the pentagonal inequality: a form of this inequality is given in Ref. [4, Eq. (61)], its maximum quantum value is 2, and almost-quantum correlations can give a value up to 2.5.

5.7 Generalised causal structures: post-quantum correlations and resources

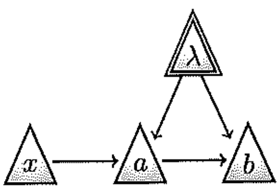
Bell scenarios, as well as steering scenarios, are particular examples of causal structures. In each of them, the main premise is that a group of parties, which are each located in a distant laboratory, need to achieve a task (or simply generate statistical data) with only limited resources shared by all of them. In the language of causal inference [Pea09], a bipartite Bell experiment can be recast as follows (see Fig. 9): there are four (observed) classical variables – $x \in \mathbb{X}$ and $y \in \mathbb{Y}$ (which correspond to Alice's and Bob's measurement choices resp.), and $a \in \mathbb{A}$ and $b \in \mathbb{B}$ (which correspond to Alice's and Bob's measurement outcomes resp.) – and an unobserved (latent) variable $\lambda \in \Lambda$ shared by both parties. The arrows in the diagrams represent the direction of causal inferences: for example, the statistical features of the variable a may be influenced by the variables x and λ, but not by y or b. From this causal perspective, one can then interpret Bell's theorem as follows: quantum statistics are incompatible with λ being a classical variable (see Section 5.7.2 for a formal statement).

In this section I will take a step further from the traditional Bell-type scenarios, and explore how nonclassicality may emerge in other types of causal structures [K]. In turn, I will use the new perspective from causal inference to develop tools to study correlations in Bell scenarios as a resource [L].

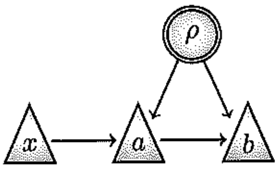
5.7.1 Nonclassical and post-quantum correlations in the Instrumental scenario

Here we study classical vs. quantum vs. post-quantum statistics in the simplest (non-trivial [HLP14]) causal structure: the so-called Instrumental scenario [Pea95, Bon01]. The causal structure of the instrumental scenario[10] is depicted in Fig. 10. This is the simplest causal

[10] The original interest in this scenario came across the study of cases of controlled medical trials where the patients don't comply with the treatment perfectly: let x be a randomly assigned treatment, a the treatment the



(a) Instrumental scenario



(b) Quantum Instrumental scenario

Figure 10: **(a) Instrumental scenario:** The causal structure corresponding to the Instrumental scenario. There are three observed classical variables x, a, and b. There is also an unobserved common cause (latent variable) λ between the variables a and b. The observed statistics are given by $\{p(ab|x)\}$. Here we find that quantum correlations may be incompatible with this causal structure. **(b) Quantum Instrumental scenario:** A new Instrumental causal structure where the unobserved classical variable λ is replaced by an unobserved quantum variable ρ (i.e., a shared quantum state). This structure is compatible with quantum statistics. However, here we find that post-quantum correlations may be incompatible with this causal structure.

structure that can possibly display a separation between classical, quantum, and post-quantum correlations, where 'simplicity' means fewer number of variables (nodes) and causal arrows (edges) [HLP14]. Indeed, the instrumental scenario is simpler than the Bell scenario. In Ref. [K] we showed that this 'possibility' displayed by the instrumental scenario is actually realised: there do exist quantum correlations in the instrumental scenario that have no classical explanation, and moreover, there exist post-quantum correlations as well which do not admit a quantum realisation.

The key insight in Ref. [K] was brought by the identification of a relation between Bell scenarios and the Instrumental scenario. More precisely, consider the following situations. On the one hand, you have access to an experimental implementation of a bipartite Bell test: a shared system between Alice and Bob, who make measurements on their respective shares, and observe the outcome statistic $\{p(ab|xy)\}$. Whenever $|\mathbb{A}| = |\mathbb{Y}|$, one can then take these state and measurement instruments to do the following: have Alice measure her share of the system, obtain an outcome a, send this outcome to Bob, who will then perform the measurement labelled by a on his share of the system, finally obtaining outcome b. This new setup, which unlike the Bell scenario allows for communication from Alice to Bob, will generate outcome statistics characterised by the conditional probability distribution $p'(ab|x)$. Now, since the physical devices used were taken from the Bell setup, it follows that $p'(ab|x) = p(ab|xa)$. On the other hand, one can take as starting point a correlation $p'(ab|x)$ in the instrumental scenario, and ask in turn whether there exists a correlation $p(ab|xy)$ in a bipartite Bell scenario such that $p'(ab|x) = p(ab|xa)$. The answer to this is yes, and follows from applying the so-called interruption technique to the Instrumental scenario [K]. Putting these two situations together, we concluded that a correlation $p(ab|xy)$ is admissible in the Bell scenario if and only if its post-selection $p'(ab|x) = p(ab|xa)$ is admissible in the Instrumental scenario. Moreover, this relationship preserves the kind of hidden latent variable that those correlations can be realised with. Namely, classical correlations in Bell scenarios map to classical correlations in the Instrumental scenario (and vice-versa), and the same happens with quantum correlations and post-quantum correlations [K].

This identification we found between the Bell scenario and the Instrumental scenario allowed

patient actually follows, and b the variable that records recovery. There could be factors λ that influence both the chance of recovery under each treatment, and also the chance of compliance with a particular treatment.

(a) Instrumental steering scenario

(b) Quantum Instrumental steering scenario

Figure 11: **(a) Instrumental steering scenario:** The causal structure corresponding to the Instrumental steering scenario. There are two observed classical variables, x and a, and one observed quantum variable σ. There is also an unobserved common cause (latent variable) λ between the variables a and σ. The observed statistics are given by the assemblage $\{\sigma^I_{a|x}\}_{a\in\mathbb{A},x\in X}$. Here we find that quantum assemblages may be incompatible with this causal structure. **(b) Quantum Instrumental steering scenario:** A new Instrumental steering causal structure where the unobserved classical variable λ is replaced by an unobserved quantum variable ρ (i.e., a shared quantum state). This structure is compatible with quantum assemblages. However, in Ref. [F] we find that post-quantum assemblages may be incompatible with this causal structure.

us to import all the "machinery" from the former and apply it to the latter. For example, take the following inequality for the instrumental scenario with $\mathbb{A} = \mathbb{B} = \{0,1\}$ and $\mathbb{X} = \{0,1,2\}$, found by Bonet [Bon01]:

$$I_{\text{Bonet}} = p(a = b|0) + p(b = 0|1) + p(01|2) \leq 2\,.$$

By replacing $p(ab|x) \to p(ab|xa)$ we related the value of I_{Bonet} with the value of the CHSH Bell inequality [CHSH69], and hence analytically found the maximum values of I_{Bonet} achievable by quantum correlations and post-quantum correlations in the Instrumental scenario. From this, contrary to what one may expect, we showed that any nonclassical correlation in the CHSH Bell scenario can be used to construct nonclassical correlations in the Instrumental scenario. For example, we showed how one can take the PR-box correlations (which feature $\mathbb{X} = \mathbb{A} = \mathbb{Y} = \mathbb{B} = \{0,1\}$) and construct a conditional probability distribution $p(ab|xy)$ for $\mathbb{X} = \{0,1,2\}$ which evaluates I_{Bonet} beyond what is possible with quantum resources.

Another important example of how the technique of Ref. [K] allows us to apply tools from Bell scenarios into the Instrumental one, pertains to relaxations of the set of quantum correlations. For example, we can now define and discuss the concept of almost-quantum correlations in Instrumental scenarios. From a more pragmatic perspective, the connection is also useful since the computational tools provided by the NPA hierarchy can now be directly applied to the Instrumental scenario. This is relevant when one aims at finding an upper-bound of the quantum violation of a particular inequality in the Instrumental scenario: one can now take a particular level of the NPA hierarchy, and ask what is the maximum value of the linear functional by $p'(ab|x)$ when its corresponding $p(ab|xy)$ belongs to that level of NPA. This is indeed a single instance of a semidefinite program.

A final comment in this section pertains to the Instrumental steering scenario, which I introduced in Section 5.5.4. From a causal perspective, one can think of the Instrumental steering scenario as one where the observed variable b is replaced by an observed quantum variable σ, which corresponds to the elements of the assemblage prepared in Bob's laboratory. This causal perspective on the Instrumental steering scenario is depicted in Fig. 11.

5.7.2 The Resource Theory of Nonclassicality of Common-Cause Boxes

Ref. [L] is another paper in this series where taking a causal perspective to the problem provided crucial insight. The question that we tackled in Ref. [L] is how to understand the nonclassicality of correlations in a Bell scenario as a *resource*, and how to quantify the resourcefulness of such nonclassicality. As an example, consider the situation where two distant parties, Alice and Bob, want to communicate securely by following a device-independent quantum cryptographic protocol. In this case, they will need nonclassical correlations in a suitable bipartite Bell setup to implement the protocol. The kind of questions that motivate us here are, e.g., if they have access to two different nonclassical correlations, $\{p_1(ab|xy)\}$ and $\{p_2(ab|xy)\}$, how can they know which is more "nonclassical" in the sense of "more useful" for this cryptographic task? The precise core problem that we tackle is at the level of the correlations in Bell setups: which correlations are fundamentally more nonclassical than the others, and how to quantify this.

The framework that we followed to develop our systematic approach to resource ordering and quantification is that of Resource Theories, and in our case we followed the particular formulation of Ref. [CFS16]. The crucial aspect is how to identify the co-called free operations in Bell scenarios, and this is where the causality perspective on the problem plays a role. Historically, the debate underpinning Bell's theorem evolved around notions of *locality*, *realism*, and *local causality* [Wis14]. In Ref. [L], however, our starting point is a reinterpretation of Bell's theorem, where the assumptions are: Reichenbach's principle (that correlations need to be explained causally), the framework of classical causal modelling, and the principle of no fine-tuning (that statistical independences should not be explained by fine-tuning of the values of parameters in the causal model). From this perspective, a violation of a Bell inequality does not lead to the traditional dilemma between realism and locality, but rather attests to the impossibility of providing a nonfine-tuned explanation of the experiment within the framework of classical causal models. Hence, if one wants to hold on to the possibility of achieving satisfactory causal explanations of correlations, then Bell's theorem implies that one must give up on the classical causal models framework. In Ref. [L] we hence work within a generalisation of causal models, that uses the framework of Generalised Probabilistic Theories (GPTs): in these Bell setups, then, the unobserved causal mediators (here common causes) may be variables which are no longer assumed classical: it could be a quantum state ρ (see Fig. 9) or even a post-quantum system. From a more fundamental perspective, this approach tells us that in an experimental situation where we have space-like separated parties, the fundamental source of nonclassicality is given by the nature of the common cause that correlates the experimental rounds of the laboratories.

This perspective on Bell's theorem allowed us to identify the set of free operations in our Resource Theory: anything that can be achieved by sharing classical common causes among the labs [L, Sec. 3]. In the context of quantum information theory, this set is known as *local operations and shared randomness* (LOSR) [DV14, GP14]. This perspective also give us a physical justification for disregarding the set of operations called *wirings and prior to input classical communication* [GA17] as a relevant candidate for the free operations in a resource theory of Bell nonclassicality.

The question of resource convertibility is here hence the following: can a resource R_1 be converted into a resource R_2 by means of LOSR operations? When the answer is *yes*, this means that R_1 is more nonclassical (more resourceful) than R_2. In the particular case of correlations $\{p(ab|xy)\}$, we showed that this question can be answered with two instances of a linear program, which provides an efficient algorithm for deciding whether a correlation is more nonclassical than another [L, Sec. 5].

In Ref. [L] we further studied *resource monotones*, which are real-valued functions over resources whose value cannot increase under any free operation in the resource theory. The intuition is that the values provided by a collection of such monotones could serve as a *measure of nonclassicality* for each of the correlations. We found that facet-defining Bell inequalities are not sufficient to capture how nonclassical a particular correlation is. Indeed, we showed that in the CHSH Bell scenario (that has only one equivalence class of facet-defining inequalities), at least 7 other monotones are required [L, Sec. 7].

In Ref. [L] we also defined two resource monotones, which allowed us to derive various global properties of the ordering of resources in the resource theory. First, we showed that there exist infinite sets of *incomparable*[11] resources. For example the quantum correlations that maximally violate the CHSH inequality, call them R_{Tsi}, cannot be converted via LOSR operations to the Hardy correlations, R_{Hardy}, and vice-versa. Moreover, we showed that the continuous set of extremal quantumly realizable correlations contains an infinite set of incomparable resources, even when restricting to the subset of the resource theory that can be realized in quantum theory. In particular, we showed that all extremal quantum resources are at the top of the order: there is no analogue of a *maximally entangled* resource for quantum correlations.

Finally, one of the fundamental contributions of Ref. [L] is to set the grounds on which to develop resource theories for other causal scenarios. Indeed, our fundamental ideas on how to leverage causal concepts to develop a resource theory can be readily applied to other causal structures. For example, Ref. [L, App. A] mentions the so called *triangle-with-settings* scenario [BRGP12, Fig. 8], and defines the sets of free resources and free operations therein. One crucial fact here is that, unlike Bell scenarios, these sets of free operations and free resources are no longer convex.

6 PRESENTATION OF TEACHING, ORGANISATIONAL, AND 'POPULARISATION OF SCIENCE' ACHIEVEMENTS

6.1 Teaching achievements

Academic teaching:

- *UBA - Universidad de Buenos Aires, Argentina.* September, 2017.
  Lecturer – Graduate course: "Nonlocality and Contextuality: Foundations and applications". 8 lectures, 32 hours total.

- *UPC - Polytechnic University of Catalunya, Spain.* January, 2013 - May, 2013.
  Teaching Assistant – Program: Master in Photonics – Course: Photonics Laboratory.

- *UNC - Universidad Nacional de Córdoba, Argentina.* March, 2007 - July, 2009.
  Teaching Assistant – Facultad de Matemática, Astronomía y Física – Undergraduate Courses:
  - Física 4: Classical Optics, Lab Sessions – Autumn 2009.
  - Physics for Computer Sciences – Spring 2008.
  - Classical Mechanics – Autumn 2008.
  - Física 3: Electricity and Magnetism – Spring 2007.
  - Física 4: Classical Optics – Autumn 2007.

Invited lectures:

[11] Two resources are incomparable if neither can be freely converted into the other one, i.e., by means of LOSR operations.

- *Bell nonlocality – quantum correlations from principles* June, 2019.
  QPL Quantum Physics and Logic – Chapman University, CA, US.

- *Contextuality* June, 2017.
  Solstice of Foundations Summer School, ETH Zurich – Zurich, Switzerland.

- *Bell Nonlocality* September, 2015.
  Hanyang University (ERICA) – Ansan, South Korea.

6.2 Organisational Achievements

Conferences, workshops, and seminars: organisation

- **Quantum Speedup Conference** December, 2020.
  ICTQT, University of Gdańsk, Poland.

- **Q-turn: changing paradigms in quantum science** November, 2018.
  Universidade Federal de Santa Catarina, Florianópolis, Brazil.
  `https://qturnworkshop.wixsite.com/2018`

- **Algorithmic Information, Induction and Observers in Physics workshop** April, 2018.
  Perimeter Institute for Theoretical Physics, Waterloo, ON, Canada.
  `https://tinyurl.com/ybgbgvwb`

- **Observers in quantum and foil theories workshop** April, 2018.
  Perimeter Institute for Theoretical Physics, Waterloo, ON, Canada.
  `http://perimeterinstitute.ca/conferences/observers-quantum-and-foil-theories`

- **Contextuality: Conceptual Issues, Operational Signatures, and Applications workshop** July, 2019.
  Perimeter Institute for Theoretical Physics, Waterloo, ON, Canada.
  `https://tinyurl.com/ybgbgvwb`

- **Quantum Information Theory weekly seminars series** September 2014 – August 2016.
  University of Bristol – Bristol, UK.
  `http://www.cs.bris.ac.uk/Research/QuantumComputing/seminars.jsp`

- **10th workshop on Quantum Physics and Logic (QPL)** July, 2013.
  ICFO–Institut de Ciències Fotòniques, Castelldefels (Barcelona), Spain.

- **1st EPS Scientific Workshop for Young Minds** September, 2012.

  ICFO–Institut de Ciències Fotòniques, Castelldefels (Barcelona), Spain.
  `https://www.icfo.eu/newsroom/news/1692-icons-workshop-for-young-minds`

6.3 Popularisation of Science Achievements

Science communication:

- **News article:** "Quantique, Le nouvel âge d'or" October, 2020.
  Interview with Pierre-Yves Bocquet on the future of quantum research in its golden age.
  Science&Vie magazine, hors série n° 292, October 2020.
  `https://tinyurl.com/y4mv7gow`

- **News article:** "What is non-classical about quantum no-cloning?" June, 2020.
  Perspective on "Contextual advantage for state-dependent cloning" by Matteo Lostaglio and Gabriel Senno, published in Quantum 4, 258 (2020).
  I discuss what is nonclassical about quantum no-cloning, and the importance of a proper account of operational features when claiming that contextuality is a resource.
  Quantum Views 4, 39 (2020).
  `https://quantum-journal.org/views/qv-2020-06-22-39/`

- **News article:** "Is Causality Fundamental?" June, 2020.
  Interview with Brendan Foster on the topic of our FQXi Large Grant "The Emergence of Agents from Causal Order".
  The Foundational Questions Institute (FQXi), 18 June 2020.
  `https://fqxi.org/community/articles/display/242`

- **News article:** "Au-delà du quantique : la découverte d'un nouveau monde" May, 2019.
  Interview with Mathilde Fontez on understanding the limits of quantum theory and entertaining possibilities beyond it.
  Science&Vie magazine n° 1220, 17 April 2019.
  `https://tinyurl.com/y9twd4mj`

- **News article:** "The quantum world is infamously weird – now we might know why" March, 2019.
  Interview with Dr. Ciaran Lee on contextuality as one of the of the key conceptual differences between quantum and classical physics.
  New Scientist issue 3221, 16 March 2019.
  `https://tinyurl.com/yac8nl6g`

- **Video**: "Part II: Generalized Probabilistic Theories." August, 2018.
  Contribution to a three-part video series for general public on 'Definition of Quantum Theory'.
  Centre for Quantum Technologies, NUS Singapore and John Templeton Foundation.
  `https://www.youtube.com/watch?v=pE-mBRZAbfM`

Public engagement:

- **Public talk:** "Quantum" November, 2017.
  Hive Waterloo Region Meetup #4 – Waterloo, ON, Canada.
  Topic: overview for the general public of what quantum theory is, and what the research job of scientist entails.
  `http://hivewr.ca/events/meetup4/`

7 OTHER SCIENTIFIC ACHIEVEMENTS

7.1 Bibliometric data

Source: Google Scholar (12.02.2021)

- Number of peer-reviewed publications: 26 (20 after PhD)
- Number of online pre-prints: 3
- Total number of citations: 834
- H-index: 13

Source: Web of Science (12.02.2021)

- Number of peer-reviewed publications: 25 (19 after PhD)
- Total number of citations: 427 (390 without self citations)
- H-index: 9

7.2 Awards

- *Stypendium ministra dla wybitnego młodego naukowca*
  Award date: June, 2020.
  Duration: 36 months

7.3 Track record before PhD

The research output from my PhD thesis may be presented in the three following topics.

1. Characterising quantum correlations in Bell scenarios.–
Bell Nonlocality [Bel64] is a form of nonclassicality displayed by the correlations observed in measurement outcomes, and which can be harnessed as a resource for information processing, e.g., in cryptographic setups [BCP$^+$14b]. In my PhD I worked on the question of what the set of correlations in Bell scenarios allowed by quantum theory is, with the goal of understanding the power they provide.

We proposed the first intrinsically multi-partite principle to characterise quantum correlations, which we called "Local Orthogonality" (LO) [1, 3]. This principle is based on a definition of orthogonality (or exclusiveness) between measurement events, and the demand that the sum of the probabilities of mutually exclusive events is less than or equal to one. These conditions imply restrictions on the correlations we have access to. We showed that LO implies a highly non-trivial structure in the space of correlations, and that its intrinsically multi-partite formulation allows one to rule out post-quantum correlations for which any bipartite principle fails [1]. Interestingly, we show that the restrictions imposed by LO take the form of Bell inequalities (called LO inequalities) which are satisfied by quantum correlations. Violations of LO inequalities hence witness post-quantum correlations [1].

An important property of LO is its connection with Graph Theory, which proves very useful when computing the constraints that LO imposes on the space of correlations [1, 3]. Indeed, we showed this problem to be equivalent to computing some graph-theoretical invariants of what we call the orthogonality graph of the scenario [1, 3].

2. A new framework for the study of contextuality.–
We developed a graph-theoretical framework for Kochen-Specker Contextuality [KS67], inspired by that of Cabello, Severini and Winter [CSW10], but which allows the study of both nonlocality and contextuality in a unified manner [2].

Using this framework we defined a postulate for quantum correlations in contextuality scenarios, which we called the Consistent Exclusivity principle (CE) [2] [Hen12, Cab13]. In particular, we proved that within our definition of Contextuality scenarios, the Local Orthogonality principle and CE are equivalent. Similarly to the LO case, we showed that the CE principle imposes a highly non-trivial structure on the probability space, which until then had been unnoticed. We formalised this structure by defining a hierarchy of sets of probabilistic models, each level satisfying stronger constraints formulated from CE [2].

In addition, we defined a hierarchy of semidefinite programs (SDP) for probabilistic models on contextuality scenarios, similar to that of Navascués, Pironio, and Acín (NPA) [NPA07, NPA08, PNA10]. We showed that this SDP hierarchy converges into the quantum set, and each level satisfies the CE principle. We also showed that these hierarchies can be understood as being in a family of hierarchies, with each member of the family converging into a different set of models (e.g., classical probabilistic models) in the contextuality scenarios [5].

Being a graph-theoretically based framework, our approach profits from graph theory. Indeed, we characterised in terms of graph theoretical invariants some sets of probabilistic models, such as the no-signaling set, all the sets in the CE hierarchy, the first level in the SPD hierarchy, and the classical set [2].

Finally, our approached allowed us to generalise almost-quantum correlations [NGHA15] to Kochen-Specker contextuality scenarios, and characterise the set in terms of a graph theoretical invariant computable by an SDP [2].

3. Bell inequalities for many-body systems.–
How to certify that a correlation observed in a Bell experiment is not classical is, in principle, a problem that can be easily solved: one merely checks whether the correlations belong to the so-called local polytope. However, in practice, this is not always the case, since the computational complexity of the problem grows exponentially with the number of parties involved, and also increases with the number of allowed measurement choices and possible outcomes. Another obstacle when working with large multi-partite systems (i.e., many-body systems) comes from an experimental perspective: measuring all the many-body correlation functions required to reconstruct the full probability vector is not necessarily an easy task. Hence, one may not tackle the problem of nonclassicality certification of correlations in many-body systems in a straightforward way.

In this project we studied how to develop tests of Bell nonclassicality that could certify nonclassical correlations in many-body systems. We focused on tests of nonclassicality that rely only on one- and two-body correlators [6, 7], simplifying the complexity of both the theoretical and the experimental issues. In addition, we further restricted the 2-body Bell inequalities to those that satisfy certain symmetries regarding the labeling of the parties: on the one hand permutational invariance [6], and on the other translational invariance [7] – this allowed us to develop software packages that do not require the use of a super-computer for performing the certification. Remarkably, we showed that these Bell inequalities are already powerful enough for detecting nonlocality in physically relevant many-body systems, such as the ground state of the Lipkin-Meshkov-Glick Hamiltonian [LMG65].

Finally, we showed that in some cases the derived inequalities are experimentally friendly, as they can be tested through measurements of global observables such as the components of the total spin, which back then were routinely measured in atomic physics with great precision [HSP10, ERIR$^+$08]. With this we showed that our nonlocality criteria may be applicable in systems where individual particles cannot be addressed.

7.4 Additional track record after PhD

Research that I carried out after my PhD, and which is not part of this Habilitation achievement, includes the following topics.

1. The complexity of compatible measurements.–
Measurement incompatibility is one of the basic aspects of quantum theory [BLM96]. In this work [8] we have instigated the study of the complexity of a set of compatible measurements, that is, we explored how complex the structure of the set of compatible – i.e. jointly measurable [HMZ16] – measurements is. In one direction we have shown that very large sets of compatible measurements can be formed starting from a parent, and in the other direction that the complexity of compatibility can be bounded, and scales no worse than linearly in the number of measurements in the set. We have also explored the typical behaviour of the boundary of compatible measurements in instances where this bound is not tight, and shown that in these cases the boundary appears to have a rich structure. We have finally raised the possibility of using randomness in the parent to reduce the complexity, and found examples where this is indeed possible.

2. Quantum Reference Frames and Their Applications to Thermodynamics.–
We constructed a quantum reference frame that allows us to approximately implement arbitrary unitary transformations on a system in the presence of any number of extensive conserved quantities, by absorbing any back action provided by the conservation laws [9]. Our reference frame hence, at the same time, acts as a battery for the conserved quantities. Our construction features a physically intuitive, clear, and implementation-friendly realisation. Indeed, the reference system is composed of the same types of subsystems as the original system and is finite for any desired accuracy. We showed that the interaction with the reference frame can be broken down into two-body terms coupling the system to one of the reference frame subsystems at a time. We applied this construction to quantum thermodynamic setups with multiple, possibly non-commuting conserved quantities, which allowed us to define explicit batteries in such cases.

3. Reference frames which separately store non-commuting conserved quantities.–

We built up on the results of Ref. [9], to construct a new reference frame that not only acts as a battery for the (possibly non-commuting) conserved quantities, but that also stores these quantities in different "batteries" [10]. More precisely, we first show that it is possible to perform an arbitrary unitary transformation on any number of spin-$\frac{1}{2}$ particles while respecting angular momentum conservation, in such a way that any changes in the three components of angular momentum are separated into different batteries. The errors in this procedure, if any, can be made arbitrarily small by making these batteries sufficiently large. We then show that this protocol may be generalized to higher spins, when the objective is to implement a spatial rotation under total angular momentum conservation. We also show that, when the dimension of the Hilbert space is a power of 2, a complete set of conserved quantities can be constructed to which our protocol may be applied: arbitrary unitary transformations on the system can be implemented while separating the changes in each conserved quantity in a different battery. Remarkably, our results also allow one to completely extract the different components of angular momentum of an unknown spin state into distinct systems (up to arbitrary precision).

4. Multipartite Composition of Contextuality Scenarios.–
Contextuality is a particular quantum phenomenon that has no analogue in classical probability theory. Given two independent systems, a natural question is how two separate contextuality

scenarios combine into a joint scenario. Under the premise that the the allowed probabilistic models satisfy the No Signalling principle [Cir80, Tsi93, PR94], Foulis and Randall defined the unique possible way to compose two contextuality scenarios [FR81]. When composing strictly-more than two test spaces, however, we showed that a variety of possible composition methods can be conceived [2]. Nevertheless, we also proved that all these formally-distinct composition methods appear to give rise to observationally equivalent scenarios, in the sense that the different compositions all allow precisely the same sets of classical and quantum probabilistic models [2]. This raises the question of whether this property of invariance-under-composition-method is special to classical and quantum probabilistic models, or if it generalizes to other probabilistic models as well. We then proved that this is indeed not the case, by showing that some composition rules give rise to scenarios with inequivalent allowed sets of almost-quantum probabilistic models [11]. We further found that the non-trivial dependence of almost-quantum models on the choice of composition method is apparently an artifact of failure of those composition rules to capture the orthogonality relations given by the Local Orthogonality principle [1]. We finally proved that almost-quantum models satisfy invariance-under-composition-method for all the constructive compositions protocols which do capture this notion of Local Orthogonality [11].

5. Tightness of correlation inequalities with no quantum violation.–
In this work [4] we studied the faces of the set of quantum correlations, i.e., the Bell and non-contextuality inequalities without any quantum violation. First we asked whether every proper (tight) Bell inequality for two parties can be violated by quantum correlations. We presented progress toward the resolution of this question by presenting paradigmatic examples of correlation Bell inequalities with no quantum violation, in the form of non-local computation games which do not constitute tight Bell inequalities and have no quantum violation. Then we moved on to contextuality scenarios as per Ref. [2], and asked whether tight non-contextuality inequalities with binary outcomes with no quantum violation exist. We answered this question in the positive for some scenarios, by showing that there exist tight non-contextuality inequalities with no quantum advantage and that are not in the Consistent Exclusivity form. A crucial step we took here was to identify the polytope of non-contextual behaviours with the CUT polytope of the suspension graph of the compatibility graph representing the measurements in the experiment.

6. A new property of the Lovász number and duality relations between graph parameters.–
Many natural graph parameters arising as combinatorial optimization problems, such as independence number or chromatic number, are not generally multiplicative under graph products, but due to their nature retain super-multiplicativity. In this work [12] we diverted from this consideration of the behaviour of graph parameters under the product of many copies of G, and looked more broadly at how they are affected by products with a generic other graph H. First, we showed that the Lovász number [Lov79] is asymptotically attained by the independence number for every graph G when activated by suitable graphs H. Then, we studied tight upper bounds on the independence number of graph products in terms of products of individual, "dual", graph parameters, and found some examples of such pairs.

7. Bell nonclassicality in many-body systems.–
We continued the work we started during my PhD [6, 7], about how to detect Bell non-locality in many-body systems. First, we explored further properties of these one- and two-body correlation Bell inequalities: we characterised their tightness, and discussed their maximal quantum violations in the general case and their scaling with the number of parties [13]. Moreover, we

provided new classes of two-body Bell inequalities which reveal nonlocality of the Dicke states—ground states of physically relevant and experimentally realizable Hamiltonians [13]. Finally, we discussed various scenarios for nonlocality detection in mesoscopic systems of trapped ions or atoms, and by atoms trapped in the vicinity of designed nanostructures [13, 14].

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